Optimizing an Absolute Gravimeter Comparison Schedule

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Abstract

Since 1980 various groups have executed comparisons of absolute gravimeters for the purpose of determining the accuracy of operational meters. While the final method of processing data and estimating meter accuracy has varied from comparison to comparison, one common fact has persisted – two meters can not observe gravity at both the same time and the same place. With this simple fact in mind, and despite variations in the final method of data processing, it has always been necessary to develop an *efficient* observation schedule for a comparison. Such a schedule must obviously depend on the **number of meters in attendance** and **number of observing piers available**.

But other factors must be considered, such as **how many observations each meter is required to make**, **how many times a meter compares to another meter** and **how many times a meter compares to itself** – all of which effect the conditioning of the equation system.

Finding the ***most efficient schedule*** with the ***greatest conditioning of the equation system*** is a problem of optimization. The number of possible combinations for even small numbers of meters and piers grows exponentially out of computational possibility if a brute force method is used. This talk discusses an efficient solution to forming such an optimized schedule, and a fast computer program which makes use of this solution.

Types of Meter-to-Meter comparisons

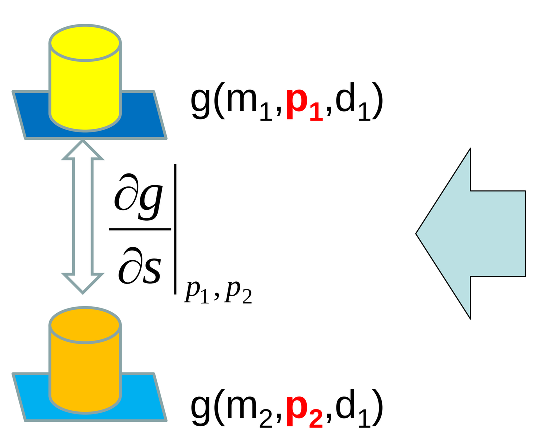
Before considering any form of optimization, two choices must be addressed in an absolute gravimeter schedule:

1) Will comparisons be across space, across time or both?

2) Will comparisons of one meter to itself be disallowed, allowed but given lower priority, or allowed and balanced with all other comparisons?

<u>Space vs Time</u>

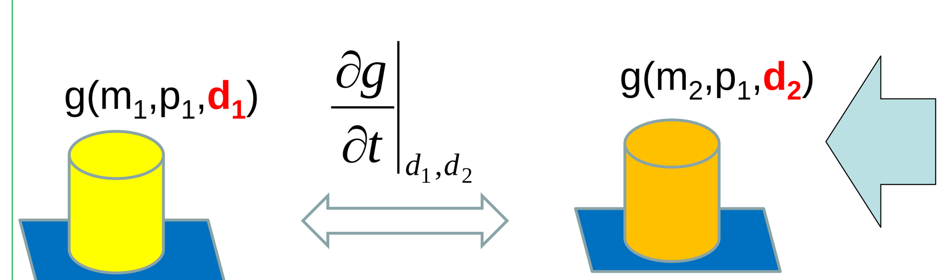
Because of the prevailing fact that 2 meters cannot be in the same space at the same time, a comparison between them must at least be separated in time or space; theoretically even both, though this adds additional uncertainty to the comparison.



$$\left.\frac{\partial g}{\partial s}\right|_{p_1,p_2}$$

Comparisons <u>across space</u> allow two meters nearby to measure "g" at identical times, but on two different piers.

The requirement for this type of comparison is that horizontal gradients of gravity between piers must be established.



$$\left.\frac{\partial g}{\partial t}\right|_{d_1,d_2}$$

Comparisons <u>across time</u> allow two meters to measure "g" at different times, but on identical piers.

The requirement for this type of comparison is that temporal gradients of gravity between days must be established.

For the purposes of this study, <u>**we restrict ourselves entirely to *comparisons across time.***</u>

<u>**Comparisons of any meter to itself ("self-self comparisons")**</u>

An absolute gravimeter comparison is an uncommon event requiring significant logistics to get observers and equipment all in one observatory for a short period of time. As such, we view the primary purpose of such comparisons to be this:

Maximize and balance as many comparisons between pairs of <u>different</u> meters as possible in the time allowed

Consider also the fact that any given meter may be compared to itself on a solo basis, without taking up valuable time and space in an official comparison involving multiple meters. As such, the problem of optimization was tackled by attempting to satisfy the following criteria, in priority order:

1) If possible, do not allow any self-self comparisons

2) If not possible, minimize the number of self-self comparisons that occur

Methodology

The goal of optimizing an absolute gravimeter comparison schedule will be to find a schedule where the following are given, and sought:

Given:
- N_g: Number of Gravimeters
- N_p : Number of Piers
- N_o: Number of Observations Required by each meter
  - Can also input N_d (Number of Days for observing) but this is computable from the above 3 variables.

Rules:
- Disallow (or at least minimize) self-self comparisons
- Fill the schedule (all piers occupied over all days)
- Balance the number of observations made by each meter
  - No meter will have more than 1 extra observation than any other meter
- Get at least 1 comparison between any pair of 2 different meters

Goal:
- Balance the number of comparisons across the schedule
  - That is, minimize the standard deviations of the meter-to-meter comparison counts

A Simple Example (3 Meters, 2 Piers, 2 Required Obs)

Using the rules to the left, the problem of finding an optimized schedule for a small comparison is shown, by way of example. With 3 meters, but only 2 piers available and a requirement that every meter make 2 observations, the minimum number of days for such a schedule is **3 days**.

Over those three days, on each day the number of possible ways to put 2 of the 3 meters on 2 piers each day is:

<u>**# Possible schedules on any day**</u> $= N_g! / (N_g - N_p)! = 3! / (3-2)! =$ **6**

With 3 days of observations, that makes 6x6x6 = 6^3 = 216 possible schedules. However, as both days (3) and piers (2) can be changed around without impacting the actual meter-to-meter comparison counts, the **number of unique comparisons that are actually possible is only 18** ($=216 / N_d! / N_p!$)

15 Meters, 11 Piers, 4 Required Obs

Nonetheless, consider larger, more realistic comparisons of, say 15 meters over 11 piers and 4 observations required. (requiring 6 days). The number of total possible schedules is $(15! / 4!)^6 = 2.6 \times 10^{64}$ though the **number of unique schedules** (divide by 6! And 11!) is "only" **9.1×10^{53}**.

Optimized Schedules versus Simple Schedules

The logistics of an absolute gravimeter comparison can be difficult, which can lead to the adoption of a simple, yet non-optimized schedule. The advantages of such simplicity should not be minimized. However, if optimization and balance in the schedule are given priority then ample time should be set aside to allow for more complicated pier assignments on a daily basis. An example of an optimized, versus a simple schedule is given below.

Given:
13 Meters, 13 Piers, 4 Observations Required per meter

A "simple" schedule

Schedule Example "Everyone rotate one pier clockwise"				
	Day 1	**Day 2**	**Day 3**	**Day 4**
Pier 1	1	13	12	11
Pier 2	2	1	13	12
Pier 3	3	2	1	13
Pier 4	4	3	2	1
Pier 5	5	4	3	2
Pier 6	6	5	4	3
Pier 7	7	6	5	4
Pier 8	8	7	6	5
Pier 9	9	8	7	6
Pier 10	10	9	8	7
Pier 11	11	10	9	8
Pier 12	12	11	10	9
Pier 13	13	12	11	10

Meter #	# of Obs	# of meter-to-meter comparisons over the entire schedule												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
1	4	0	3	2	1	0	0	0	0	0	0	1	2	3
2	4		0	3	2	1	0	0	0	0	0	0	1	2
3	4			0	3	2	1	0	0	0	0	0	0	1
4	4				0	3	2	1	0	0	0	0	0	0
5	4					0	3	2	1	0	0	0	0	0
6	4						0	3	2	1	0	0	0	0
7	4							0	3	2	1	0	0	0
8	4								0	3	2	1	0	0
9	4									0	3	2	1	0
10	4										0	3	2	1
11	4											0	3	2
12	4												0	3
13	4													0

An "optimized" schedule

Schedule Example "Optimize for Balanced Comparisons"				
	Day 1	**Day 2**	**Day 3**	**Day 4**
Pier 1	1	9	4	13
Pier 2	2	3	1	10
Pier 3	3	8	6	4
Pier 4	4	2	5	11
Pier 5	5	7	3	9
Pier 6	6	5	12	1
Pier 7	7	1	11	8
Pier 8	8	13	10	5
Pier 9	9	12	8	2
Pier 10	10	4	7	12
Pier 11	11	10	9	6
Pier 12	12	11	13	3
Pier 13	13	6	2	7

Meter #	# of Obs	# of meter-to-meter comparisons over the entire schedule												
		1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
1	4	0	1	1	1	1	1	1	1	1	1	1	1	1
2	4		0	1	1	1	1	1	1	1	1	1	1	1
3	4			0	1	1	1	1	1	1	1	1	1	1
4	4				0	1	1	1	1	1	1	1	1	1
5	4					0	1	1	1	1	1	1	1	1
6	4						0	1	1	1	1	1	1	1
7	4							0	1	1	1	1	1	1
8	4								0	1	1	1	1	1
9	4									0	1	1	1	1
10	4										0	1	1	1
11	4											0	1	1
12	4												0	1
13	4													0

Criteria	Simple	Optimized
Observations	Balanced (Every meter makes 4 observations)	Balanced (Every meter makes 4 observations)
Number of self-self comparisons	Perfect (0 of 13)	Perfect (0 of 13)
Number of missing meter-to-meter comparisons	**Non-optimal: 39 missing of 78 possible**	Optimal: 0 missing of 78 possible
Balance of meter-to-meter combinations	**Unbalanced:** **Minimum 0** **Maximum 3** **Standard Deviation 1.16**	**Balanced:** **Minimum 1** **Maximum 1** **Standard Deviation 0.00**

Approach to finding the unique optimal schedule.

Initially, a purely mathematical approach was sought to this problem. However, at this time, the exact formulation remains elusive. Nonetheless, a highly efficient near-optimal direct approach has been coded up in FORTRAN and is operational. The name of that program is **COAGS.F** (<u>**C**</u>omparison <u>**O**</u>f <u>**A**</u>bsolute <u>**G**</u>ravimeter <u>**S**</u>cheduler). It's approach is to build up the first schedule by assigning meter #i to pier #i on day 1. After that, it builds up one day at a time by maximizing the number of new meter-to-meter combinations gained by each assignment, while maintaining balance between meters of their number of observations. While this approach leads frequently to *truly* optimal solutions, it has been shown to arrive at *nearly* optimal solutions as the number of gravimeters is increased.

A truly mathematical solution to the unique optimal schedule is still sought, and ideas on its solution are most welcome.

By way of example, the schedule at right was generated by COAGS.F for the combination of 17 meters, 11 piers and 4 required observations (necessitating at least 7 days of observations). The program took less than 1 second to generate the schedule. The logic is found below.

	Day 1	Day 2	Day 3	Day 4	Day 5	Day 6	Day 7
Pier 1	1	17	13	3	8	14	4
Pier 2	2	7	17	14	11	12	13
Pier 3	3	15	12	16	7	10	1
Pier 4	4	1	15	11	2	5	17
Pier 5	5	12	9	1	6	7	8
Pier 6	6	14	8	15	13	16	5
Pier 7	7	13	5	4	16	9	12
Pier 8	8	4	10	12	17	6	16
Pier 9	9	3	14	10	5	4	11
Pier 10	10	11	6	13	3	2	7
Pier 11	11	16	2	8	9	17	15

```
fill first day (meter "i" on pier "i" for piers 1-npier)
do iday=2,ndays
1 continue
    do ipier = 1,npiers
        skip past this pier if already occupied on this day
        do imeter=1,nmeters
            skip past this meter if it yields a self-self comparison
            skip past this meter if it already leads the pack in number of observations
            nm2m(ipier,imeter) = number of new meter-to-meter comparisons on this day for this combo
        enddo
    enddo
    find which imeter/ipier combo yielded the maximum nm2m(imeter,ipier) and assign imeter to ipier on iday
    if (# available piers on this day > 0) goto 1
enddo
```

Meter #	# of Observations	# of meter-to-meter comparisons over the entire schedule																
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
1	4	0	1	2	2	2	1	2	2	1	1	1	2	1	1	2	1	2
2	4		0	1	1	1	1	2	1	1	1	4	1	2	1	2	1	3
3	4			0	2	1	1	2	1	1	3	2	1	2	2	1	1	1
4	5				0	3	1	1	2	2	2	2	2	2	2	1	2	3
5	5					0	2	2	2	3	1	2	2	2	2	2	2	1
6	4						0	2	3	1	2	1	2	2	1	1	2	1
7	5							0	1	2	2	2	4	3	1	1	2	1
8	5								0	2	1	1	2	2	2	2	3	3
9	4									0	1	2	2	1	1	1	2	1
10	4										0	2	2	1	1	1	2	1
11	5											0	1	2	2	2	1	3
12	5												0	2	1	1	3	2
13	5													0	3	1	2	2
14	4														0	1	1	2
15	4															0	3	2
16	5																0	2
17	5																	0

Criteria	Simple
Observations	Balanced (Every meter makes 4 or 5 observations)
Number of self-self comparisons	Perfect (0 of 17)
Number of missing meter-to-meter comparisons	Optimal: 0 of 153
Balanced meter-to-meter combinations	**Near-optimal (possibly optimal?):** **Minimum 1** **Maximum 4** **Standard Deviation 0.70**

The reason that the above schedule is called "Near-optimal" is that it is hypothesized (but not proven) that a schedule could be created with nothing but 1 and 2 meter-to-meter comparison counts (41 1's and 95 2's, with no 3's or 4's as above). Should that be the case, the standard deviation would drop to 0.46 from the current 0.70. However, proof that such a schedule exists has not yet been established.

Conclusions and Future Work

A simple program, COAGS.F, exists which efficiently seeks optimal schedules for absolute gravimeter comparisons, where "optimal" is taken to mean "balanced meter-to-meter comparison counts, while minimizing self-self comparisons, over the minimum number of days". While this program often finds the truly optimal solution, it does not implement a unique mathematical solution to the optimization problem. Only when such a solution is found will the program be assured of always achieving optimal results.

Ideas for collaboration on seeking a mathematical solution to the problem of optimization are welcome.