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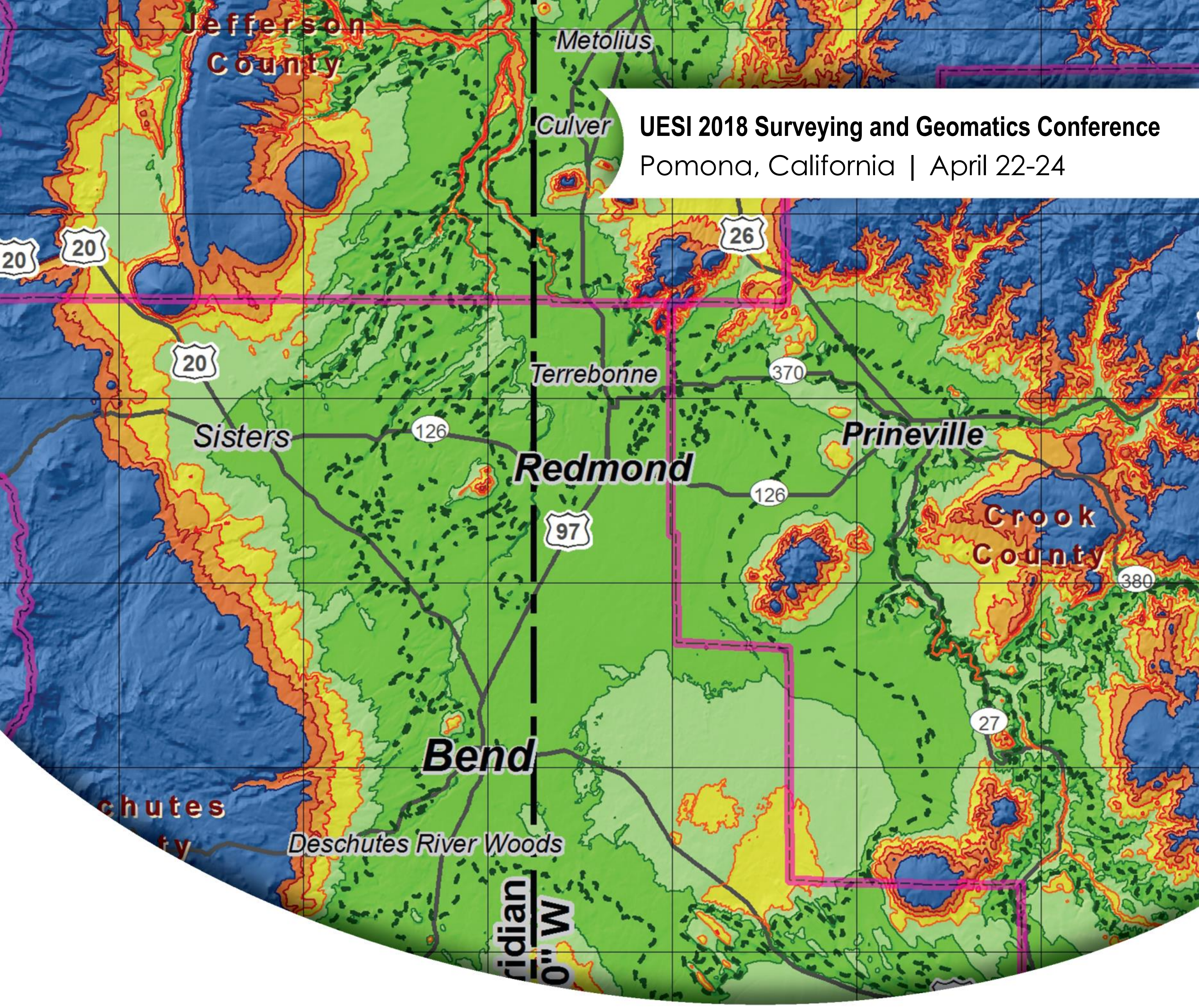

Ground Truth for the Future

Low Distortion Projections and the State Plane Coordinate System of 2022

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Workshop description

Map projections are distorted — it is a Fact of Life. The crux of the problem is *linear distortion*: the difference between true horizontal "ground" distance and its projected representation. This difference often exceeds 1 foot per mile (20 cm/km) for State Plane and other existing published coordinate systems. Such linear distortion can be problematic for various geospatial products and services, including engineering plans, survey plats, construction staking, as-built surveys, and facilities management. Linear distortion cannot be eliminated, but it can be minimized using low distortion projections (LDPs) – although some situations can prove challenging for designing LDPs that perform satisfactorily. This workshop shows how LDPs can be designed to achieve optimal performance even over relatively large areas with variable topographic relief. Importantly, the design procedures are based on the same conformal map projection types used for the new State Plane Coordinate System of 2022 (SPCS2022): Lambert Conformal Conic, Transverse Mercator, and Oblique Mercator. The workshop also provides an overview of the history of State Plane and current plans for development of SPCS2022, including proposed options for states and territories to adopt LDPs as part of SPCS2022. Beyond consistency with SPCS2022, another benefit of using those existing map projection types is that they are compatible with engineering, surveying, and GIS data. Because they can be rigorously georeferenced, LDPs can be used directly to represent conditions "at ground" in GIS and CAD platforms. A resulting notable benefit is that LDP datasets can coexist with other geospatial data without resorting to approximate "best-fit" transformations or other "rubber-sheeting" acts of desperation.

Speaker biography

Michael L. Dennis, PE, RLS, is a geodesist at NOAA's National Geodetic Survey (NGS) where he performs research and assists in development of products and services that define, maintain, and provide access to the National Spatial Reference System (NSRS). His current main duty at NGS is managing the State Plane Coordinate System of 2022 project, although he is also involved in evaluation of data processing and survey network adjustment procedures, development of standards and guidelines, and public outreach. Mr. Dennis is also a Professional Engineer and Surveyor with private sector experience, including ownership of a consulting and surveying firm. He is a member of the American Society of Civil Engineers in the Utility Engineering and Surveying Institute where he serves as Vice Chair of the Surveying and Geomatics Division. His other professional memberships include the American Association for Geodetic Surveying, the American Society for Photogrammetry and Remote Sensing, the National Society of Professional Surveyors, and the Arizona Professional Land Surveyors Association. In addition to his professional duties, Mr. Dennis is currently pursuing a PhD in Geomatics Engineering and GIS at Oregon State University.

Map projection types and conformality

When a map projection is associated with a specific geometric reference frame (i.e., g geodetic datum or geographic coordinate system), it is called a *projected coordinate systems* (PCS). A PCS definition must always include a projection type, geometric reference frame, and linear unit.

Thousands of map projection types have been developed, and about a hundred are commonly used for a wide range of geospatial applications. Fortunately, the list of projections that are useful for surveying and engineering is much shorter, because they should meet the following requirements:

1. Appropriate for large-scale mapping (i.e., not just for covering large portions of the Earth)
2. Widely available and well-defined in commercial geospatial software packages
3. Conformal

Based on these three criteria, the number of conformal map projections appropriate for survey engineering applications reduces to the four in Table 1: Transverse Mercator (TM), Lambert Conformal Conic (LCC), Oblique Mercator (OM), and Stereographic. Table 1 also indicates which of the projections are used in the following well-known PCSs: State Plane Coordinate System (SPCS), Universal Transverse Mercator (UTM), and Universal Polar Stereographic (UPS) systems. These projections types are shown in Figure 1.

Table 1. Conformal projections used for large-scale engineering and surveying applications.

Projection	Type	Usage*	Comments
Transverse Mercator (TM)	Cylindrical	SPCS, UTM	Often used for areas elongate in north-south direction. Perhaps the most widely used projection for large-scale mapping. Also called the Gauss-Krüger projection.
Lambert Conformal Conic (LCC)	Conical	SPCS	Often used for areas elongate in east-west direction. Also widely used for both large- and small-scale mapping. Includes both the one-parallel and two-parallel versions (which are mathematically identical).
Oblique Mercator (OM)	Cylindrical	SPCS	Often used for areas elongate in oblique direction. Not used as often as the TM and LCC projections, but widely available in commercial software. A common implementation is the Hotine OM (also called “rectified skew orthomorphic”).
Stereographic (oblique and polar aspects)	Planar (azimuthal)	UPS	Suitable for small areas, but for large areas scale error is greater than TM, LCC, or OM because it does not match Earth curvature in any direction. Polar aspect (origin at Earth’s poles) used for polar regions. Can be implemented as “ordinary” or “double” stereographic, but resulting coordinates are not the same.

*SPCS = State Plane Coordinate System; UTM = Universal Transverse Mercator; UPS = Universal Polar Stereographic

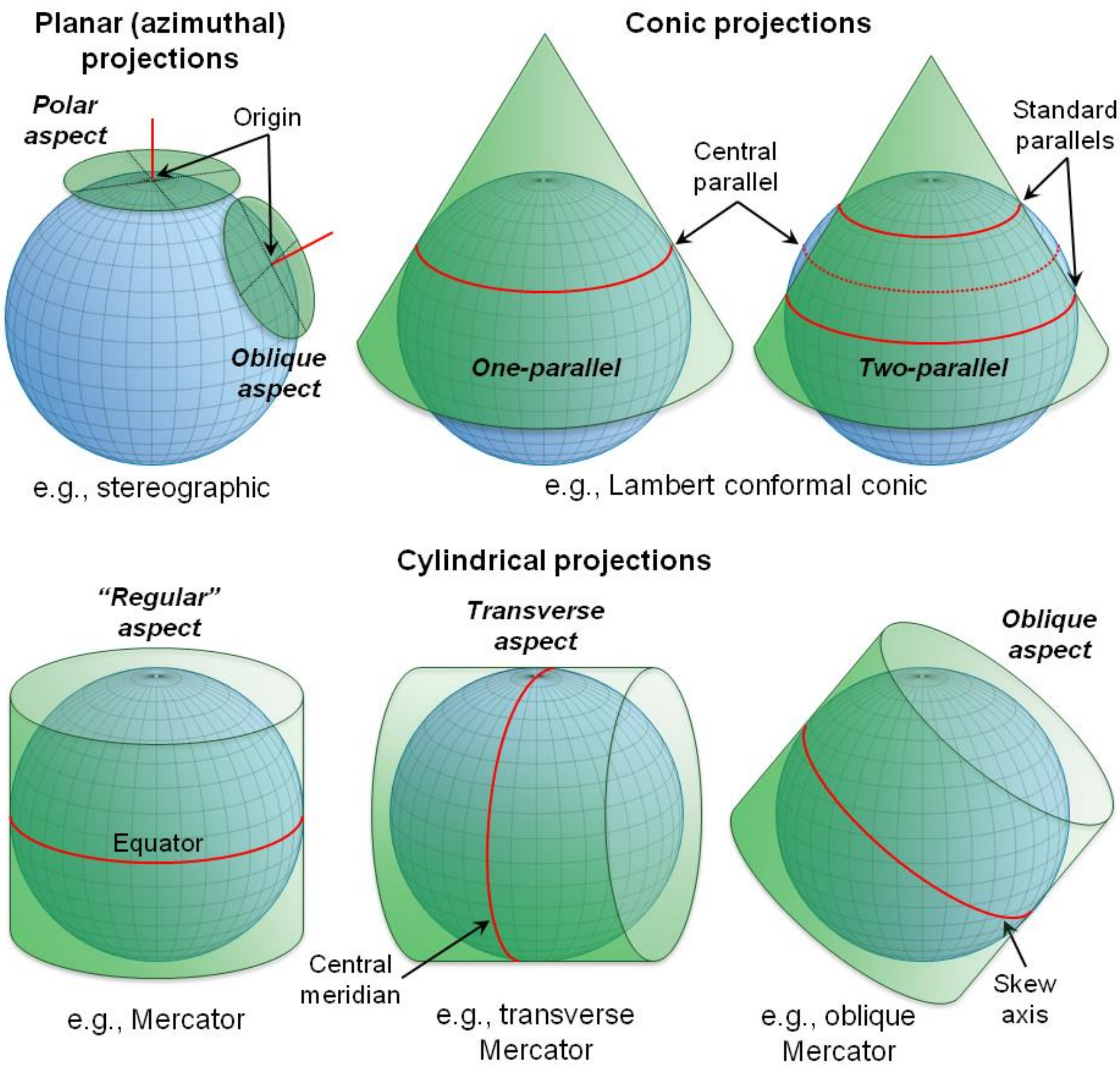


Figure 1. Map projection developable surfaces and their projection axes.

For all non-conformal projections (such as equal area projections), meridians and parallels generally do not intersect at right angles, and scale error varies with direction, so there is no unique linear distortion at a point. These characteristics make non-conformal projections inappropriate for most surveying and engineering applications.

The "flat" surface upon which coordinates are projected is called the *developable surface*. There are three types – plane, cylinder, and cone – as shown in Figure 1. Each of these is "flat" in the sense that it can be represented as a plane without distortion, because it has an infinite radius of curvature in at least one direction. Conceptually, the cylinder and cone can be "cut" parallel to their central axis (which is the direction of infinite radius of curvature) and laid flat without changing the relationship between the projected coordinates. Another way to think of it is that there is only one developable surface, the cone: a cone of infinite height is a cylinder, and a cone of zero height is a plane.

Each of the projection types listed in Table 1 is usually defined with a set of five or six parameters (although in some cases an LCC and OM can require seven or eight parameters, respectively). One is k_0, the projection scale (factor) on the *projection axis*. The projection axis is the line along which projection scale is minimum and constant with respect to the reference ellipsoid, as shown in Figure 1. It is the *central meridian* (λ_C) for the TM, the *central parallel* (φ_C) for the LCC, and the *skew axis* for the OM. Actually, the scale is not quite constant along the OM skew axis but is minimum at a single point (the local origin) and increases slowly along the axis with distance from the origin. The stereographic projection does not have a projection axis *per se* but rather a single point of minimum scale at its origin. For the two-parallel LCC, k_0 is defined as less than 1 implicitly, by the distance between the north and south standard parallels (the further apart the standard parallels, the smaller is k_0)

The k_0 value defines the scale relationship between the ellipsoid and conformal developable surfaces, as listed below and shown in Figure 2:

- **k_0 < 1.** The developable surface is inside ("below") the ellipsoid and intersects the ellipsoid along two curves on either side of the projection axis, beyond which the developable surface is outside ("above") the ellipsoid. In this case the projection is called *secant* because it cuts through the ellipsoid. Secant is the most common projection configuration for published PCSs (such as SPCS, UTM, and UPS) because it covers the largest region with the least absolute scale error with respect to the ellipsoid. Positive and negative scale errors are balanced for secant projection zone as shown in Fig. 2, with approximately the middle 71% of the developable surface below the ellipsoid and the outer 14.5% on either side above the ellipsoid. The "zone" is the area on the Earth where the PCS is used.

- **k_0 = 1.** The developable surface is tangent to the ellipsoid. That is, it touches the ellipsoid along its projection axis (or at a single point for the Stereographic projection).

- **k_0 > 1.** The developable surface is above the ellipsoid and does not intersect the ellipsoid surface anywhere. This approach is used to place the developable surface near the topographic surface, which is typically above the ellipsoid. The intent is to decrease linear distortion of the projected coordinates with respect to the ground surface, rather than the ellipsoid surface.

In addition to the projection axis scale, at least four other parameters are needed to fully define the projections listed in Table 1. Two of these are the latitude and longitude of its *geodetic origin* (φ_0, λ_0). The geodetic origin may or may not be located on the projection axis. It is always on the central meridian of the TM ($\lambda_0 = \lambda_C$) but often is not on the central parallel of the LCC projection ($\varphi_0 \neq \varphi_C$), in which case at least six parameters are required to define an LCC. The other two parameters are the projected coordinate values of the geodetic origin, often called the *grid origin* and specified as *false northing* (N_0) and *false easting* (E_0) in this document. Grid origin values are usually selected such that projected coordinates are positive within the zone. An additional (sixth) parameter called the *skew axis azimuth* (α_0) is required for the OM projection to specify the orientation of its skew axis (α_0 can also be specified implicitly by using a two-point definition).

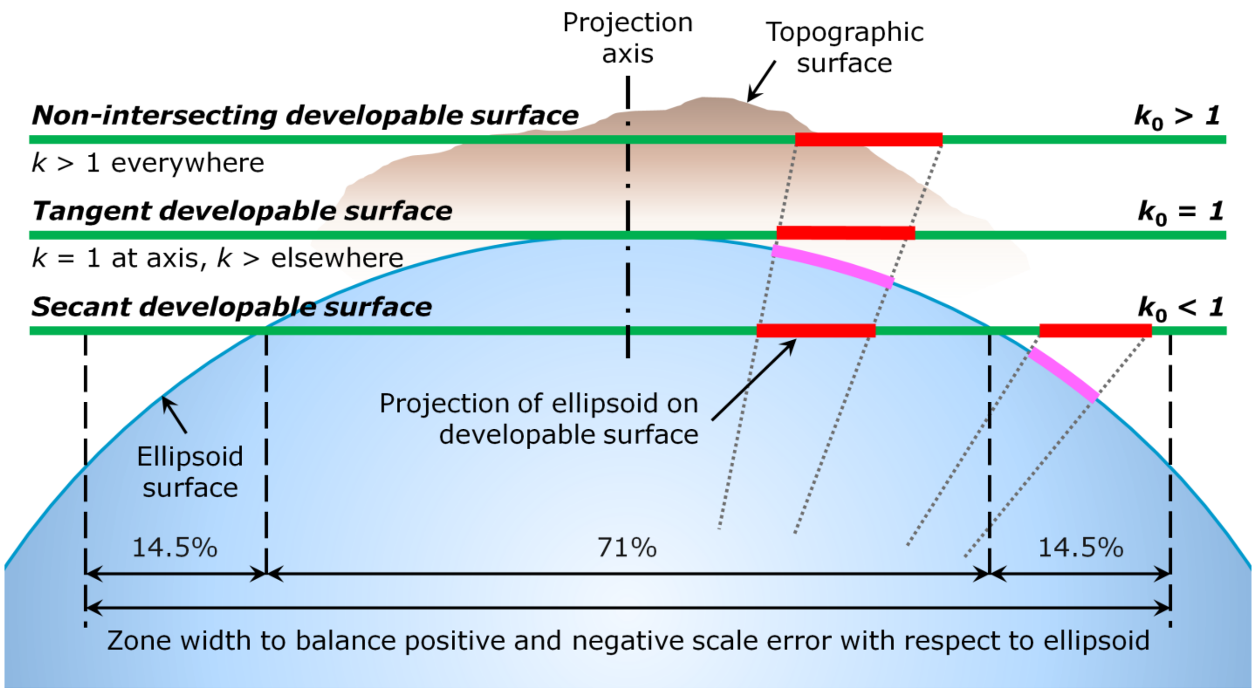


Figure 2. Secant, tangent, and non-intersecting projection developable surfaces.

Map projection distortion

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the "true" relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a ***F***act of ***L***ife. The best we can do is decrease the effect.

There are two general types of map projection distortion, *linear* and *angular*:

1. Linear distortion. Although formally defined infinitesimally at a point, it can be thought of as the finite difference in distance between a pair of grid (map) coordinates when compared to the true ("ground") distance, denoted here by $\boldsymbol{\delta}$.
   - Can express as a ratio of distortion length to ground length:
     - E.g., feet of distortion per mile; parts per million (= mm per km)
     - *Note:* 1 foot / mile = 189 ppm = 189 mm / km

- Linear distortion can be positive or negative:
  - POSITIVE distortion means the grid (map) length is LONGER than the "true" horizontal (ground) length.
  - NEGATIVE distortion means the grid (map) length is SHORTER than the "true" horizontal (ground) length.

2. Angular distortion. For conformal projections, it equals the *convergence* (*mapping*) *angle*, ***γ***. The convergence angle is the difference between projected grid (map) north and true (geodetic) north – a useful property for surveying applications.
   - Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian.
   - Magnitude of the convergence angle increases with distance from the central meridian, and its rate of change increases with increasing latitude, as shown in Table 2.
   - For the OM projection, there is no true central meridian (i.e., longitude along which the convergence angle is zero). However, the meridian passing through the local origin has a convergence very close to zero (and is zero at the origin), and the values in this table can be used to estimate the convergence angle for the OM local origin.
   - Usually convergence is not as much of a concern as linear distortion, and it can only be minimized by staying close to the projection central meridian (or limiting surveying and mapping activities to equatorial regions of the Earth). Note that the convergence angle is zero everywhere for the regular Mercator projection, but this projection is not suitable for large-scale mapping in non-equatorial regions due to its extreme distortion.

Table 2. Convergence angles at various latitudes, at a distance of 1 mile (1.6 km) east (positive) and west (negative) of central meridian for TM and projection (and LCC projection with central parallel equal to latitude in table).

Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM	Latitude	Convergence 1 mi from CM
0°	0° 00’ 00”	30°	±0° 00’ 30”	60°	±0° 01’ 30”
5°	±0° 00’ 05”	35°	±0° 00’ 36”	65°	±0° 01’ 51”
10°	±0° 00’ 09”	40°	±0° 00’ 44”	70°	±0° 02’ 23”
15°	±0° 00’ 14”	45°	±0° 00’ 52”	75°	±0° 03’ 14”
20°	±0° 00’ 19”	50°	±0° 01’ 02”	80°	±0° 04’ 54”
25°	±0° 00’ 24”	55°	±0° 01’ 14”	85°	±0° 09’ 53”

One can think of linear distortion as due to the projection "developable surface" (plane, cone, or cylinder) departing from the reference ellipsoid. Although no ellipsoidal forms of conformal projections are perspective (i.e., cannot be created geometrically), it is still useful to think of linear distortion increasing as the "distance" of the developable surface from the ellipsoid increases. In that sense, linear distortion is entirely a function of "height" with respect to the ellipsoid.

Although total linear distortion is (conceptually) due to departure of the developable surface from the ellipsoid, it is convenient to separate it into two components: one due to Earth curvature and one due to height above or below the reference ellipsoid. Indeed, this "total" distortion is often computed as the product of these two components and called the "combined" scale error (or factor). The relative magnitude of each at a point of interest depends on its horizontal distance from the projection axis and its ellipsoid height.

Figure 3 provides a conceptual illustration of distortion as a geometric departure of the developable surface from the reference ellipsoid. Table 3 gives the range of distortion due to curvature for various projection zone widths, and Table 4 gives change in distortion due to change in height, but total distortion is always a combination of both.

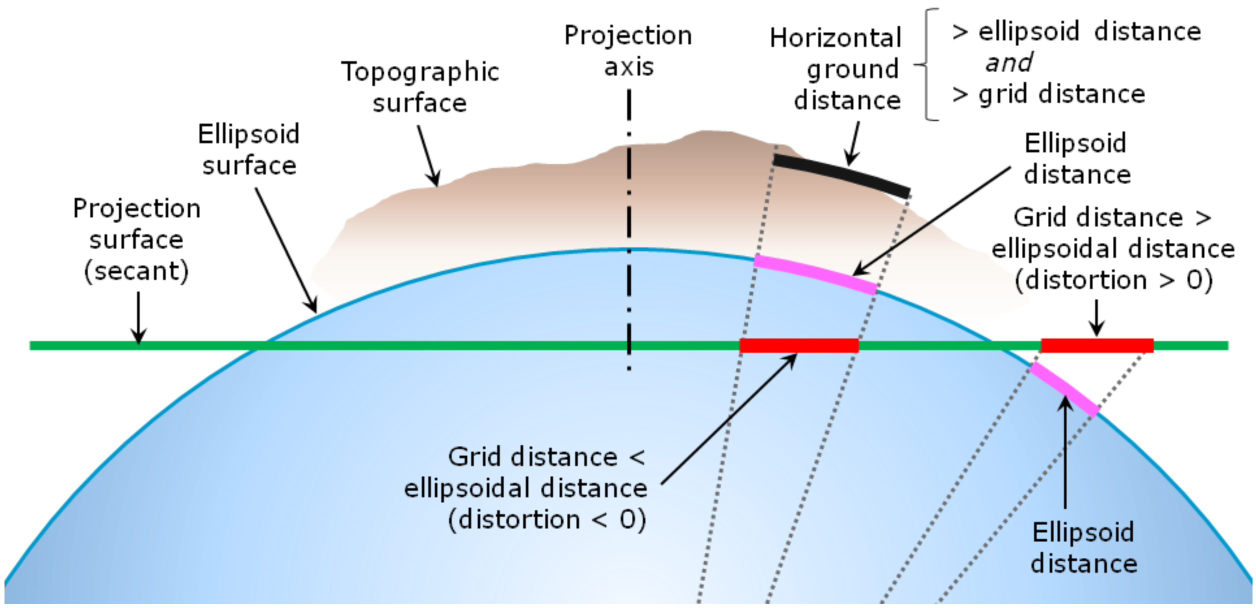


Figure 3. Linear distortion of secant map projection with respect to ellipsoid and topography.

Table 3. Maximum range in linear distortion due to Earth curvature for various zone widths (perpendicular to projection axis).

Zone width for secant projections (i.e., balanced positive and negative distortion)	Maximum range in linear distortion, $\Delta(\delta + 1) = \Delta k$		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
16 miles (25 km)	±1 ppm	±0.005 ft/mile	1 : 1,000,000
35 miles (57 km)	±5 ppm	±0.026 ft/mile	1 : 200,000
50 miles (81 km)	±10 ppm	±0.053 ft/mile	1 : 100,000
71 miles (114 km)	**±20 ppm**	**±0.11 ft/mile**	**1 : 50,000**
112 miles (180 km)	±50 ppm	±0.26 ft/mile	1 : 20,000
~158 miles (255 km) e.g., SPCS*	±100 ppm	±0.53 ft/mile	1 : 10,000
~317 miles (510 km) e.g., UTM[†]	±400 ppm	±2.11 ft/mile	1 : 2500

*State Plane Coordinate System; [†]Universal Transverse Mercator

Table 4. Change in projection linear distortion due to change in ellipsoid height.

Change in ellipsoid height, Δh	Change in linear distortion, $\Delta(\delta + 1) = R_G / (R_G + \Delta h)$		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
±100 feet (30 m)	±4.8 ppm	±0.025 ft/mile	~1 : 209,000
±400 feet (120 m)	**±19 ppm**	**±0.10 ft/mile**	**~1 : 52,000**
±1000 feet (300 m)	±48 ppm	±0.25 ft/mile	~1 : 21,000
+2500 feet (750 m)*	–120 ppm	–0.63 ft/mile	~1 : 8400
+3300 feet (1000 m)**	–158 ppm	–0.83 ft/mile	~1 : 6300
+14,400 feet (4400 m)[†]	–690 ppm	–3.6 ft/mile	~1 : 1450

*Approximate mean topographic ellipsoid height of the conterminous US (CONUS)
** Approximate mean topographic ellipsoid height in CONUS west of 100°W longitude
[†] Approximate maximum topographic ellipsoid height in CONUS

Rules of thumb for ±5 ppm distortion:

- Due to curvature: within ±5 ppm for area 35 miles wide (perpendicular to projection axis).
- Due to change in topographic height: ±5 ppm for range in height of ±100 ft.

Methods for creating low-distortion projected coordinate systems

1. **Design a Low Distortion Projection (LDP) for a specific geographic area.**

   Use a conformal projection referenced to the existing geometric reference frame.

   Described in detail in next section.

2. **Scale the reference ellipsoid "to ground".**

   A map projection referenced to this new "datum" is then designed for the project area.

   Problems: Main problem is that method is more complex but performs no better than LDP.

   - Requires a new ellipsoid for every coordinate system. Therefore the five or six projection parameters plus two ellipsoid parameters are required, for a total of seven or eight parameters to define each system.
   - Coordinates must be transformed to the new ellipsoidal system prior to being projected. So projection algorithm must include a datum transformation, and this can make these systems more difficult to implement.
   - The transformed latitudes of points can differ substantially from the original values, by more than 3 feet for heights greater than 1000 ft. This can cause incorrect projected coordinates if original geodetic coordinates are not transformed prior to projecting.

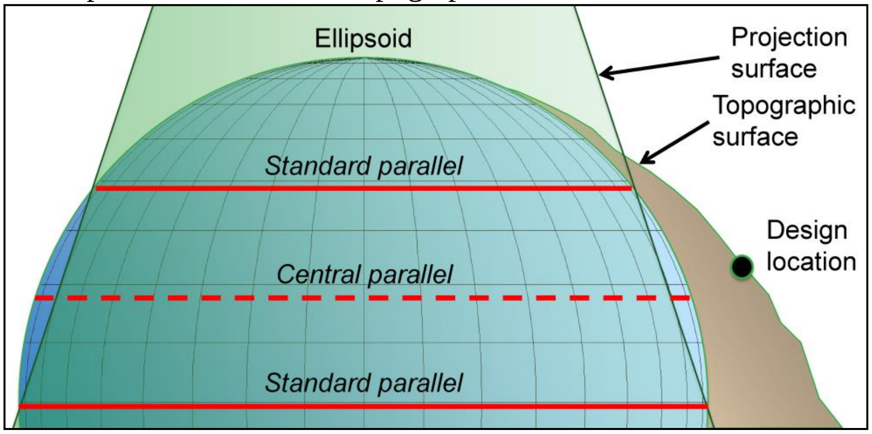
3. **Scale an existing published map projection "to ground".**

   Referred to as "modified" State Plane when an existing SPCS projection definition is used.

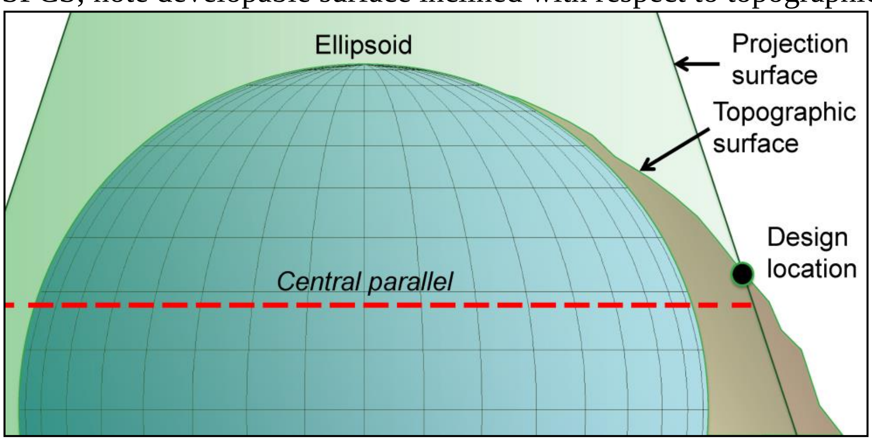
   Problems:

   - Generates coordinates with values similar to "true" State Plane (can cause confusion).
     - Can eliminate this problem by translating grid coordinates to get smaller values.
   - Often yields "messy" parameters when a projection definition is back-calculated from the scaled coordinates (e.g., to import the data into a GIS).
     - More difficult to implement in a GIS, and may cause problems due to rounding or truncating of "messy" projection parameters (especially for large coordinate values).
     - Can reduce this problem through judicious selection of "scaling" parameters.
   - Does ***not*** reduce the convergence angle (it is same as that of original SPCS definition). Likewise, *arc-to-chord correction* is the same as original SPCS (used along with convergence angle for converting grid azimuths to geodetic azimuths).
   - ***MOST IMPORTANT:* Usually does not minimize distortion over as large an area as the other two methods.**
     - Extent of low-distortion coverage generally *decreases* as distance from projection axis *increases*.
     - State Plane axis usually does ***not*** pass through the project area and in addition may be oriented in a direction that decreases the area of low distortion coverage.
     - Figure 4 illustrates this problem with "modified" SPCS as compared to an LDP.

(a) Typical SPCS situation (for LCC projection). Projection is secant to ellipsoid, with developable surface below topographic surface.



(b) SPCS scaled "to ground" at design location. Central parallel in same location as original SPCS; note developable surface inclined with respect to topographic surface.



(c) LDP design. Note central parallel moved north to align developable surface with topographic surface, thereby reducing distortion over a larger region.

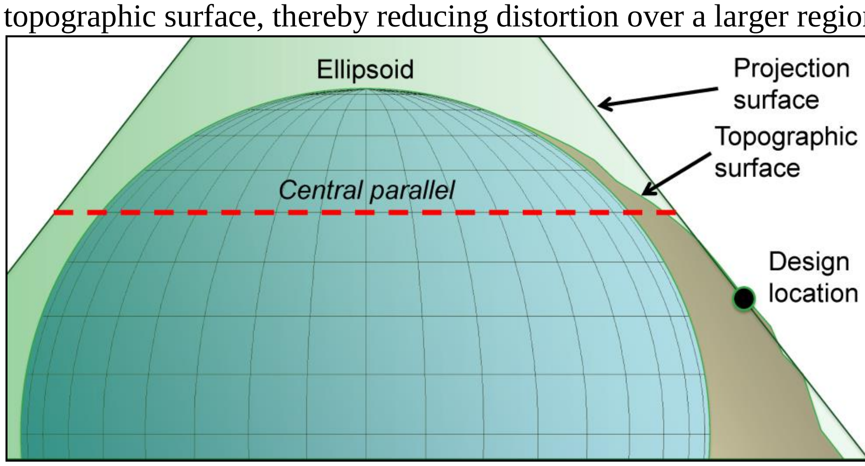


Figure 4. Comparison of (a) SPCS, (b) "modified" SPCS, and (c) LDP.

Six steps illustrating Low Distortion Projection (LDP) design

The design objective is usually to minimize linear distortion over the largest area possible. These goals are at odds with one another, so LDP design is an ***optimization problem***. It is important to also realize that the most difficult part is often not technical but psychological. There is little value in designing an LDP for a region without first getting concurrence and buy-in from the many stakeholders impacted by the design. This includes surveyors, engineers, GIS professionals, as well as public and private organizations that make use of geospatial data in the design area. Getting stakeholders involved early in the process will increase the likelihood that the LDP will be adopted and actually used.

The following six steps are intended to illustrate commonly encountered situations in LDP design. These steps are provided to teach the design concepts; in the actual design process some of these "steps" can be omitted or modified, especially when designing for large areas. But these steps work well for small areas (< ~30 miles or 50 km wide perpendicular to the projection axis).

1. **Define distortion objective for area of interest and determine *representative* ellipsoid height, h_0 (<u>not</u> elevation)**

   NOTE: *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height. It is even possible to skip this step entirely, and instead start the process with a projection scale of 1. However, considering height helps illustrate the concepts behind design the process.

   - A common objective for "low distortion" is ±20 ppm (±0.1 ft/mile), but this may not be achievable due to range of topographic height and/or size of design area. The following "rules of thumb" can help guide the initial design. However, often it is possible to achieve better results than these guidelines indicate, because both height and areal extent affect distortion simultaneously, and one can be used to compensate for the other.
     - <u>Size of design area</u>. **Distortion due to curvature is within ±5 ppm for an area 35 miles wide**. Note that this width is perpendicular to the projection axis (e.g., east-west for TM and north-south for LCC projections). The effect is not linear; range of distortion due to curvature increases rapidly with increasing zone width and is proportional to the square of the zone width, i.e., doubling the zone width increases the distortion by about a factor of four (for this ±5 ppm case, doubling zone width to 70 miles quadruples the distortion range to about ±20 ppm).
     - <u>Range in topographic ellipsoid height</u>. **Distortion due to change in topographic height is about ±5 ppm for a ±100 ft range in height.** Note that this is linear for the topographic height ranges on Earth. Thus a range of ±400 ft in height corresponds to a range of about ±20 ppm distortion.
   - The *average* height of an area may not be appropriate (e.g., because mountains are in the design area).

  - There is no need to estimate height to an accuracy of better than about ±20 ft (±6 m); this corresponds to about 1 ppm distortion. In addition, the initial projection scale determined using this height will likely be refined later in the design process.

2. Choose projection type and place projection axis near centroid of project area

NOTE: *This is just to get the design process started.* In cases where the topography generally changes in one direction, offsetting the projection axis can yield substantially better results. As with step #1, there is no need to spend a lot of effort on this step, since the effect of the projection type and axis location is evaluated later in the design process.

- Select a well-known and widely used ***conformal*** projection, such as the Transverse Mercator (TM), Lambert Conformal Conic (LCC), or Oblique Mercator (OM).
  - When minimizing distortion, it will not always be obvious which projection type to use, but for small areas (< ~30 miles or ~50 km wide), both the TM and LCC will usually provide similar and satisfactory results. However, significantly better performance can be obtained in many cases when a projection is used with its axis perpendicular to the general topographic slope of the design area (more on this below).
  - In nearly all cases, a two-parallel LCC should ***not*** be used for an LDP with the NAD 83 datum definition (but note that some software may not support a one-parallel LCC). A two-parallel LCC should not be used because the reason there are two parallels is to make the projection secant to the ellipsoid (i.e., the central parallel scale is less than 1). This is at odds with the usual objective of scaling the projection so that the developable surface is at the topographic surface, which is typically well above the ellipsoid, particularly in areas where reduction in distortion is desired.
  - The OM projection can be very useful for minimizing distortion over large areas, especially areas that are elongate in an oblique direction. It can also be useful in areas where the topographic slope varies gradually and more-or-less uniformly in an oblique direction. The disadvantage of this projection is that it is more difficult to use for designs that account for topographic slope, since both the projection skew axis location and orientation must be simultaneously optimized. Such designs would be extremely difficult to perform manually but can be optimized using mathematical methods (such as least squares). There is also more than one version of the OM projection; the Hotine OM, also called rectified skew orthomorphic (RSO), is the most common version of the OM used in the U.S.
  - The oblique stereographic projection can also be used, but it is unlikely that it will perform better than the TM, LCC, or OM projections since it does not curve with the Earth in any direction. Situations where it would provide the lower distortion than the other projections would only rarely (if ever) be encountered. In addition, there are two common versions ("original" and "double" stereographic), but they do not yield the same coordinates and so care must be taken to ensure the one used for design is the same used in subsequent applications (coordinates differ by about 1 foot 20 miles from the projection origin).

  - When choosing a projection, universal commercial software support, although desirable, is not an essential requirement. In rare cases where third parties must use a coordinate system based on a projection not supported in their software, it is possible for them to get on the coordinate system implicitly, for example by using a 2-D best-fit conformal transformation based on LDP coordinates at common points (e.g., the so-called horizontal "calibration" or "localization" process).
- Placing the projection axis near the design area centroid is often a good first step in the design process (or, for the OM projection, parallel to the long axis of the design area).
  - In cases where topographic height increases more-or-less uniformly in one direction, dramatically better performance can be achieved by offsetting the projection axis from the project centroid. In such cases a projection type should be chosen such that its projection axis is perpendicular to the topographic slope (e.g., for topography sloping east-west, a TM projection should be used; for slope north-south, an LCC projection should be used). The axis is located such that the developable surface best coincides with the topographic surface (as shown in Figure 4 for an LCC).
  - Often the central meridian of the projection is placed near the east-west "middle" of the project area in order to minimize convergence angles (i.e., the difference between geodetic and grid north). The central meridian is the projection axis only for the TM projection; its location has no effect on linear distortion for the LCC projection.

3. Scale projection axis to the representative ground height, h_0

NOTE: *This is just to get the design process started.* Ellipsoid height by itself is unlikely to yield the final design scale, except for small areas, due to curvature and/or systematic change in topographic height. This step can also be skipped by simply using $k_0 = 1$, but the following provides the concepts (as well as some mathematical information for step #4).

- Compute map projection axis scale factor "at ground": $k_0 = 1 + \dfrac{h_0}{R_G}$
  - For TM projection, k_0 is the central meridian scale factor.
  - For one-parallel LCC projection, k_0 is the standard (central) parallel scale factor.
  - For OM projection, k_0 is the scale at the local origin.
- R_G is the geometric mean radius of curvature, $R_G = \dfrac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$

  and φ = geodetic latitude of point, and for the GRS 80 ellipsoid:

  a = semi-major axis = 6,378,137 m (exact) = 20,925,646.325 international ft
  = 20,925,604.474 US survey ft

  e^2 = first eccentricity squared = $f(2 - f)$ = 0.00669438002290

  f = geometric flattening = 1 / 298.257222101

- Alternatively, can initially approximate R_G using Table 5, since k_0 will likely be refined in Step #4:

Table 5. Geometric mean radius of curvature at various latitudes for the GRS 80 ellipsoid (rounded to nearest 1000 feet and meters).

Lat	feet	(meters)	Lat	feet	(meters)	Lat	feet	(meters)
0°	20,855,000	(6,357,000)	**35°**	20,902,000	(6,371,000)	**65°**	20,971,000	(6,392,000)
10°	20,860,000	(6,358,000)	**40°**	20,913,000	(6,374,000)	**70°**	20,980,000	(6,395,000)
15°	20,865,000	(6,360,000)	**45°**	20,926,000	(6,378,000)	**75°**	20,987,000	(6,397,000)
20°	20,872,000	(6,362,000)	**50°**	20,938,000	(6,382,000)	**80°**	20,992,000	(6,398,000)
25°	20,880,000	(6,364,000)	**55°**	20,950,000	(6,385,000)	**85°**	20,995,000	(6,399,000)
30°	20,890,000	(6,367,000)	**60°**	20,961,000	(6,389,000)	**90°**	20,996,000	(6,400,000)

4. Compute distortion throughout project area and refine design parameters

- Distortion computed at a point (at ellipsoid height h) as $\delta = k\left(\frac{R_G}{R_G + h}\right) - 1$
    - Where k = projection grid point scale factor (i.e., distortion with respect to the ellipsoid at a point). Note that computation of k is rather involved, and is often done in commercial software. However, if your software does not compute k, or if you want to check the accuracy of k computed by your software, equations for doing so for the TM and LCC projections are provided in section "Projection grid scale factor and convergence angle computation" later in this document.
    - Multiply δ by 1,000,000 to get distortion in parts per million (ppm).
- Best approach is to compute distortion over entire area and generate a distortion map and compute distortion statistics (this helps ensures low-distortion coverage is achieved where it is desired).
    - Often requires repeated evaluation using different k_0 values and different projection axis locations.
    - May also warrant trying different projection types.
- General approach for computational refinement:
    - Compute distortion statistics, such as mean, range, and standard deviation for all points in the design rea.
    - Changing the projection scale only affects the mean distortion; it has essentially no effect on the variability (standard deviation and range).
    - The only way to reduce distortion variability is by moving the projection axis and/or changing the projection type. The usual objective is to minimize the distortion range.

Once this is done, the scale can be changed so that the mean distortion is near zero (this will have no effect on the distortion range or standard deviation).

- ○ Finally, check to ensure the desired distortion is achieved in important areas, and check to ensure overall performance is satisfactory, by using a map showing distortion everywhere in the design area. It may be worthwhile to give greater weight to distortion in populated areas (such as cities), rather than using a uniform weight for all areas.

5. Keep the definition SIMPLE and CLEAN!

- Define k_0 to no more than SIX decimal places, e.g., 1.000175 (exact).
  - *Note:* A change of one unit in the sixth decimal place (±1 ppm) equals distortion caused by a 20 ft (6 m) change in height.
  - For large areas with variable relief, scale defined to five decimal places (±10 ppm) is often sufficient.
- Define the central meridian and latitude of grid origin to nearest whole arc-minute. Using arc minutes evenly divisible by 3 will result in exact values in decimal degrees (e.g., 121°33’00” W = −121.55°), although some prefer using the nearest 5 arc minutes (as done for State Plane 1983 and 1927).
- Define grid origin using whole values. Often it is desired to use values with as few digits as possible (e.g., false easting = 50,000 for a system with maximum easting coordinate value < 100,000), although there are many different options for selecting values. Note that the grid origin definition has no effect whatsoever on map projection distortion.
  - It is strongly recommended that the coordinate values everywhere in the design area be distinct from other coordinate system values for that area (such as State Plane and UTM) in order to reduce the risk of confusing the LDP with other systems. For multi-zone LDPs, it could similarly be helpful to keep coordinates between the zones distinct, if possible.
  - Often it is desirable to define grid origins such that the northings and eastings do not equal one another anywhere in the design area.
  - In some applications, there may be an advantage to using other criteria for defining the grid origin. For example, it may be desirable for all coordinates in the design area to have the same number of digits (such as six digits, i.e., between 100,000 and 999,999). In other cases it may be useful to make the coordinates distinct from State Plane by using larger rather than smaller coordinates, especially if the LDP covers a very large area. In multi-zone systems, it may also be helpful to define grid origins such that the values correlate to zone numbers (e.g., a false easting of 3,500,000 m for a zone designated as #3). This approach was used for the Kansas Regional Coordinate System (e.g., Dennis, 2017).

6. *Explicitly* define linear unit and geometric reference system (i.e., geodetic datum)

- <u>Linear unit</u>, e.g., meter (*or* international foot, *or* US survey foot, or…?)

  - The international foot is shorter than the US survey foot by 2 ppm. Because coordinate systems typically use very large values, it is critical that the type of foot used be identified (the values differ by 1 foot per 500,000 feet).
  - Because of the possibility of confusion between the international and US survey foot, it is recommended that the design parameters for the LDP be in meters (this approach is used in most State Plane zones). Output coordinates can then be specified for which type of foot is desired. It can be difficult to detect an implementation that used the incorrect type of foot, since they differ by only 2 ppm.
- <u>Geometric reference system (datum)</u>, e.g., North American Datum of 1983 (NAD 83)
  - The reference system realization (i.e., "datum tag") should not be included in the coordinate system definition (just as it is not included in State Plane definitions). However, the datum tag *is* an essential component for defining the spatial data used within the coordinate system (as shown in a metadata example later in this document). For NAD 83, the NGS convention is to give the datum tag in parentheses after the datum name, usually as the year in which the datum was "realized" as part of a network adjustment. Common datum tags for NGS control are listed below:
    - "2011" for the current NAD 83 (2011) epoch 2010.00 realization, which is referenced to the North America tectonic plate.
    - "2007" for the (superseded) NSRS2007 (National Spatial Reference System of 2007) realization. Functionally equivalent to the superseded "CORS" datum tag and referenced to an epoch date of 2002.00 for most of the coterminous US.
    - "199*x*" for the various supersede HARN (or HPGN) realizations, where *x* is the last digit of the year of the adjustment (usually done for a particular state). HARN is "High Accuracy Reference Network" and HPGN is "High Precision Geodetic Network".
- <u>*Note regarding the State Plane Coordinate System of 2022 (SPCS2022)*</u>*:* NGS will replace NAD 83 with new geometric datums in 2022. For North America, it will be called the North American Terrestrial Reference Frame of 2022 (NATRF2022); there will also be a TRF for the Caribbean, Pacific, and Mariana tectonic plates. The GRS 80 ellipsoid will continue to be used for the 2022 datum. In North America, horizontal coordinates will change by less than 2 m (6.5 ft). Ellipsoid heights will change by less than ±2 m everywhere. A change in height of 2 m will change linear distortion by 0.3 ppm. Since the change to the 2022 datum will have negligible impact on the distortion of LDPs designed with respect to NAD 83, those LDPs could continue to be used with the 2022 datum. However, to avoid confusion it would be prudent to change the grid coordinates so that LDP coordinates based on the 2022 datum are significantly different from those referenced to the NAD 83. Such a change will not affect distortion but would reduce risk of accidentally referencing the wrong datum.

  NGS is currently early in the process of defining SPCS2022. The references section of this document includes recently released NGS documents about SPCS2022:

  - A report on the history, status, and possible future of State Plane (Dennis, 2018).

- Draft SPCS2022 policy and procedures (NGS, 2018a and 2018b, respectively). Note that these draft documents, as currently written, allow for the use of LDPs for SPCS2022 zones. However, the LDPs must be defined by others than NGS.
- A Federal Register Notice (NGS, 2018c) that solicits public comment on the draft SPCS2022 policy and procedures. It also asks for input on (and defines) "special purpose" zones. The public comment period is through August 31, 2018.

- *Note regarding the relationship between NAD 83 and WGS 84:* For the purposes of entering the LDP projection parameters into vendor software, the datum should be defined as NAD 83 (which uses the GRS 80 reference ellipsoid for all realizations). Some commercial software implementations assume there is no transformation between WGS 84 and NAD 83 (i.e., all transformation parameters are zero). Other implementations use a non-zero transformation, and in some cases both types are available. The type of transformation used will depend on specific circumstances, although often the zero transformation is the appropriate choice (even though it is not technically correct). Check with software technical support to ensure the appropriate transformation is being used for your application. Additional information about WGS 84 is available from the National Geospatial-Intelligence Agency (NGA, 2014b).
- *Note regarding the vertical component of a coordinate system definition:* The vertical reference system (datum) is an essential part of a three-dimensional coordinate system definition. But LDPs are restricted exclusively to horizontal coordinates. Although the vertical component is essential for most applications, it is not part of an LDP and must be defined separately. It should be specified as part of the overall coordinate system metadata (as shown in the metadata example later in this document). A complete three-dimensional coordinate system definition must include a vertical "height" component. Typically the vertical part consists of ellipsoid heights relative to NAD 83 (when using GNSS) and/or orthometric heights ("elevations") relative to the North American Vertical Datum of 1988 (NAVD 88). These two types of heights are related (at least in part) by a hybrid geoid model, such as GEOID12B, and often a vertical adjustment or transformation to match local vertical control for a project. The approach used for the vertical component usually varies from project to project and requires professional judgment to ensure it is defined correctly. Providing such instructions is beyond the scope of this document.

Design example for a Low Distortion Projection (LDP)

The LDP design example is for the southern Deschutes River valley of central Oregon (shown in Figure 5). This example follows the design of the Bend-Redmond-Prineville zone in the Oregon Coordinate Reference System (OCRS). The design process is illustrated in the six steps below.

- First three steps are mainly to initiate the design; step 4 is where the design is optimized to minimize distortion over the largest area possible.
- Overall design objective is ±20 ppm for the region and ±10 ppm within the three largest towns (Bend, Redmond, and Prineville).
- Towns of Sisters, Culver, and Madras are also used for evaluation.

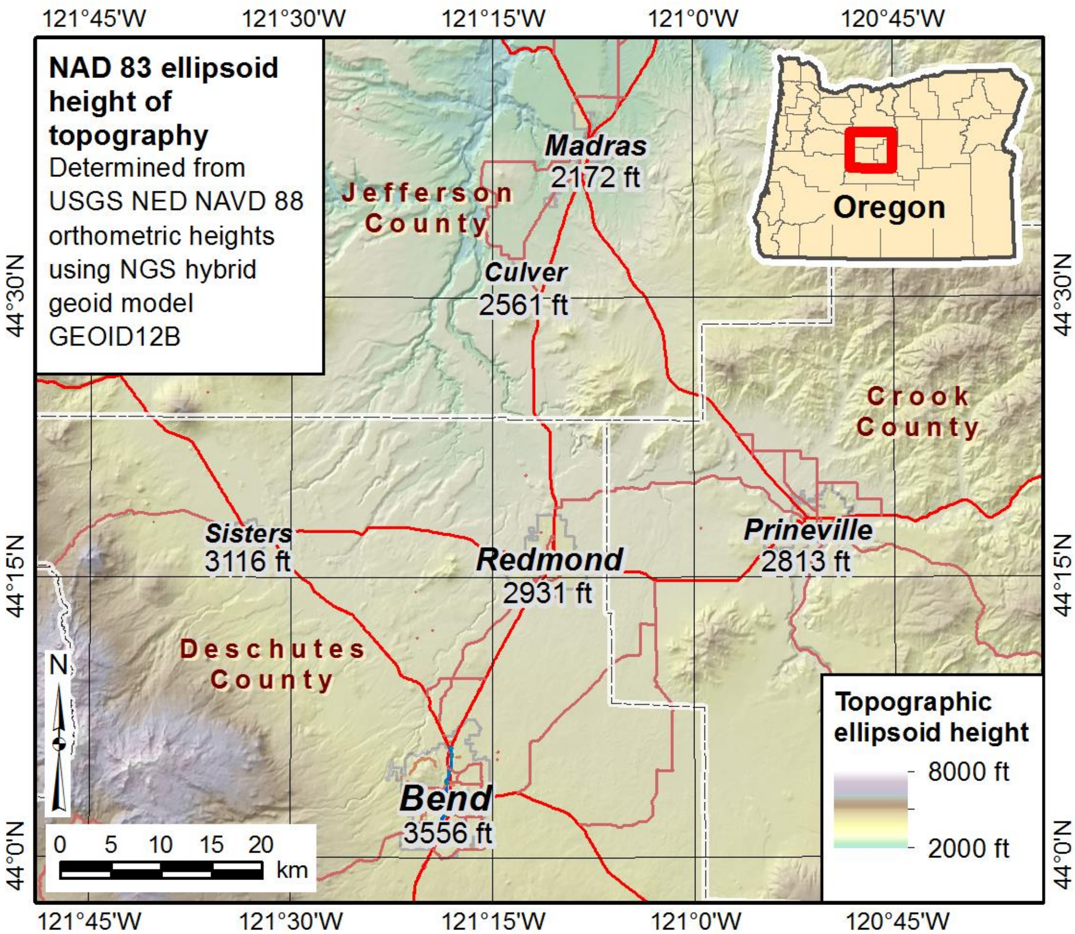


Figure 5. LDP design area, showing topographic ellipsoid heights of towns.

1. **Define distortion objective for area of interest and determine *representative* ellipsoid height, h_0 (<u>not</u> elevation)**

   To get the process started, ellipsoid heights were obtained at arbitrary locations in each of the six towns using NAVD 88 orthometric heights from the USGS National Elevation Dataset with GEOID12B hybrid geoid heights. These values are given in Table 6, for a mean topographic ellipsoid height of h_0 **= 2858 ft** ("representative" value for initial design).

   <u>Size of design area</u>. The overall design area is about 45 miles long north-south, and about 35 miles wide east-west. Based on ±10 ppm distortion for a zone width of 50 miles in Table 3, it appears the design distortion can be achieved, at least with respect to Earth curvature.

   <u>Range in topographic ellipsoid height</u>. The height range in Table 6 is 1384 ft (i.e., ±692 ft), which corresponds to about ±33 ppm based on ±4.8 ppm per ±100 ft in Table 4 – not an encouraging observation, considering the design objectives of ±20 ppm overall and especially of ±10 ppm in Bend, Redmond, and Prineville.

Table 6. The six locations (towns) in the project region used to perform LDP design.

Location	NAD 83 latitude	NAD 83 longitude	Topographic height at location (feet)		
			NAVD 88 orthometric	GEOID12B hybrid geoid	NAD 83 ellipsoid
Bend	44°03'29"N	121°18'55"W	3625	-68.8	**3556**
Redmond	44°16'21"N	121°10'26"W	3000	-69.5	**2931**
Prineville	44°17'59"N	120°50'04"W	2880	-67.5	**2813**
Sisters	44°17'27"N	121°32'57"W	3186	-70.1	**3116**
Culver	44°31'32"N	121°12'47"W	2631	-69.8	**2561**
Madras	44°38'00"N	121°07'46"W	2242	-70.0	**2172**
		Mean	**2927**	**-69.3**	**2858**
		Range	**1383**	**2.6**	**1384**
		Std deviation	**±473**	**±1.0**	**±473**

2. Choose projection type and place projection axis near centroid of project area

Upon initial inspection, it is not clear which projection type would be best, so will evaluate both TM and LCC. To get the process started, the projection axes placed near the center of the region.

For the TM projection, the initial central meridian is set at $\boldsymbol{\lambda_0}$ **= 121° 15' 00" W**.

For the LCC projection, the initial central parallel is set at $\boldsymbol{\varphi_0}$ **= 44° 20' 00" N**.

Because the design area is somewhat longer north-south than east-west (45 vs. 35 miles), the TM projection may be the better choice. On the other hand, the topographic height overall decreases from north to south, which tends to favor the LCC projection. The performance of these projections will be evaluated as part of the design process.

3. Scale projection axis to representative ground height, $\boldsymbol{h_0}$

First compute Earth radius at mid-latitude of φ = 44° 20' 00" N (same as central parallel for initial LCC design):

$$R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi} = \frac{20{,}925{,}646.325\times\sqrt{1-0.00669438002\overline{3}}}{1-0.006694380023\times\left[\sin(44.333333°)\right]^2} = \mathbf{20{,}923{,}900\ ift}$$

Thus the central meridian scale factor scaled to the representative ellipsoid height is

$$k_0 = 1+\frac{h_0}{R_G} = 1+\frac{2858}{20{,}923{,}900} = \mathbf{1.00014}\ \text{(rounded to five decimal places)}$$

Based on these results, the following initial TM and LCC projections are defined (will check and refine as necessary in step #4). Only the characteristics affecting distortion need to be specified at this point. Other parameters, such as false northings and eastings, will be specified after a design is selected based on distortion performance.

Projection:	**Transverse Mercator**	**Lambert Conformal Conic**
Projection axis:	λ_0 = 121° 15' 00" W	φ_0 = 44° 20' 00" N
Projection axis scale:	k_0 = 1.00014	k_0 = 1.00014

4. Compute distortion throughout project area and refine design parameters

Distortion can also be computed at discrete points. These points can be NGS control points, other surveyed points, or any point with a reasonable accurate topographic ellipsoid height. For this design example, the heights at the given location for each of the six towns are used, which are accurate to about ±10 ft (corresponding to distortion accuracy of ±0.5 ppm). A computation example for each of the two initial LDP designs is provided for the point representing the town of Bend using the values from Table 6:

Bend: φ = 44° 03' 29" N, λ = 121° 18' 55" W, h = 3556 ft,

where linear distortion is computed as $\delta = k\left(\dfrac{R_G}{R_G + h}\right) - 1$

and geometric mean radius of curvature as $R_G = \dfrac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$ = 20,923,218 ift.

The value of k can be computed using various geospatial software packages. If such software is not available, it can be computed using the equations given later in this document. The value obtained for the TM is k = 1.000 140 336, and for the LCC is k = 1.000 151 486.

Using these values gives the following values of distortion at the point in Bend:

TM: $\delta = 1.000140336 \times \left(\dfrac{20{,}923{,}218}{20{,}923{,}218 + 3556}\right) - 1$ = 0.999 970 387 – 1 = **–29.6 ppm**

LCC: $\delta = 1.000151486 \times \left(\dfrac{20{,}923{,}218}{20{,}923{,}218 + 3556}\right) - 1$ = 0.999 981 534 – 1 = **–18.5 ppm**

Despite using the mean topographic height of the six towns for determining the projection scale, the distortion magnitude for both projections exceeds the ±10 ppm criterion for Bend. This could be fixed for the point in Bend by increasing the projection scale by, say, 20 ppm to k_0 = 1.00016, which would change the values to –9.6 ppm and +1.5 ppm for the TM and LCC projections, respectively. However, this would also increase the distortion at the other points by 20 ppm, yielding a maximum in Madras of +57.3 ppm and +69.9 ppm for the TM and LCC projections, respectively. Such distortion is much too large, so a different approach is needed.

For a given projection, variability can only be changed by changing the location of the projection axis. In this case, simply changing the projection scale alone will not achieve the desired result. Changing their locations will change the variability of the distortion in the design area. We can assess the variability by the distortion range and standard deviation. The results of doing that for the TM and LCC projections are shown in tables 7 and 8, respectively. In addition to changing the projection axis locations, in all design alternatives the axis scale was also changed so that the mean distortion was within ±10 ppm.

As shown in Table 7, the distortion standard deviation and range of the TM design alternatives both decrease as the projection axis (central meridian) location is moved east, with a minimum range of 49.5 ppm at λ_0 = 120° 40' W. However, the changes are generally modest, with no substantial improvement from the initial design.

In contrast, Table 8 shows that the change in distortion standard deviation and range of the LCC design alternatives is significant as the projection axis (central parallel) location is changed. The standard deviation and range decrease from ±24.6 and 68.3 from the initial design to a minimum of ±7.6 ppm and 19.4 ppm (for φ_0 = 44° 45' N).

Table 7. Distortion performance for six different TM projection alternatives (initial design values are *italicized*).

TM axis scale	*Initial 1.00014*	1.00013	1.00013	1.00012	1.00011	1.00010
TM axis longitude	*121°15'W*	121°10'W	121°00'W	120°45'W	120°40'W	120°35'W
Location	**Linear distortion for TM projection (parts per million)**					
Bend	*-29.6*	-38.2	-32.1	-24.7	-26.7	-27.7
Redmond	*0.4*	-10.1	-7.7	-6.0	-9.9	-12.7
Prineville	*19.1*	4.2	-2.3	-13.9	-22.2	-29.5
Sisters	*-1.9*	-7.5	4.7	21.1	22.1	24.1
Culver	*17.7*	7.8	11.1	14.3	10.8	8.4
Madras	*37.3*	26.3	27.5	27.3	22.8	19.3
Mean	*7.2*	-2.9	0.2	3.0	-0.5	-3.0
Range	*66.9*	64.5	59.6	52.1	49.5	53.6
Std deviation	*±23.0*	±21.6	±20.0	±20.9	±22.0	±23.5

Table 8. Distortion performance for six different LCC projection alternatives (initial design values are *italicized*; final values are **bold**).

LCC axis scale	*Initial 1.00014*	1.00013	1.00012	**Final 1.00012**	1.00011	1.00010
LCC axis latitude	*44°20'N*	44°30'N	44°35'N	**44°40'N**	44°45'N	44°50'N
Location	**Linear distortion (parts per million)**					
Bend	*-18.5*	-10.4	-8.2	**6.1**	12.4	20.9
Redmond	*0.5*	-2.2	-5.4	**3.5**	4.4	7.5
Prineville	*5.7*	1.7	-2.2	**6.0**	6.3	8.7
Sisters	*-8.6*	-12.3	-16.0	**-7.5**	-7.0	-4.4
Culver	*23.2*	7.7	-1.9	**0.6**	-4.8	-8.0
Madras	*49.9*	28.9	16.6	**16.4**	8.3	2.3
Mean	*8.7*	2.2	-2.9	**4.2**	3.3	4.5
Range	*68.3*	41.2	32.5	**23.9**	19.4	28.9
Std deviation	*±24.6*	±15.0	±10.8	**±7.8**	±7.6	±10.4

Although the standard deviation and range are minimum for φ_0 = 44° 45' N, the distortion was becoming excessive in the southern end of the design region, as exemplified by the distortion of +12.4 ppm in Bend. For this case, the central parallel is far enough north that distortion in the southern part of the design area was changing too rapidly with change in latitude. Because of this affect, as well as inspection of performance in other areas of the design region (as shown on the distortion maps), a design with φ_0 = 44° 40' N and k_0 = 1.00012 was selected for the final design (values in bold in Table 8). This design has distortion less than 10 ppm in Bend, Redmond, and Prineville, and variability is also less for these towns.

Evaluating distortion values at discrete points is typically not sufficient for optimizing an LDP design. A more comprehensive evaluation can be done by computing distortion on a regular grid. Distortion can then be visualized and analyzed everywhere, as shown in the map in Figure 6 for the final LDP design. The area with ±20 ppm distortion is also shown in Figure 7, for both the initial and final LCC designs. Note the improvement in low-distortion coverage gained by moving the central parallel north rather than leaving it at the center of the design area.

5. Keep the definition SIMPLE and CLEAN!

The LCC projection parameters affecting distortion were defined in the previous step and are given again in this step, along with the other needed parameters that do NOT affect linear distortion.

- LCC k_o defined to *exactly* FIVE decimal places: $\boldsymbol{k_0}$ **= 1.00012 (exact)**
- Both central parallel and central meridian are defined to nearest whole arc-minute. .

  $\boldsymbol{\varphi_0}$ **= 44° 40' 00" N = 31.666666666667°** and $\boldsymbol{\lambda_0}$ **= 121° 15' 00" W = −121.25°**

  The central meridian (λ_0) was selected as a clean value near the east-west center of the design area (has no effect on distortion).

  For an LCC projection, the latitude of grid origin must also be specified; it is the latitude where the false northing is defined (i.e., the northing on the central meridian at that latitude). It also has no effect on distortion, and it was set equal to the central parallel. This was done for simplicity and consistency, so that the LCC projection is defined with five parameters, same as the number of parameters used for a TM projection.
- Grid origin is defined using clean whole values with as few digits as possible:

  $\boldsymbol{N_0}$ **= 130,000.000 m** and $\boldsymbol{E_0}$ **= 80,000.000 m**

  Metric values were used to avoid confusion between international and US survey feet in the defined parameters (as also done in Oregon State Plane). These values were selected to keep grid coordinates positive but as small as possible throughout the design area (and also distinct from State Plane and UTM coordinates).

6. *Explicitly* define linear unit and geometric reference system (i.e., geodetic datum)

- Linear unit is the **meter**, and geometric reference system (geodetic datum) is **NAD 83**
  - Although the projection parameters are defined in meters, the output coordinates are typically provided in international feet, as is done for Oregon State Plane.
  - Note that the geometric reference system definition is NAD 83 without a realization ("datum tag") specified such as "2011", per the previous discussion on LDP design. Exactly the same approach is used for State Plane; it is always referenced to "generic" NAD 83. Only the coordinates themselves are referenced to a specific realization. But that has no effect on the projection or ellipsoid parameters; the ellipsoid parameters are the same for all realizations of NAD 83.
- The projection parameters, linear unit, and geodetic datum can be used directly to create a coordinate system definition that is compatible with most surveying, engineering, GIS, and other geospatial software. For example, this can be done for Esri software by creating a projection file (*.prj), or for Trimble software by using Coordinate System Manager to augment the coordinate system database file (*.csd).

The final design projection parameters are shown in Table 9, which are the values adopted for this as the Bend-Redmond-Prineville Zone of the Oregon Coordinate Reference System (OCRS); see Armstrong, et al. (2017) for more information.

Comparison to State Plane and "modified" State Plane
Table 9 also includes projection parameters for the State Plane Coordinate System of 1983, Oregon South Zone (SPCS 83 OR S), both as defined and "modified" (scaled "to ground") for Bend. SPCS 83 OR S was scaled by applying a scale factor of 1.000 160 760 so that distortion in Bend is the same as the OCRS. The modified SPCS 83 projection parameters were calculated from the scale factor, resulting in a "messy" definition (i.e., the false easting and scale have trailing digits after the decimal). This can make such systems problematic to use in geospatial software via formal projection definitions. For modified SPCS, the geodetic parameters (latitude and longitude) are unaffected; the central parallel latitude and scale for a two-parallel LCC is a computed value (implicitly defined by the two standard parallels).

The performance of "modified" State Plane systems is usually inferior to a carefully designed LDP. Table 10 gives distortion for the same six towns from the OCRS LDP designed in this example, and for SPCS 83 OR S, using both the original and "modified" definitions (given in Table 9). For modified SPCS 83, note that although the distortion in Bend is the same (+6 ppm), the distortion elsewhere is much greater, with a mean of +136 ppm (versus +4 ppm for the OCRS zone). Note also that both the original and modified version have the same distortion range and standard deviation (274 and ±97 ppm, respectively). Compare this to the much lower variability of the OCRS zone, with a range of 24 ppm and standard deviation of ±7 ppm.

Table 9. Comparison of OCRS, SPCS 83, and "modified" SPCS 83 projection parameters, for OCRS Bend-Redmond-Prineville Zone and SPCS 83 Oregon South Zone, and "equivalent" back-calculated modified SPCS 83 scaled to match OCRS distortion in Bend.

Lambert Conformal Conic projection parameters	OCRS Bend-Redmond-Prineville Zone	SPCS 83 Oregon South Zone	"Modified" SPCS 83 Oregon South Zone (for Bend)
Central standard parallel	44° 40' 00" N	43° 10' 06.91956…" N (computed)	43° 10' 06.91956…" N (computed)
North standard parallel	n/a	44° 00' 00" N	44° 00' 00" N
South standard parallel	n/a	42° 20' 00" N	42° 20' 00" N
Latitude of grid origin	44° 40' 00" N	41° 40' 00" N	41° 40' 00" N
Central meridian longitude	121° 15' 00" W	120° 30' 00" W	120° 30' 00" W
False northing	130,000 m (exact)	0 m (exact)	0 m (exact)
False easting	80,000 m (exact)	1,500,000 m (exact)	1,500,241.14 m (computed*)
Central parallel scale factor	1.00012 (exact)	0.999 894 607 592 09… (computed)	1.000 055 350 649 21… (computed*)

*Computed by applying 1.000 160 760 scale factor to original SPCS 83 central parallel scale.

Table 10. Comparison of distortion for OCRS, SPCS 83 OR S, and modified SPCS 83 OR S (modified such that distortion is same as OCRS in Bend).

Location	Linear distortion (parts per million)		
	OCRS Bend-Redmond-Prineville Zone	**SPCS 83 Oregon South Zone**	**"Modified" SPCS 83 Oregon South Zone**
Bend	6.1	-154.7	6.1
Redmond	3.5	-59.4	101.4
Prineville	6.0	-44.4	116.3
Sisters	-7.5	-62.0	98.8
Culver	0.6	53.8	214.6
Madras	16.4	119.1	279.9
Mean	**4.2**	**-24.6**	**136.2**
Range	**23.9**	**273.8**	**273.8**
Std deviation	**±7.8**	**±96.7**	**±96.7**

Despite the popularity of "modified" SPCS, the performance is almost always inferior to a carefully designed LDP. This is illustrated in the maps in Figure 8 for the SPCS 83 Oregon South Zone, for both original and "modified" by scaling "to ground" such that it gives the same distortion in Bend as the final LDP (+6 ppm). The difference in performance with the final LDP in Figure 7 is striking, even though both are based on the LCC projection. For both original and scaled SPCS, low distortion (±20 ppm) is only achieved in a narrow band more-or-less parallel to the projection axis (located 60 miles south of Bend). Scaling SPCS has essentially no effect on the width of the band; it is merely shifted so that it is centered on Bend. This is a vivid example of how changing the projection scale has virtually no impact on variability.

Compatibility of design with multiple software platforms

The projection parameters, linear unit, and geodetic datum can be used directly to create a coordinate system definition that is compatible with most surveying, engineering, GIS, and other geospatial software. For example, this can be done for Esri software by creating a projection file (*.prj), or for Trimble software by using Coordinate System Manager to augment the coordinate system database file (*.csd).

Computation of grid point scale factor and "ground" distances

Not all geospatial software computes the grid point scale factor, k, which is essential for computing total distortion. Equations to compute k for the TM and LCC projections are given in the next section.

There is often interest in assessing linear distortion by computing "ground" distances and comparing them to distances computed from projected coordinates. Two methods for computing such ground distances are also given in the section following the one giving equations for computing k. They are based on defining "ground distance" as the (curved) distance parallel to the ellipsoid at the mean ellipsoid height of the endpoints.

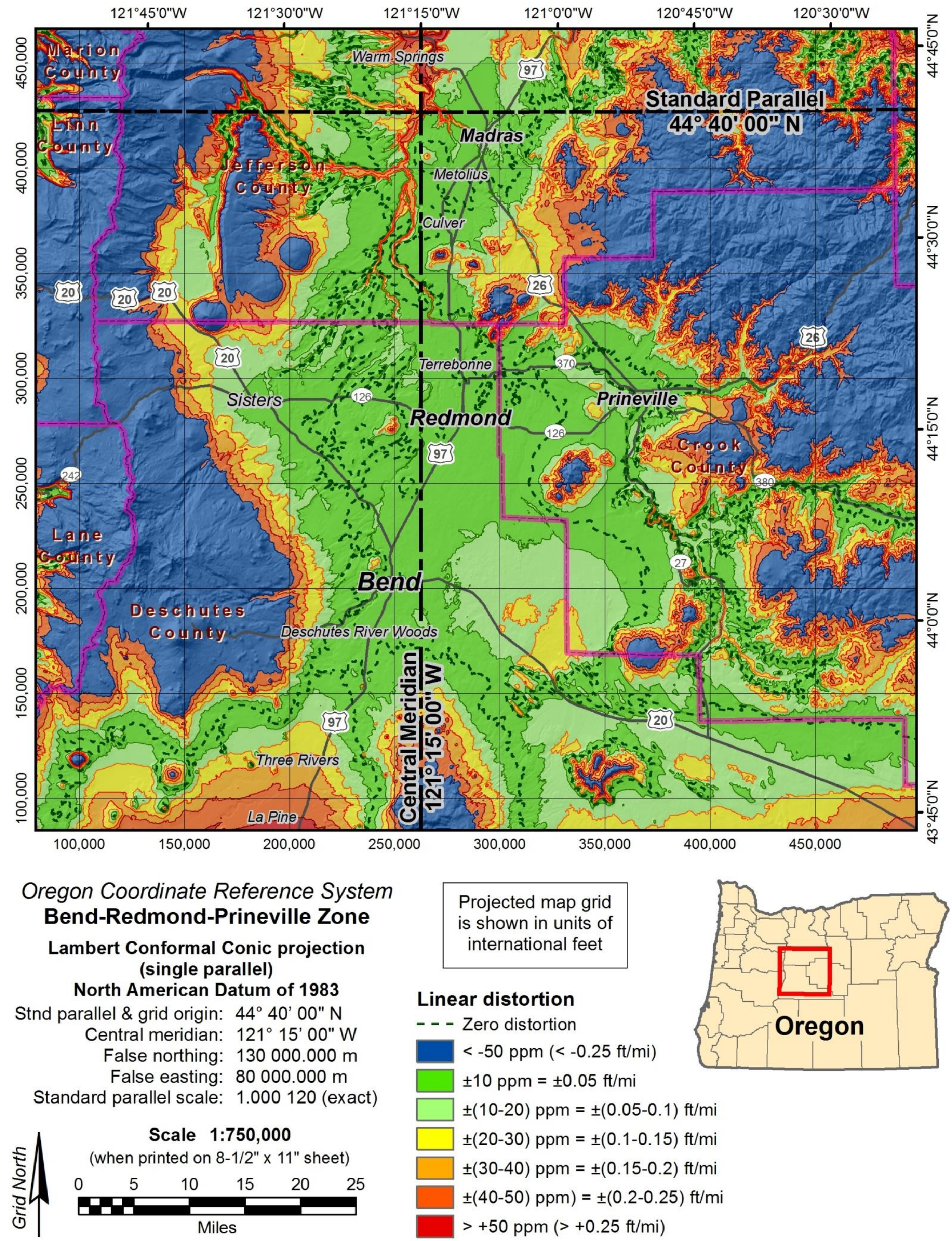


Figure 6. Linear distortion for OCRS Bend-Redmond-Prineville Zone.

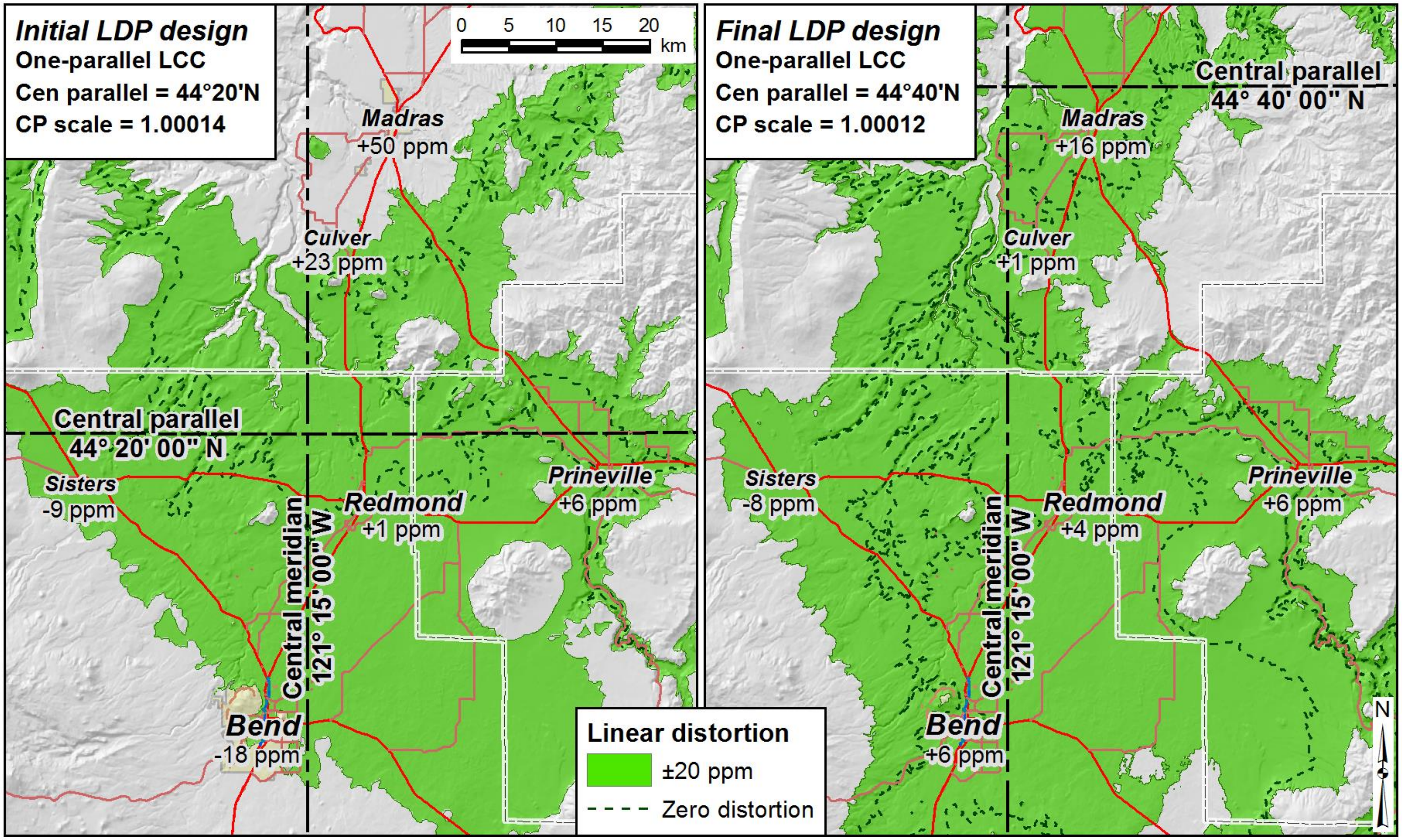


Figure 7. Areas with ±20 ppm distortion in example for initial and final LCC LDP designs.

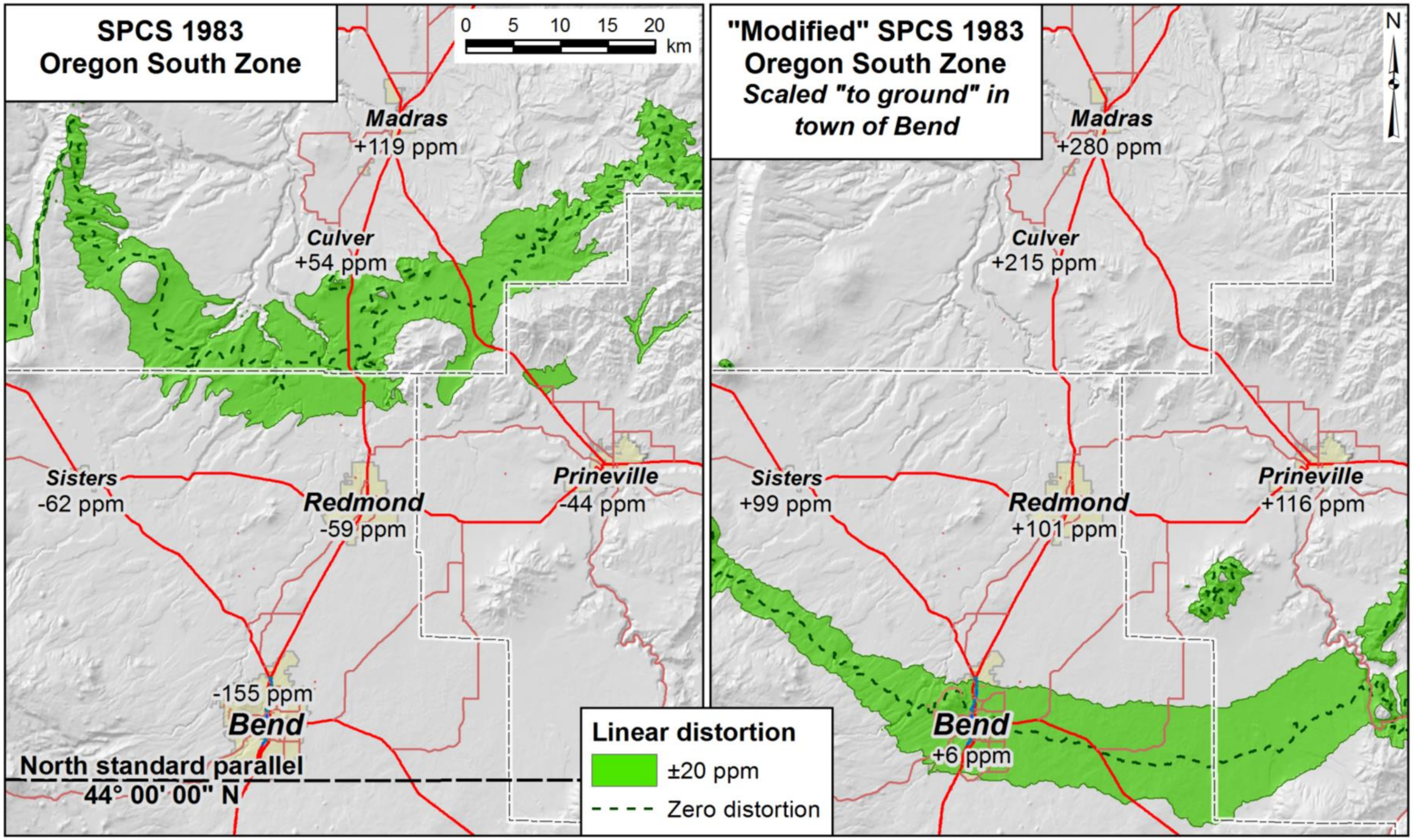


Figure 8. Areas with ±20 ppm distortion for original and "modified" SPCS 83 OR S Zone.

Projection grid scale factor and convergence angle computation

The projection grid point scale factor, k, is required to compute map projection distortion for a point on the ground. Because some surveying, engineering, and GIS software does not provide k, formulas for computing it are given below for the Transverse Mercator and Lambert Conformal Conic projections. These equations were derived from those provided in *NOAA Manual NOS NGS 5* "State Plane Coordinate System of 1983" by James Stem (1990). Equations for computing the convergence angle of these projections are also provided.

For the ***transverse Mercator*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 32-35):

$$k = k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \left[1 + \frac{(\Delta\lambda \cos\varphi)^2}{12} \left(5 - 4\tan^2\varphi + \frac{e^2 \cos^2\varphi}{1-e^2} \left(9 - 24\tan^2\varphi \right) \right) \right] \right\}$$

where $\Delta\lambda = \lambda_0 - \lambda$ (in radians, for negative west longitude)

λ = geodetic longitude of point

λ_0 = central meridian longitude

and all other variables are as defined previously.

The following shorter equation can be used to approximate k for the Transverse Mercator projection. It is accurate to better than 0.02 part per million (at least 7 decimal places) if the computation point is within about ±1° of the central meridian (about 50 to 60 miles between latitudes of 30° and 45°):

$$k \approx k_0 \left\{ 1 + \frac{(\Delta\lambda \cos\varphi)^2}{2} \left(1 + \frac{e^2 \cos^2\varphi}{1-e^2} \right) \right\}$$

Note that this equation may not be sufficiently accurate for computing k throughout a UTM system zone (at the zone width of ±3° from the central meridian the error can exceed 1 ppm).

An even simpler equation can be used to approximate the grid scale factor, which utilizes the grid coordinate easting value and is about twice as accurate as the previous equation (i.e., better than 0.01 part per million if the computation point is within about ±1° of the central meridian):

$$k \approx k_0 + \frac{(E_0 - E)^2}{2(k_0 R_G)^2}$$

where E = Easting of the point where k is computed (in same units as R_G)

E_0 = False easting (on central meridian) of projection definition (in same units as R_G)

R_G = Earth geometric mean radius of curvature

For the ***Lambert Conformal Conic*** projection, the grid scale factor at a point can be computed as follows (modified from Stem, 1990, pp. 26-29):

$$k = k_0 \frac{\cos\varphi_C}{\cos\varphi}\sqrt{\frac{1-e^2\sin^2\varphi}{1-e^2\sin^2\varphi_C}}\ \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\frac{1+\sin\varphi_C}{1-\sin\varphi_C}-\ln\frac{1+\sin\varphi}{1-\sin\varphi}+e\left(\ln\frac{1+e\sin\varphi}{1-e\sin\varphi}-\ln\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right)\right]\right\}$$

Where k_0 = projection grid scale factor applied to central parallel (tangent to ellipsoid if $k_0 = 1$)

φ_C = geodetic latitude of central parallel = standard parallel for one-parallel LCC

$e = \sqrt{e^2} = \sqrt{2f - f^2}$ = first eccentricity of the reference ellipsoid

and all other variables are as defined previously. To use this equation for a two-parallel LCC, the two-parallel LCC must first be converted to an equivalent one-parallel LCC by computing φ_C and k_0. The equations to do this are long, but are provided here for the sake of completeness. For a two-parallel LCC, the central parallel is

$$\varphi_C = \sin^{-1}\left[\frac{2\ln\left(\dfrac{\cos\varphi_S}{\cos\varphi_N}\sqrt{\dfrac{1-e^2\sin^2\varphi_N}{1-e^2\sin^2\varphi_S}}\right)}{\ln\left(\dfrac{1+\sin\varphi_N}{1-\sin\varphi_N}\right)-\ln\left(\dfrac{1+\sin\varphi_S}{1-\sin\varphi_S}\right)+e\left[\ln\left(\dfrac{1+e\sin\varphi_S}{1-e\sin\varphi_S}\right)-\ln\left(\dfrac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right)\right]}\right],$$

and the central parallel scale factor is

$$k_0 = \frac{\cos\varphi_N}{\cos\varphi_C}\sqrt{\frac{1-e^2\sin^2\varphi_0}{1-e^2\sin^2\varphi_N}}$$

$$\times\ \exp\left\{\frac{\sin\varphi_C}{2}\left[\ln\left(\frac{1+\sin\varphi_N}{1-\sin\varphi_N}\right)-\ln\left(\frac{1+\sin\varphi_C}{1-\sin\varphi_C}\right)+e\left(\ln\left[\frac{1+e\sin\varphi_C}{1-e\sin\varphi_C}\right]-\ln\left[\frac{1+e\sin\varphi_N}{1-e\sin\varphi_N}\right]\right)\right]\right\},$$

where φ_N and φ_S = geodetic latitude of northern and southern standard parallels, respectively, and all other variables are as defined previously.

Convergence angles. For the TM, the convergence angle can be approximated as $\gamma = -\Delta\lambda\sin\varphi$ (where all variables are as defined previously; the units of γ are the same as the units of $\Delta\lambda$). This equation is accurate to better than ±00.2” if the computation point is within ~1° of the central meridian. For any LCC, the convergence angle is exactly $\gamma = -\Delta\lambda\sin\varphi_C$.

Methods for computing horizontal "ground" distance

Two methods are given below for computing horizontal "ground" distances using geodetic information. The first method is done by scaling the ellipsoid distance (geodesic) using the average of the ellipsoid heights at the endpoints, as follows:

$$D_{grnd} = s\left(1 + \frac{\bar{h}}{\bar{R}_G}\right)$$

where s is the ellipsoid distance (geodesic)

$\bar{h}$ is the average ellipsoid height of the two points

$\bar{R}_G$ is the geometric mean radius of curvature at the midpoint latitude of the two points

The NGS Geodetic Tool Kit inversing tools can be used to compute the ellipsoid distance (geodesy.noaa.gov/TOOLS/Inv_Fwd/Inv_Fwd.html).

The second method for computing a horizontal ground distance can be done by using a GPS (GNSS) vector directly. Neglecting Earth curvature, this distance can be computed as:

$$D_{grnd} = \sqrt{\Delta X^2 + \Delta Y^2 + \Delta Z^2 - \Delta h^2}$$

where ΔX, ΔY, ΔZ are the GPS vector components, as Earth-Centered, Earth-Fixed (ECEF) Cartesian coordinate deltas

Δh = change in ellipsoid height between vector end points

Note that this method can also be used with end point coordinates (rather than a GPS vector), by converting the latitude, longitude, and ellipsoid heights to X, Y, Z ECEF coordinates, and then using the difference in ECEF coordinates. The NGS Geodetic Tool Kit *XYZ Conversion* tool can be used for this purpose (geodesy.noaa.gov/TOOLS/XYZ/xyz.shtml).

Curvature increases the horizontal ground distance, but for distances of less than 20 miles (about 30 km), the error due to the increase is less than 1 part per million (ppm), i.e., less than 0.1 ft (3 cm). The straight-line horizontal distance can be multiplied by the following curvature correction factor to get the approximate curved horizontal ground distance:

$$C_C = \frac{2\bar{R}_G \sin^{-1}\left(\dfrac{D_{grnd}}{2\bar{R}_G}\right)}{D_{grnd}}$$

where all variables are as defined previously. With the curvature correction, for distances of less than 100 miles (160 km) the error is less the 0.005 ppm, i.e., less than 0.003 ft (1 mm). The mean Earth radius of curvature can be computed, or it can be estimated from Table 5.

Surveying & mapping spatial data requirements & recommendations

These should be explicitly specified in surveying and mapping projects

1. Completely define the coordinate system

a. Linear unit (e.g., international foot, U.S. survey foot, meter)
   i. Use same linear unit for horizontal and vertical coordinates
b. Geodetic datum (recommend North American Datum of 1983)
   i. Should include "datum tag", e.g., 1986, 1991, 1998, 2007, 2011, as necessary, as well as epoch date for modern high-accuracy positions, e.g., 2010.00
   ii. WGS 84, ITRF/IGS, and NAD 27 are usually **NOT** recommended
c. Vertical datum (e.g., North American Vertical Datum of 1988)
   i. If GPS used for elevations, recommend using a modern geoid model (e.g., GEOID12B)
   ii. Recommend using NAVD 88 rather than NGVD 29 when possible
d. Map projection type and parameters (e.g., Transverse Mercator, Lambert Conformal Conic)
   i. Special attention required for low-distortion grid (a.k.a. "ground") coordinate systems
      1) Avoid scaling of existing coordinate systems (e.g., "modified" State Plane)

2. Require *direct* referencing of the NSRS (National Spatial Reference System)

a. Ties to published control strongly recommended (e.g., National Geodetic Survey control)
   i. Relevant component of control must have greater accuracy than positioning method used
      1) E.g., network accuracies that meet project needs, 2nd order (or better) for vertical control
b. NGS Continuously Operating Reference Stations (CORS) can be used to reference the NSRS
   i. Free Internet GPS post-processing service: OPUS (Online Positioning User Service)

3. Specify *accuracy* requirements (*not* precision)

a. Use objective, defensible, and robust methods (published ones are recommended)
   i. Mapping and surveying: National Standard for Spatial Data Accuracy (NSSDA)
      1) Require occupations ("check shots") of known high-quality control stations
   ii. Surveys performed for establishing control or determining property boundaries:
      1) Appropriately constrained and over-determined least-squares adjusted control network
      2) Beware of "cheating" (e.g., using "trivial" GPS vectors in network adjustment)

4. Documentation is *essential* (metadata!)

a. Require a report detailing methods, procedures, and results for developing final deliverables
   i. This must include any and all post-survey coordinate transformations
      1) E.g., published datum transformations, computed correction surfaces, "rubber sheeting"
b. Documentation should be complete enough that someone else can reproduce the product
c. For GIS data, recommend that accuracy and coordinate system information be included as feature attributes (not just as separate, easy-to-lose and easy-to-ignore metadata files)

Example of surveying and mapping documentation (*metadata*)

Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geometric reference frame: North American Datum of 1983 (2011) epoch 2010.00

Vertical datum: North American Vertical Datum of 1988 (see below)

System: Oregon Coordinate Reference System

Zone: Bend-Redmond-Prineville

Projection: Lambert Conformal Conic (one-parallel)
Standard parallel and latitude of grid origin: 44° 40' 00" N
Longitude of central meridian: 121° 15' 00" W
Northing at grid origin: 130,000.000 m (~426,509.18635 ift)
Easting at central meridian: 80,000.000 m (~262,467.19160 ift)
Scale factor on central meridian: 1.000 12 (exact)

All distances and bearings shown hereon are projected (grid) values based on the preceding projection definition. The projection was defined to minimize the difference between projected (grid) distances and horizontal ("ground") distances at the topographic surface within the design area of this coordinate system.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Orthometric heights (elevations) were transferred to the site from NGS control station "C 30" (PID QD0823) using GNSS with NGS geoid model "GEOID12B" referenced to the current published 1st order NAVD 88 height of this station (1049.170 m).

The survey was conducted using GNSS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Accuracy estimates are at the 95% confidence level and are based on an appropriately constrained and weighted least-squares adjustment of redundant observations.

Point #1, NGS control station C 30 (PID QD0823), constrained (off site)		**Estimated accuracy**
Latitude = 44° 06' 53.98076'' N	Northing = 225,363.515 ift	Horiz = ±0.024 ift
Longitude = 121° 17' 27.31006'' W	Easting = 251,718.529 ift	Ellipsoid ht = ±0.076 ift
Ellipsoid height = 3372.940 ift	Elevation = 3442.159 ift	Elevation FIXED

Point #1002, 1/2" rebar with aluminum cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 31.96763'' N	Northing = 223,132.860 ift	Horiz = ±0.034 ift
Longitude = 121° 16' 51.33054'' W	Easting = 254,342.973 ift	Ellipsoid ht = ±0.086 ift
Ellipsoid height = 3395.610 ift	Elevation =3464.760 ift	Elevation = ±0.094 ift

Point #1006, 1/2" rebar with plastic cap, derived coordinates		**Estimated accuracy**
Latitude = 44° 06' 28.79196'' N	Northing = 222,811.061 ift	Horiz = ±0.047 ift
Longitude = 121° 16' 45.17852'' W	Easting = 254,791.795 ift	Ellipsoid ht = ±0.088 ift
Ellipsoid height = 3391.047 ift	Elevation = 3460.184 ift	Elevation = ±0.097 ift

Selected References

National Geodetic Survey (geodesy.noaa.gov) selected web pages

State Plane Coordinate System: geodesy.noaa.gov/SPCS/

NGS Coordinate Conversion and Transformation Tool (NAT): geodesy.noaa.gov/NCAT/

Control station datasheets: geodesy.noaa.gov/datasheets/

The Geodetic Tool Kit: geodesy.noaa.gov/TOOLS/

Online Positioning User Service (OPUS): geodesy.noaa.gov/OPUS/

Continuously Operating Reference Stations (CORS): geodesy.noaa.gov/CORS/

The NGS Geoid Page: geodesy.noaa.gov/GEOID/

NGS State Geodetic Advisors: geodesy.noaa.gov/ADVISORS/

Documents on map projections and the State Plane Coordinate System

Armstrong, M.L., Thomas, J., Bays, K., and Dennis, M.L., 2017. *Oregon Coordinate Reference System Handbook and Map Set*, version 3.01, Oregon Department of Transportation, Geometronics Unit, Salem, Oregon, <ftp://ftp.odot.state.or.us/ORGN/Documents/ocrs_handbook_user_guide.pdf>

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NGA (National Geospatial-Intelligence Agency), 2014b. *Department of Defense World Geodetic System of 1984: Its Definition and Relationships with Local Geodetic Systems*, version 1.0.0, NGA.STND.0036_1.0.0_WGS8 (National Geospatial-Intelligence Agency Standardization

Document), 2017 pp. <earth-info.nga.mil/GandG/publications/NGA_STND_0036_1_0_0_WGS84/NGA.STND.0036_1.0.0_WGS84.pdf>

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