48th Annual Alaska Surveying & Mapping Conference

GPS, Geodesy, and the Ghost in the Machine

A Workshop for Surveyors and GIS Professionals

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NOAA's National Geodetic Survey

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WORKSHOP ABSTRACT

Positions determined using GPS and GNSS equipment are determined using complex algorithms that are often hidden within proprietary software — the "ghost in the machine". Users unfamiliar with the computational process can unwittingly generate positional errors ranging from centimeters to kilometers. This workshop seeks to shed light on the GPS "black box" by 1) Explaining the main geodetic principles and terminology behind GPS; 2) Reducing blind reliance on GPS and GIS software; and 3) Providing practical information and tools for the GPS user. Topics include geodetic and vertical datums, map projections, "ground" coordinate systems, the geoid and gravity, accuracy assessment, GIS and survey data compatibility, and documentation (metadata). National Geodetic Survey (NGS) products and services are used along with numerous examples of positioning errors to illustrate the peril of neglecting geodetic principles, with particular emphasis on the situation in Alaska. A workbook is provided that includes step-by-step GPS and geodetic computations.

ACKNOWLEDGEMENTS

The creation of this workbook would not have been possible without the support and inspiration provided by Gabriel Bey and Rick Bunger. Special thanks also go to Dru Smith (Chief Geodesist of the National Geodetic Survey), Dave Minkel (NGS Arizona Geodetic Advisor), and to all members of the Geospatial Committee of the Arizona Professional Land Surveyors Association.

Today, GPS has thrust surveyors into the thick of geodesy which is no longer the exclusive realm of distant experts. Thankfully, in the age of microcomputers, the computational drudgery can be handled with software packages. Nevertheless, it is unwise to venture into GPS believing that knowledge of the basics of geodesy is, therefore, unnecessary. It is true that GPS would be impossible without computers, but blind reliance on the data they generate eventually leads to disaster.

Jan Van Sickle (2001, p. 126)

Note: This workbook is intended to accompany a presentation. Therefore some of the material in this workbook may appear incomplete or be unclear if it is used without attending the presentation.

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LIST OF EQUATIONS

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LIST OF SYMBOLS

a	Semi-major axis of geodetic ellipsoid; *also* semi-major axis of error ellipse
A	Astronomic azimuth
b	Semi-minor axis of geodetic ellipsoid, $b = a\left(1 - f\right)$; *also* semi-minor axis of error ellipse
c_X^n	Value used to scale an *n*-dimensional standard error to a confidence level of *X*%
C	Geopotential number
C_C	Horizontal curvature correction factor (multiplied with straight horizontal distance)
C_{CR}	Vertical correction for curvature and refraction (added to vertical distance)
CEP	Circular Error Probable
DC_{AB}	Dynamic correction applied to leveled height difference from point *A* to *B*
D_{grnd}	Horizontal ground distance (parallel to ellipsoid)
D_S	Slope distance
e	First eccentricity of geodetic ellipsoid, $e = \sqrt{e^2} = \sqrt{2f - f^2}$
e^2	First eccentricity squared of geodetic ellipsoid, $e^2 = 2f - f^2$
E	Easting coordinate (in the *x*-direction)
E_{95}^N	Error (accuracy) north component at accuracy at 95% confidence
E_{95}^E	Error (accuracy) east component at accuracy at 95% confidence
E_{95}^h	Error (accuracy) ellipsoid height component at accuracy at 95% confidence

E_0	False easting (on central meridian) of map projection definition
f	Geometric flattening of geodetic ellipsoid
g	Gravity at the topographic surface of the Earth
$\overline{g}$	Mean gravity on the plumbline between the topographic surface and the geoid
h	Ellipsoid height
H	Orthometric height ("elevation")
H^D	Dynamic height
i	Instrument or GPS base station antenna height above station
K	Constant for computing Helmert mean gravity, $K = 2{,}358{,}000 \text{ s}^2 = 1 / (4.24 \times 10^{-7} \text{ s}^{-2}) = 1 / (\frac{1}{2} F - 2\pi G\rho)$, where F is the vertical gradient of gravity, G is the universal gravitational constant and ρ is the topographic density (assumed constant 2670 kg/m^3)
k	Conformal map projection grid scale factor
k_o	Grid scale factor on the central meridian for the Transverse Mercator projection (or on the central parallel for the Lambert Conformal Conic projection)
L	Laplace correction
N	Northing coordinate (in the y-direction)
N_0	False northing (where central meridian crosses latitude of grid origin) of map projection
N_G	Geoid height = geoid separation = geoid undulation
Δn_{AB}	Leveled height difference observed from point A to B
OC_{AB}	Orthometric correction applied to leveled height difference from point A to B
r	Prism rod or rover antenna height above station
R_G	Geometric mean radius of curvature of geodetic ellipsoid
R_M	Radius of curvature in the meridian of geodetic ellipsoid
R_N	Radius of curvature in the prime vertical of geodetic ellipsoid
R_α	Radius of curvature of geodetic ellipsoid in a specific azimuth, α
s	Geodesic distance ("horizontal" distance on the ellipsoid)
SEP	Spherical Error Probable
t	Grid azimuth
$(t - T)$	Arc-to-chord ("second term") correction for converting grid to geodetic azimuths
X	Earth-Centered, Earth-Fixed Cartesian coordinate in the X-direction (in equatorial plane and passing through Prime Meridian, i.e., 0° longitude)
Y	Earth-Centered, Earth-Fixed Cartesian coordinate in the Y-direction (in equatorial plane and perpendicular to X-axis, i.e., passing through 90°E longitude)
Z	Earth-Centered, Earth-Fixed Cartesian coordinate in the Z-direction (parallel to Earth's conventional spin axis and perpendicular to equatorial plane)

α_{AB}	Geodetic azimuth from point *A* to point *B*
$\tilde{\alpha}_{AB}$	Approximate geodetic azimuth from point *A* to point *B*
γ	Convergence angle
γ_0	Normal gravity on the GRS 80 ellipsoid at 45° latitude
δ	Map projection distortion
Δ	Denotes discrete change in a quantity, usually as final value minus initial value (e.g., for change in northing coordinate, $\Delta N = N_2 - N_1$)
$\Delta\lambda''$	Change in longitude in arc-seconds
$\Delta\varphi''$	Change in latitude in arc-seconds
ζ	Geodetic zenith angle
η	East-west component of the deflection of the vertical (in the prime vertical plane)
θ	Horizontal error ellipse rotation angle
λ	Geodetic longitude
λ_0	Longitude of central meridian for map projection
ν	Zenith angle
ξ	North-south component of the deflection of the vertical (in the meridian plane)
π	Irrational number pi (ratio of circle circumference to diameter)
ρ	Horizontal correlation
σ_E	Standard error (east component)
σ_N	Standard error (north component)
σ_{NE}	Horizontal covariance
φ	Geodetic latitude (on ellipsoid or sphere)
φ_0	Latitude of grid origin for map projection; central parallel for conical map projection
φ_{N}	Latitude of north standard parallel for conical map projection
φ_{S}	Latitude of south standard parallel for conical map projection
ψ	Angle between two points on a sphere with vertex at center of sphere

TABLE OF USEFUL NUMERICAL VALUES

Symbol	Description	Numerical values (exact values shown in **BOLD**)
a	GRS-80 ellipsoid semi-major axis (identical to WGS-84 value)	**6,378,137 m** = 20,925,646.325 459 ift = 20,925,604.474 167 sft
f	GRS-80 geometrical flattening WGS-84 geometrical flattening	1 / 298.257 222 101 1 / 298.257 223 563
b	GRS-80 ellipsoid semi-minor axis WGS-84 ellipsoid semi-minor axis	6,356,752.314 140 m = 20,855,486.594 949 ift = 20,855,444.883 876 sft 6,356,752.314 245 m = 20,855,486.595 293 ift = 20,855,444.884 319 sft
e^2	GRS-80 first eccentricity squared WGS-84 first eccentricity squared	0.006 694 380 022 901 0.006 694 379 990 141
ift	International Foot	**1 ift ≡ 0.3048 m** (2 ppm shorter than sft)
sft	US Survey Foot	**1 sft ≡ 1200/3937 m** (2 ppm longer than ift)
ppm	Parts per million	Value multiplied by one million (analogous to "percent" which is "parts per hundred")
rad	Radian (angular measure)	**1 rad = 180° / π** (i.e., 1 rad ≈ 57.295 779 513°)
π	Pi (irrational number)	π = 3.141 592 653 589 793 238 462 643 383…
γ_0	Normal gravity on the GRS 80 ellipsoid at 45° latitude	9.806199 m/s^2 32.172569 ift/s^2 = 32.172505 sft/s^2

Section 1

GPS, GEODESY, AND THE PERILS OF MODERN POSITIONING

Exercise 1.1: Computation of coordinates from total station data

Total stations determine three-dimensional coordinates by measuring three quantities: 1) *slope distance*, 2) *horizontal angle*, and 3) *zenith angle*.

Grid coordinates (northing and easting) and elevation can be computed from a total station using the following formulas (designated as Equation 1.1):

Equation 1.1 Computation of grid coordinates from total station data

$$\boxed{\begin{aligned} N &= N_0 + D_S \cos\alpha \sin\nu \\ E &= E_0 + D_S \sin\alpha \sin\nu \\ H &= H_0 + D_S \cos\nu + i - r \end{aligned}}$$

where N, E, and H are the northing, easting, and height (elevation) coordinates to be determined

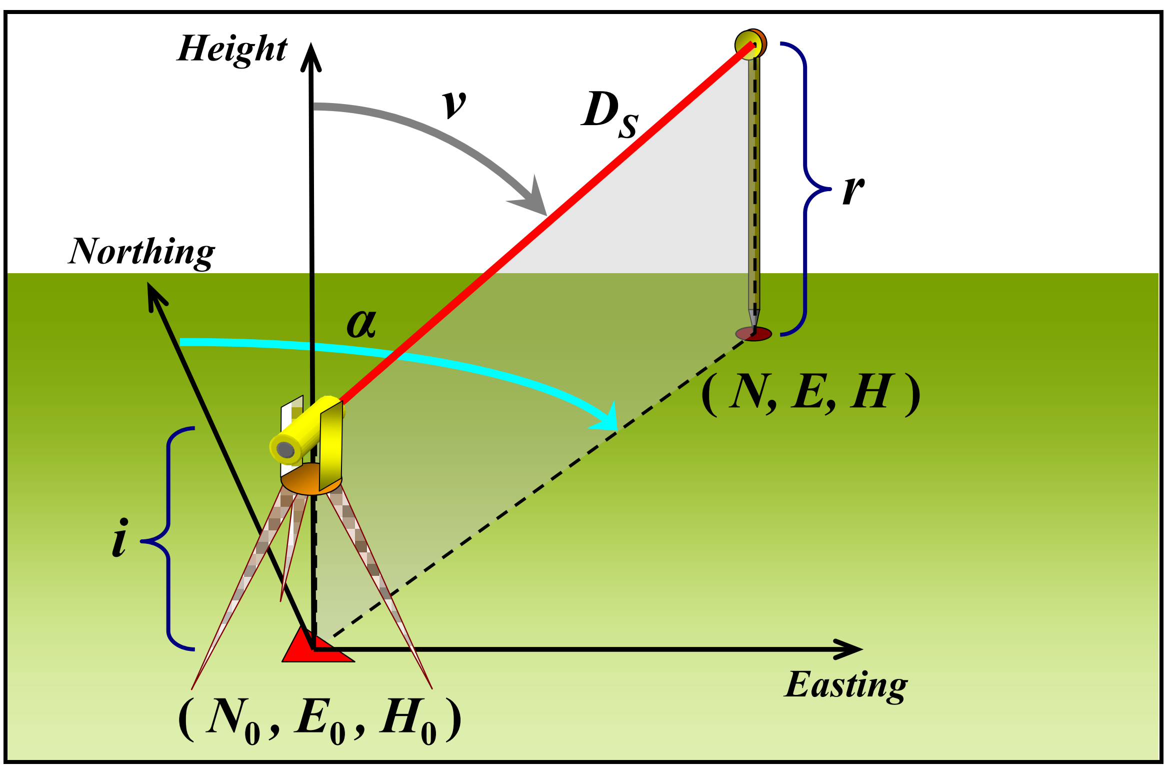
N_0, E_0, and H_0 are the northing, easting, and height of the instrument setup point

D_S is the observed slope distance

α is the observed horizontal angle (azimuth)

ν is the observed zenith angle

i and r are the instrument and the prism rod heights, respectively.



Example computation

Given: A total station set up with i = 5.32 ft over starting point with N_0 = 5000.00 sft, E_0 = 5000.00 sft, and H_0 = 100.00 sft. The horizontal circle is set so that it reads azimuth directly, and the following observations are made to a point with prism rod of height r = 6.56 ft:

D_S = 336.84 sft $\qquad$ α = 152°17'23" $\qquad$ v = 83°48'50"

Find: The coordinates and elevation of the observed point.

Computations:

$N = \quad N_0 \quad + \quad D_S \quad \times \quad \cos\alpha \quad \times \quad \sin v$

N = ____________ + ______________ × cos(________________) × sin(________________)

N = ____________ + ______________ × ____________________ × ____________________

***N* =** ________________

$E = \quad E_0 \quad + \quad D_S \quad \times \quad \sin\alpha \quad \times \quad \sin v$

E = ____________ + ______________ × sin(________________) × sin(________________)

E = ____________ + ______________ × ____________________ × ____________________

***E* =** ________________

$H = \quad H_0 \quad + \quad D_S \quad \times \quad \cos v \quad + \quad i \quad - \quad r$

H = ____________ + ______________ × cos(________________) + __________ − __________

H = ____________ + ______________ × ____________________ + __________ − __________

***H* =** ________________

Solution:

N = 5000.00 + 336.84 × cos(152°17'23") × sin(83°48'50")

N = 5000.00 + 336.84 × (−0.88531023) × 0.99417711

***N* = 4703.53 sft**

E = 5000.00 + 336.84 × sin(152°17'23") × sin(83°48'50")

E = 5000.00 + 336.84 × 0.46500086 × 0.99417711

***E* = 5155.72 sft**

H = 100.00 + 336.84 × cos(83°48'50") + 5.32 − 6.56

H = 100.00 + 336.84 × 0.10775836 + 5.32 − 6.56

***H* = 135.06 sft**

GPS: A geodetic tool

A comparison between total stations and GPS

Both GPS and total stations determine three-dimensional coordinates, but they differ in virtually every other respect, to wit:

- **Observations**
  - Total stations directly observe slope distance[1], horizontal angle, and zenith angle
    - Total station EDM sends and receives signal that it uses for computing distance
  - GPS receivers observe the carrier phase (fractional wavelength), and Doppler shift of the signals transmitted from the satellites, as well as the navigation signals encoded on the carrier wave
    - GPS only receives signals from the satellites (a one-way ranging system)
    - Based on the navigation signal (encoded on the carrier wave sent by the GPS satellites) and its own internal clock, a GPS receiver computes the pseudorange to each satellite
- **Measurements**
  - The vector components from a total station to the prism are directly measured
    - Total station measures both distance[1] and angles
  - The vector components between GPS antennas are computed, NOT observed
    - This has implications for error propagation and control network design
    - GPS does NOT measure angles
- **Computations**
  - Coordinates can be determined from total station observations using simple plane trigonometry
  - Geodetic methods MUST be used to compute coordinates from GPS vectors
- **Reference frame**
  - Total stations are referenced to the gravity vector (plumbline) passing through the vertical axis of the instrument
  - GPS is referenced to a world-wide coordinate system (in common with the satellites) with its origin located at the Earth's center of the mass

Geodesy: The science of positioning

Geodesy is a quantitative scientific field dealing with the size and shape of the Earth (or other planetary bodies), precise determination of coordinates and relationship between coordinates on the Earth, and includes study of the Earth's gravity field. Basically, it is the science behind surveying, mapping, and navigation, and it is essential for using GPS.

The bottom line: GPS is a geodetic tool that *requires* geodesy to perform computations and it is explicitly referenced to the *entire* Earth.

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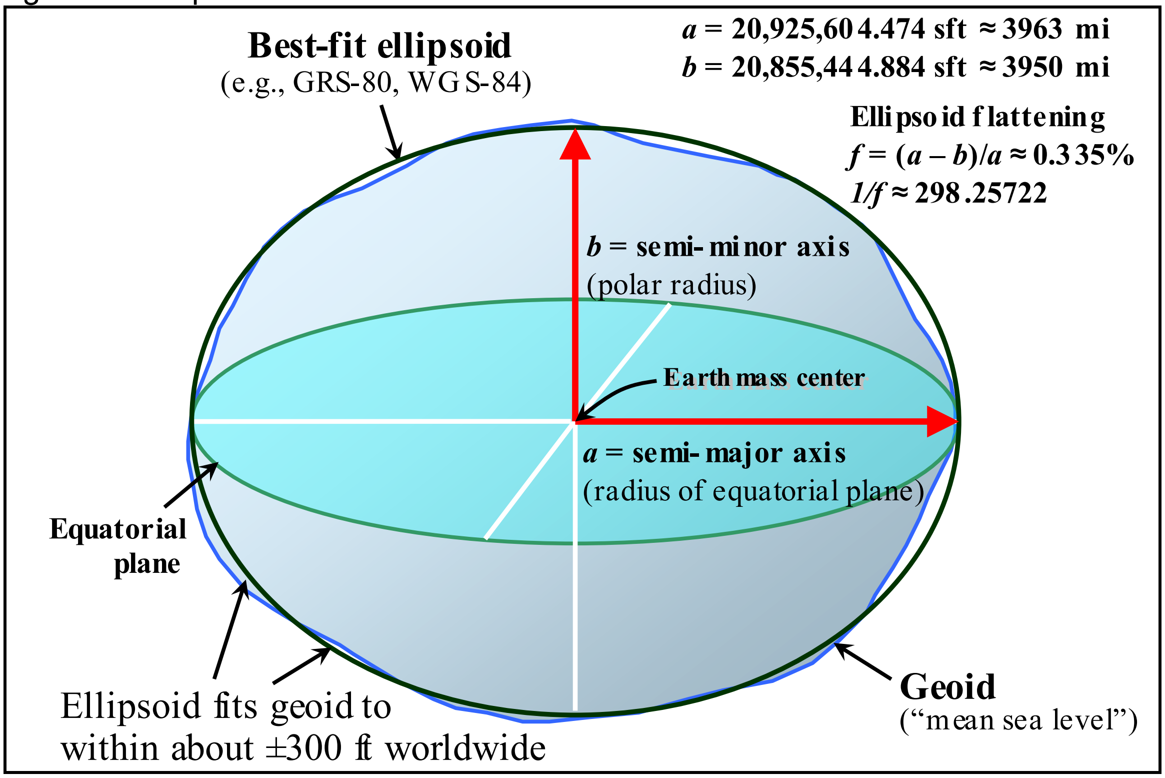
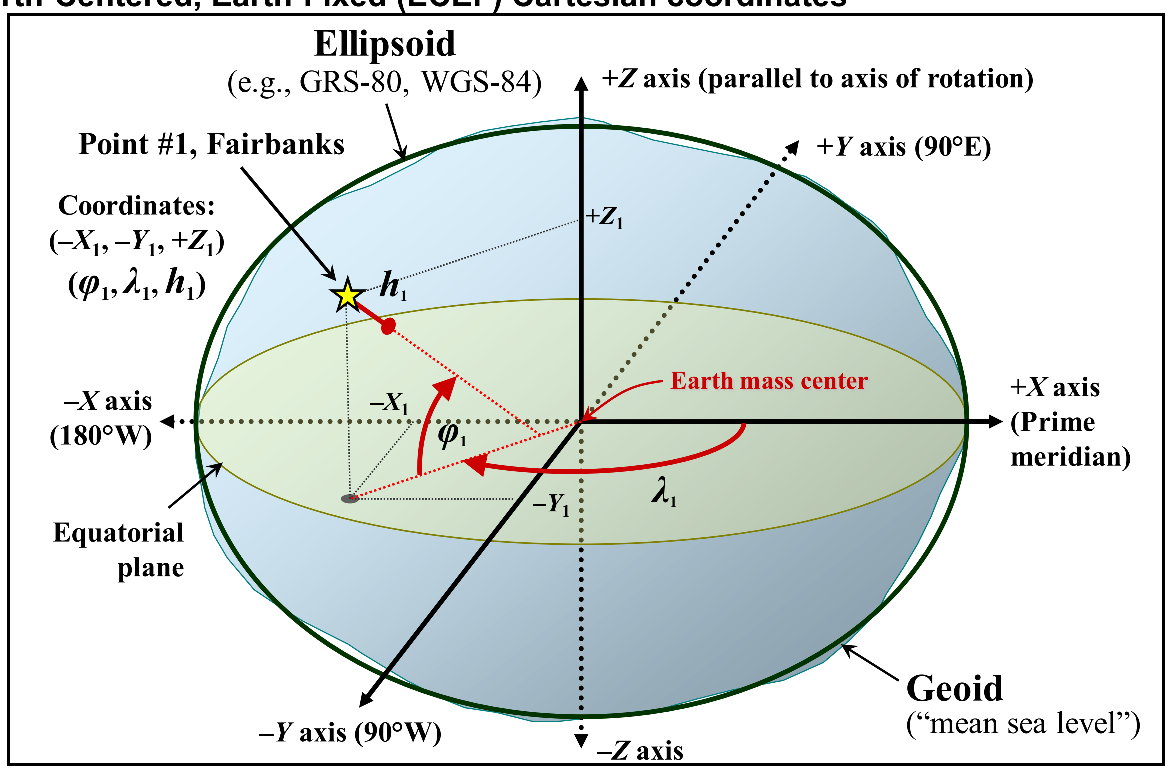
[1] Technically, an EDM does not "observe" distance, but rather it observes the (atmospherically-impacted) time difference between a sent and received signal, and computes the distance.

The geodetic ellipsoid of revolution



Earth-Centered, Earth-Fixed (ECEF) Cartesian coordinates



Exercise 1.2: Geodetic ellipsoid parameters and computations

The shape of the geodetic ellipsoid of revolution is completely defined by two numbers. By convention, these are usually a, the semi-major axis, and $1/f$, the inverse geometric flattening. These can be used to compute other commonly used ellipsoid parameters, such as the following two:

Equation 1.2 Ellipsoid semi-minor axis

$$\boxed{b = a\left(1 - f\right)}$$

Equation 1.3 Ellipsoid first eccentricity squared

$$\boxed{e^2 = 2f - f^2}$$

Example computations

Given: The following parameters for the GRS-80, WGS-84, and Clarke 1866 ellipsoids:

Ellipsoid	GRS-80	WGS-84	Clarke 1866
Semi-major axis, *a*	6,378,137 m (exact)	6,378,137 m (exact)	6,378,206.4 m (exact)
Inverse flattening, 1/*f*	298.257 222 101	298.257 223 563	294.978 698 214

Find: The semi-minor axis (in US Survey Feet) of these ellipsoids.

Computations:

Semi-minor axis = $a \quad \times \quad \left(1 - f\right) \quad \times$ unit conversion

GRS-80: $b = \underline{\qquad\qquad\qquad} \times \left[1 - \left(\dfrac{1}{\underline{\qquad\qquad\qquad}}\right)\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right)$

$b = \underline{\qquad\qquad\qquad\qquad}$ **sft**

WGS-84: $b = \underline{\qquad\qquad\qquad} \times \left[1 - \left(\dfrac{1}{\underline{\qquad\qquad\qquad}}\right)\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right)$

$b = \underline{\qquad\qquad\qquad\qquad}$ **sft**

Clarke 1866: $b = \underline{\qquad\qquad\qquad} \times \left[1 - \left(\dfrac{1}{\underline{\qquad\qquad\qquad}}\right)\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right)$

$b = \underline{\qquad\qquad\qquad\qquad}$ **sft**

Solution:

GRS-80: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257222101}\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right) =$ **20,855,444.8840 sft**

WGS-84: $b = 6{,}378{,}137 \text{ m} \times \left[1 - \dfrac{1}{298.257223563}\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right) =$ **20,855,444.8843 sft**

Clarke 1866: $b = 6{,}378{,}206.4 \text{ m} \times \left[1 - \dfrac{1}{294.978698214}\right] \times \left(\dfrac{3937 \text{ sft}}{1200 \text{ m}}\right) =$ **20,854,892.0172 sft**

Exercise 1.3: Computation various ellipsoidal radii of curvature

Because the ellipsoid of revolution is non-spherical, its surface curvature (measured in a plane intersecting the ellipsoid) at any given point, depends on the orientation of that plane to the ellipsoid. The radii of curvature at a point on (but not above or below) the ellipsoid in the *meridian* (north-south) and *prime vertical* (east-west) are frequently used in geodesy:

Equation 1.4 Meridian radius (north-south)

$$\boxed{R_M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\varphi\right)^{3/2}}}$$

Equation 1.5 Prime vertical radius (east-west)

$$\boxed{R_N = \frac{a}{\sqrt{1-e^2\sin^2\varphi}}}$$

where φ is the geodetic latitude at the point where the radius is computed.

a is the ellipsoid semi-major axis (= 20,925,604.4742 sft for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

R_M and R_N are used to compute other commonly used ellipsoidal radii, such as the following two:

Equation 1.6 Radius of curvature in a specific azimuth, α

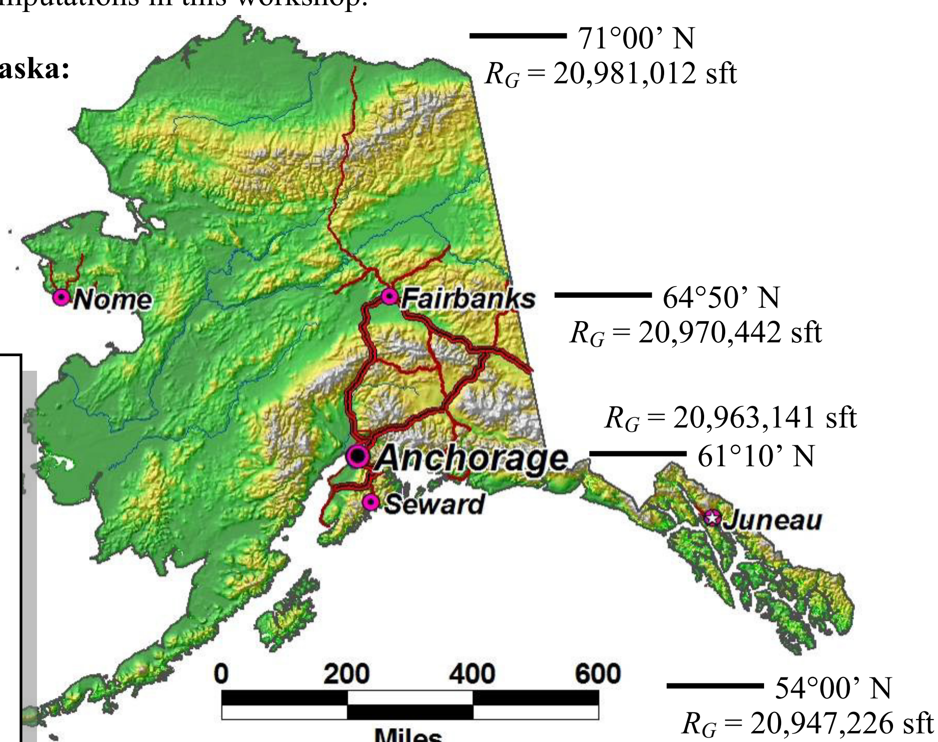
$$\boxed{R_\alpha = \frac{R_M\ R_N}{R_M\sin^2\alpha + R_N\cos^2\alpha}}$$

Equation 1.7 Geometric mean radius of curvature

$$\boxed{R_G = \sqrt{R_M\ R_N} = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}}$$

R_G is essentially the "average" radius of curvature at a point on the ellipsoid, and is the one that will be used for radius computations in this workshop.

Some R_G values for Alaska:



Rules of thumb:
Geometric mean radius of curvature *increases* northward:

- **By ~22 ft per mile** (at latitude of 70°)
- **By ~30 ft per mile** (at latitude of 60°)
- **By ~35 ft per mile** (at latitude of 50°)

Example computation

Given: A point at latitude φ = 61°13’49.09296”N (midway between points ZAN A and ZAN B).

Find: The radii of curvature in the meridian, prime vertical, at an (approximate) azimuth of α = 109°23’49” (from ZAN A to ZAN B), and the geometric mean radius (for the GRS-80 ellipsoid).

Computations: First convert latitude and azimuth to decimal degrees:

$$\varphi = 61 + 13/60 + 49.09296/3600 = 61.2303036000°$$

$$\alpha = 109 + 23/60 + 49/3600 = 109.39693°$$

Now compute following function of latitude (since it appears in most of the equations):

$$1 - e^2 \sin^2\varphi = 1 - 0.006694380023 \times [\sin(61.23030360000°)]^2 = 0.99485630743$$

Now compute the various radii:

$$R_M = \frac{\underline{\hspace{4cm}} \times \left(1 - \underline{\hspace{3.5cm}}\right)}{\left(\underline{\hspace{3.5cm}}\right)^{3/2}} = \underline{\hspace{4cm}}$$

$$R_N = \frac{\underline{\hspace{5cm}}}{\sqrt{\underline{\hspace{3.5cm}}}} = \underline{\hspace{4cm}}$$

$$R_\alpha = \frac{\underline{\hspace{3.5cm}} \times \underline{\hspace{3.5cm}}}{\underline{\hspace{3.5cm}} \times \left[\sin(\underline{\hspace{2cm}})\right]^2 + \underline{\hspace{3.5cm}} \times \left[\cos(\underline{\hspace{2cm}})\right]^2}$$

$$= \underline{\hspace{4cm}}$$

$$R_G = \frac{\underline{\hspace{3.5cm}} \times \sqrt{1 - \underline{\hspace{3.5cm}}}}{\underline{\hspace{4.5cm}}} = \underline{\hspace{4cm}}$$

Solution:

$$R_M = \frac{20{,}925{,}604.4742 \times (1 - 0.0066943800229)}{(0.99485630743)^{3/2}} = \mathbf{\underline{20{,}946{,}929.368\ sft}}$$

$$R_N = \frac{20{,}925{,}604.4742}{\sqrt{0.99485630743}} = \mathbf{\underline{20{,}979{,}630.421\ sft}}$$

$$R_\alpha = \frac{20{,}946{,}929.368 \times 20{,}979{,}630.421}{20{,}946{,}929.368 \times [\sin(109.39693°)]^2 + 20{,}979{,}630.421 \times [\cos(109.39693°)]^2}$$

$$= \mathbf{\underline{20{,}976{,}018.570\ sft}}$$

$$R_G = \frac{20{,}925{,}604.4742 \times \sqrt{1 - 0.0066943800229}}{0.99485630743} = \mathbf{\underline{20{,}963{,}273.518\ sft}}$$

Check: $R_G = \sqrt{R_M\ R_N} = \sqrt{20{,}946{,}929.368 \times 20{,}979{,}630.421} = \underline{20{,}963{,}273.518\ \text{sft}}$ ✓

The NGS Datasheet

```
1        National Geodetic Survey,   Retrieval Date = MARCH 12, 2014
 TT2845 ***********************************************************************
 TT2845  DESIGNATION -  CE 314 U OF A RESET
 TT2845  PID         -  TT2845
 TT2845  STATE/COUNTY-  AK/FAIRBANKS NORTH STAR
 TT2845  COUNTRY     -  US
 TT2845  USGS QUAD   -  FAIRBANKS D-2
 TT2845
 TT2845                         *CURRENT SURVEY CONTROL
 TT2845  ______________________________________________________________________
 TT2845* NAD 83(2011) POSITION- 64 51 21.30585(N) 147 49 08.66783(W)   ADJUSTED
 TT2845* NAD 83(2011) ELLIP HT-   168.829 (meters)        (06/27/12)   ADJUSTED
 TT2845* NAD 83(2011) EPOCH   -  2010.00
 TT2845* NAVD 88 ORTHO HEIGHT -   159.371  (meters)     522.87  (feet) ADJUSTED
 TT2845  ______________________________________________________________________
 TT2845  NAD 83(2011) X  - -2,300,080.306 (meters)                     COMP
 TT2845  NAD 83(2011) Y  - -1,447,368.715 (meters)                     COMP
 TT2845  NAD 83(2011) Z  -  5,751,055.481 (meters)                     COMP
 TT2845  LAPLACE CORR    -           0.95  (seconds)                   DEFLEC12A
 TT2845  GEOID HEIGHT    -           9.46  (meters)                    GEOID12A
 TT2845  DYNAMIC HEIGHT  -         159.634 (meters)     523.73  (feet) COMP
 TT2845  MODELED GRAVITY -    982,231.4    (mgal)                      NAVD 88
 TT2845
 TT2845  VERT ORDER      -  FIRST     CLASS II
 TT2845
 TT2845  FGDC Geospatial Positioning Accuracy Standards (95% confidence, cm)
 TT2845  Type                                          Horiz  Ellip  Dist(km)
 TT2845  -------------------------------------------------------------------
 TT2845  NETWORK                                        0.86   1.84
 TT2845  -------------------------------------------------------------------
 TT2845  MEDIAN LOCAL ACCURACY AND DIST (026 points)   1.05   1.84   508.74
 TT2845  -------------------------------------------------------------------
 TT2845  NOTE: Click here for information on individual local accuracy
 TT2845  values and other accuracy information.
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 TT2845.The horizontal coordinates were established by GPS observations
 TT2845.and adjusted by the National Geodetic Survey in June 2012.
 TT2845
 TT2845.NAD 83(2011) refers to NAD 83 coordinates where the reference
 TT2845.frame has been affixed to the stable North American tectonic plate. See
 TT2845.NA2011 for more information.
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 TT2845.The horizontal coordinates are valid at the epoch date displayed above
 TT2845.which is a decimal equivalence of Year/Month/Day.
 TT2845
 TT2845.The orthometric height was determined by differential leveling and
 TT2845.adjusted by the NATIONAL GEODETIC SURVEY in June 1991.
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 TT2845.Photographs are available for this station.
 TT2845
 TT2845.The X, Y, and Z were computed from the position and the ellipsoidal ht.
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 TT2845.The Laplace correction was computed from DEFLEC12A derived deflections.
 TT2845
 TT2845.The ellipsoidal height was determined by GPS observations
 TT2845.and is referenced to NAD 83.
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 TT2845.The dynamic height is computed by dividing the NAVD 88
 TT2845.geopotential number by the normal gravity value computed on the
 TT2845.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
 TT2845.degrees latitude (g = 980.6199 gals.).
 TT2845
 TT2845.The modeled gravity was interpolated from observed gravity values.
```

The NGS Datasheet *(continued)*

```
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TT2845;UTM  06      - 7,192,648.384   461,168.110   MT  0.99961846   -0 44 29.3
TT2845
TT2845!             -  Elev Factor  x  Scale Factor =   Combined Factor
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TT2845
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TT2845.NGS no longer adjusts projects to the NAD 27 or NGVD 29 datums.
TT2845.See file dsdata.txt to determine how the superseded data were derived.
TT2845
TT2845_U.S. NATIONAL GRID SPATIAL ADDRESS: 6WVS6116892648(NAD 83)
TT2845
TT2845_MARKER: DD = SURVEY DISK
TT2845_SETTING: 7 = SET IN TOP OF CONCRETE MONUMENT
TT2845_SP_SET: SET IN TOP OF CONCRETE MONUMENT
TT2845_STAMPING: CE 314 1941
TT2845_MARK LOGO: NONE
TT2845_PROJECTION: FLUSH
TT2845_MAGNETIC: O = OTHER; SEE DESCRIPTION
TT2845_STABILITY: C = MAY HOLD, BUT OF TYPE COMMONLY SUBJECT TO
TT2845+STABILITY: SURFACE MOTION
TT2845_SATELLITE: THE SITE LOCATION WAS REPORTED AS SUITABLE FOR
TT2845+SATELLITE: SATELLITE OBSERVATIONS - December 08, 2004
TT2845
TT2845  HISTORY     - Date     Condition         Report By
TT2845  HISTORY     - 1951     MONUMENTED        CGS
TT2845  HISTORY     - 1965     GOOD              CGS
TT2845  HISTORY     - 20011012 GOOD              R+MCON
TT2845  HISTORY     - 20041208 GOOD              COMPDA
TT2845
TT2845                         STATION DESCRIPTION
TT2845
TT2845'DESCRIBED BY COAST AND GEODETIC SURVEY 1965, IN COLLEGE.
TT2845'AT COLLEGE, AT THE UNIVERSITY OF ALASKA, ABOUT 500 FEET EAST OF BENCH
TT2845'MARK MAGNETIC STA, AT THE SOUTHEAST SIDE OF A CIRCULAR CONCRETE
TT2845'SIDEWALK AROUND A CIRCULAR WATER FOUNTAIN, 109 FEET NORTHWEST OF THE
TT2845'MOST EASTERLY OF THE NORTH SIDE ENTRANCES TO BUNNELL BUILDING WHICH IS
TT2845'BUILDING 303, 87.4 FEET SOUTHEAST OF THE SOUTHEAST SIDE OF THE WATER
TT2845'FOUNTAIN, SET IN THE TOP OF A 3 X 4-FOOT CONCRETE POST INSCRIBED
TT2845'AAC-SM JULY 4 1915 LD S915, AND PROJECTING 2.4 FEET.
TT2845
TT2845                         STATION RECOVERY (2001)
TT2845
TT2845'RECOVERY NOTE BY R + M CONSULTANTS INCORPORATED 2001 (JDN)
TT2845'DESCRIBED BY R+M CONSULTANTS 2001 (JDN). THE STATION IS LOCATED
TT2845'APPROXIMATELY 4.8 KM (3MI) WEST NORTHWEST OF DOWNTOWN FAIRBANKS AT THE
TT2845'UNIVERSITY OF ALASKA FAIRBANKS CAMPUS. OWNERSHIP--STATE OF ALASKA.
TT2845
TT2845                         STATION RECOVERY (2004)
TT2845
TT2845'RECOVERY NOTE BY COMPASSDATA INC 2004 (RL). RECOVERED IN GOOD CONDITION.
```

The NGS Geodetic Toolkit

NGS Geodetic Tool Kit

National Geodetic Survey

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NGS Geodetic Tool Kit

On-line interactive computation of geodetic values

See the text version of an article about the NGS Geodetic Toolkit that appeared in the *Professional Surveyor* magazine, May 2003 Volume 23, Number 4.

See all the Professional Surveyor Articles about the NGS Geodetic Toolkit.

These utilities require Internet Explorer version 6.0+ and Netscape version 6.0+

To learn more about a particular online program, click on its link for a description:

- DEFLEC99
- DEFLEC09
- DEFLEC12A
- DYNAMIC_HT
- GEOID12A
- GEOID12
- GEOID09
- GEOID06
- GEOID03
- GEOID99
- G99SSS
- USGG2012
- USGG2009
- USGG2003
- HTDP
- IGLD85
- Inverse/Forward/Invers3D/Forwrd3D
- Leveling Online Computations User Service (LOCUS)
- LVL_DH
- Magnetic Declination
- NADCON
- NAVD 88 Modelled Gravity
- Online Adjustment User Services
- Online Adjustment Utilities User Services
- OPUS
- State Plane Coordinates
- Surface Gravity Prediction
- Tidal and Orthometric Elevations
- U.S. National Grid
- Universal Transverse Mercator Coordinates
- VERTCON
- XYZ Coordinate Conversion

Or... Know what you want to do?

Select a function from this list below:

Select a Toolkit Shortcut

For more information contact NGS Information Services:
by email or call (301) 713-3242, Monday - Friday, 7:00 AM - 4:30 PM eastern time.

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Section 2

ELLIPSOIDAL DATUM DEFINITIONS AND REFERENCE COORDINATES

How are the data connected to the Earth?

Examples of georeferencing errors for Alaska

Table 2.1 Examples of various positioning error sources and their magnitudes for Alaska due only to ellipsoidal datum definition and reference coordinate problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Alaska	Error magnitudes
Using NAD 27 when NAD 83 required	Varies from ~200 to 780 feet (horizontal)
Using "WGS 84" when NAD 83 required (e.g., by using WAAS corrections or CORS ITRF coordinates)	3.6 to 4.4 feet (horizontal) −0.4 to 3.3 feet (vertical)
Using DMA-published three-parameter datum transformation between NAD 27 and WGS 84 for NAD 83 projects	~60 to 180 feet (horizontal)
Using NADCON to transform coordinates between NAD 27 and NAD 83	~4 feet (horizontal)
Using NAD 83(1986) coordinates when NAD 83(1992) coordinates required	~0.7 to 3.3 feet (horizontal) in Anchorage
Using NAD 83(1992) "HARN" when NAD 83(NSRS2007) coordinates required (epoch 2007.00)	Up to 2.8 feet (horizontal) −1.4 to +2.1 feet (vertical)
Using NAD 83(NSRS2007) epoch 2007.00 when NAD 83(2011) epoch 2010.00 coordinates required.	Up to 9.3 feet (horizontal) −0.8 to +2.1feet (vertical)
Using only 7 of 14 published NGS parameters to transform between NAD 83(2011) epoch 2010.00 and WGS84/IGS/ITRF, i.e., ignoring reference (zero) time of 1997.00	~1.0 foot (horizontal) ~0.1 foot (vertical) in Fairbanks
Using published NGS 14-parameter transformation between NAD 83(2011) epoch 2010.00 and WGS84/IGS/ITRF epoch 2005.00 and (i.e., ignoring velocities for 5 year time change)	~0.4 foot (horizontal) in Fairbanks
Assuming current (March 2014) OPUS IGS08 coordinates are the same as published IGS08 (epoch 2005.00) coordinates (i.e., ignoring velocities for 9.3 year time change)	~0.8 foot (horizontal) in Fairbanks
Autonomous (uncorrected) GPS single-point positioning precision (at 95% confidence)	~10 to 20 ft (horizontal) ~20 to 50 ft (vertical)

The NGS Datasheet as a geodetic reference coordinate source

Recommend using Datasheets with GPS-derived coordinates, because they give ellipsoid height (as well as ECEF coordinates).

```
TT2845 ***********************************************************************
TT2845  DESIGNATION -  CE 314 U OF A RESET
TT2845  PID         -  TT2845
TT2845  STATE/COUNTY-  AK/FAIRBANKS NORTH STAR
TT2845  COUNTRY     -  US
TT2845  USGS QUAD   -  FAIRBANKS D-2
TT2845
TT2845                         *CURRENT SURVEY CONTROL
TT2845
TT2845* NAD 83(2011) POSITION- 64 51 21.30585(N) 147 49 08.66783(W)   ADJUSTED
TT2845* NAD 83(2011) ELLIP HT-   168.829 (meters)        (06/27/12)   ADJUSTED
TT2845* NAD 83(2011) EPOCH   -  2010.00
TT2845* NAVD 88 ORTHO HEIGHT -   159.371  (meters)     522.87   (feet) ADJUSTED
TT2845
TT2845  NAD 83(2011) X  - -2,300,080.306 (meters)                     COMP
TT2845  NAD 83(2011) Y  - -1,447,368.715 (meters)                     COMP
TT2845  NAD 83(2011) Z  -  5,751,055.481 (meters)                     COMP
TT2845  LAPLACE CORR    -          0.95  (seconds)                    DEFLEC12A
TT2845  GEOID HEIGHT    -          9.46  (meters)                     GEOID12A
TT2845  DYNAMIC HEIGHT  -        159.634 (meters)      523.73   (feet) COMP
TT2845  MODELED GRAVITY -    982,231.4   (mgal)                       NAVD 88
TT2845
TT2845  VERT ORDER      -  FIRST     CLASS II
TT2845
```

= φ and λ
= h
= X
= Y
= Z

Some things to note about NGS Datasheets:

- Many conventional stations do not have accurate elevations, so cannot be used with geoid model to determine accurate ellipsoid heights
- Conventionally (optically) determined control is almost always less accurate than survey-grade GPS, so using such control for surveys is not advised
    - Only GPS stations have positional accuracies given as linear "network" values in centimeters (relative "order/class" system no longer used)
- NAD 83(2011) epoch of 2010.00 gives the coordinates at a date of January 1, 2010
    - Previous realization of NAD 83 in Alaska was epoch 2007.00 for NAD 83(NSRS2007) passive marks and epoch 2003.00 for NAD 83(CORS96) CORS. For NAD 83(2011), both passive marks and CORS have the same epoch.
        - Different epochs for passive marks and CORS can be problematic in parts of Alaska due to tectonic motion (for example, NAD 83 horizontal velocities in Fairbanks are about 0.017 ft/year to the SE).

OPUS output as a geodetic reference coordinate source

The *Online Positioning User Service* is an excellent alternative to the NGS Datasheets if there are no high-quality GPS-derived NGS control stations locally available.

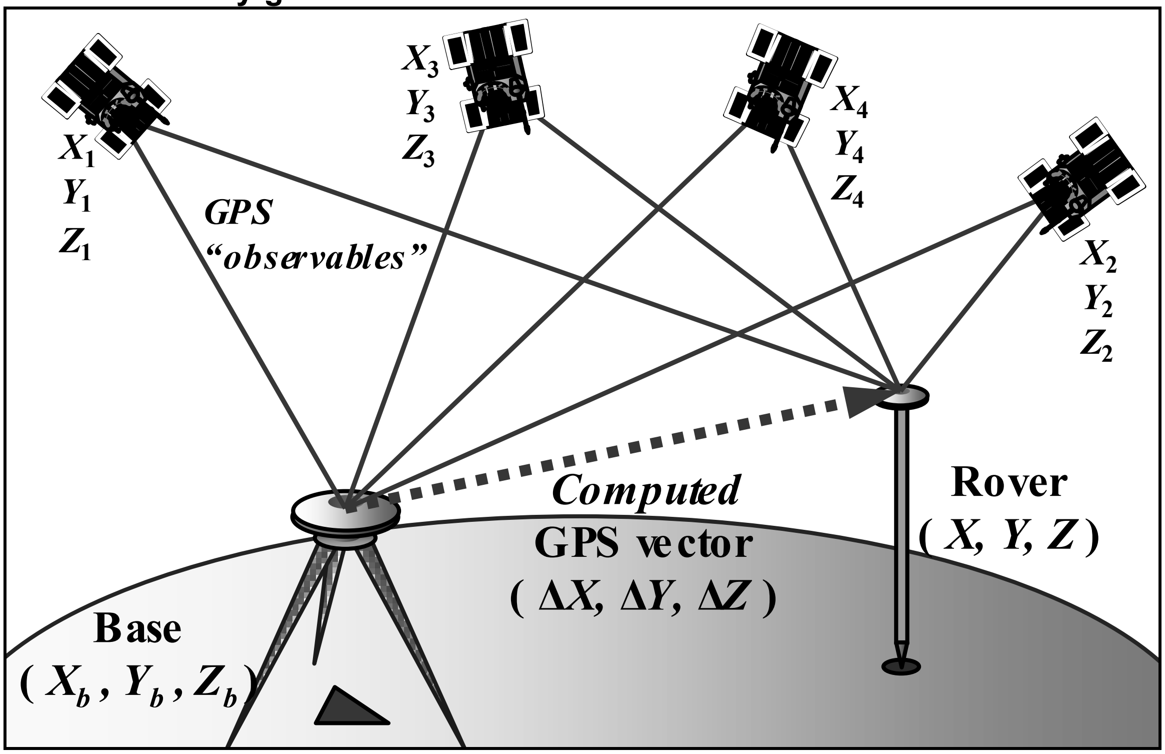
- More accurate than conventional (optical) control (*note that as of 2011, NGS no longer takes in or makes use of classical optically derived horizontal control*).
- Requires logging raw GPS data (observables) at the receiver for at least 2 hours (or as little as 15 minutes using the "Rapid Static" option)
  - This can easily be done at a GPS base while performing a survey

```
                              NGS OPUS SOLUTION REPORT
                              ========================

All computed coordinate accuracies are listed as peak-to-peak values.
For additional information: http://www.ngs.noaa.gov/OPUS/about.jsp#accuracy

      USER: michael.dennis@noaa.gov                    DATE: March 13, 2014
RINEX FILE: clgo045a.14o                               TIME: 17:44:13 UTC


  SOFTWARE: page5  1209.04 master93.pl 022814      START: 2014/02/14  00:00:00
 EPHEMERIS: igs17795.eph [precise]                  STOP: 2014/02/14  05:00:00
  NAV FILE: brdc0450.14n                        OBS USED: 15820 / 16553   :  96%
  ANT NAME: TRM29659.00     NONE             # FIXED AMB:    65 /    73   :  89%
ARP HEIGHT: 0.000                            OVERALL RMS: 0.012(m)


 REF FRAME: NAD_83(2011)(EPOCH:2010.0000)              IGS08 (EPOCH:2014.1208)

         X:     -2299608.699(m)   0.013(m) = X     -2299609.729(m)   0.013(m)
         Y:     -1444754.305(m)   0.002(m) = Y     -1444753.276(m)   0.002(m)
         Z:      5751925.446(m)   0.008(m) = Z      5751925.756(m)   0.008(m)

       LAT:   64 52 25.58665      0.007(m) = φ    64 52 25.58141      0.007(m)
     E LON:  212  8 22.34943      0.008(m)       212  8 22.24164      0.008(m)
     W LON:  147 51 37.65057      0.008(m) = λ   147 51 37.75836      0.008(m)
    EL HGT:           195.736(m)  0.011(m) = h           196.155(m)   0.011(m)
 ORTHO HGT:           186.254(m)  0.020(m) [NAVD88 (Computed using GEOID12A)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 06)         SPC (5003 AK 3)
Northing (Y) [meters]     7194664.227         1212524.907
Easting (X)  [meters]      459233.259          411836.663
Convergence  [degrees]    -0.77904780         -1.68451886
Point Scale                0.99962035          0.99999514
Combined Factor            0.99958974          0.99996452

US NATIONAL GRID DESIGNATOR: 6WVS5923394664(NAD 83)


                              BASE STATIONS USED
PID       DESIGNATION                        LATITUDE    LONGITUDE DISTANCE(m)
DL6659 SUAF SURVEYORSEXCH UAF CORS ARP      N645131.288 W1475008.768    2049.1
DK4095 FAI1 FAIRBANKS WAAS CORS ARP         N644834.672 W1475050.320    7178.8
DO1818 AC71 DELTAJUNC_AK2003 CORS ARP       N640257.498 W1454248.943  138273.2
```

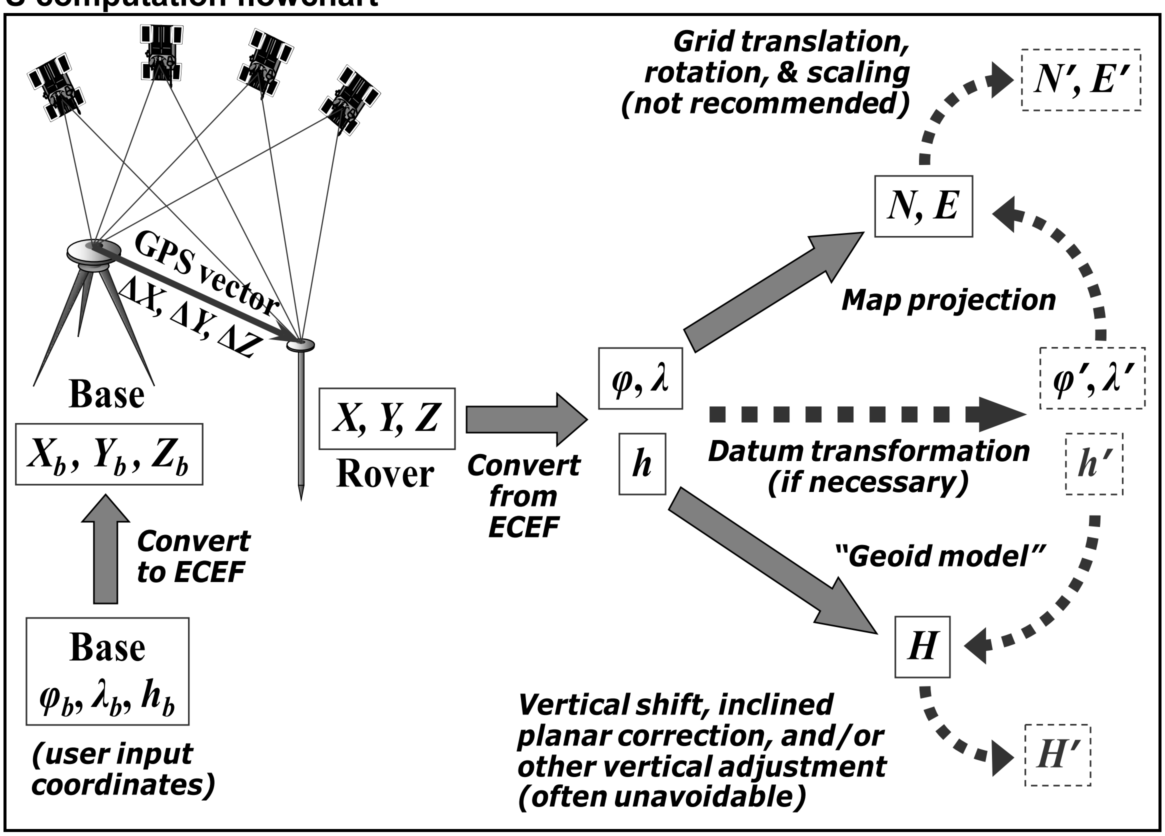
Some things to note about OPUS output:

- Gives coordinates as both NAD 83(2011) epoch 2010.00 and IGS08 current epoch.
    - IGS08 epoch is for midpoint time GPS data file submitted to OPUS (in this example, epoch of 2014.1208 = February 14, 2014).
        - This is NOT same as the *published* IGS08 coordinates for CORS, which were determined for an epoch of 2005.00. So OPUS coordinates will differ from published by the date difference times the IGS08 horizontal station velocity (about 0.08 ft/year to the SSW in Fairbanks, so for this case about 0.7 ft in 9 years).
        - IGS08 can be considered essentially equivalent to ITRF 2008 and WGS 84 (G1674) to within about 1-2 cm (as long as the coordinates refer to the same epoch).
- Slightly different results will be obtained depending on which GPS orbits were used.
    - Final orbits available after about two weeks.
    - "Rapid" orbits available in 17 hours, and are nearly as accurate as final orbits.
- Values to right of coordinates are accuracy estimates in meters, e.g., for latitude 0.007 (m).
    - These are based on the maximum difference between the 3 positions computed by OPUS.
    - Can estimate precision if have multiple OPUS solutions on a single point.
- Detailed ("extended") output also available.
    - Gives additional information such as CORS details, coordinate transformations, velocities, actual vector components, GPS solution statistics, and internal precision estimates.

Differential "survey-grade" GPS



GPS computation flowchart



Exercise 2.1: Computation **of coordinates using GPS vector components**

Below are equations for computing geodetic coordinates of a new station using the GPS vector from a base station of known geodetic coordinates.

Equation 2.1 Converting latitude, longitude, and height to ECEF coordinates

$$\boxed{\begin{aligned} X &= (R_N + h)\cos\varphi\ \cos\lambda \\ Y &= (R_N + h)\cos\varphi\ \sin\lambda \\ Z &= \left[R_N(1-e^2)+h\right]\sin\varphi \end{aligned}}$$ (Leick, 2004, p. 371)

where X, Y, and Z are the ECEF coordinates of a point

φ, λ, and h are the latitude, longitude, and ellipsoid height of the point, respectively

$\boxed{R_N = a\left(1-e^2\sin^2\varphi\right)^{-1/2}}$ is the prime vertical radius of curvature (Leick, 2004, p. 369)

a is the ellipsoid semi-major axis (= 20,925,646.325 459 ift for the GRS-80 ellipsoid)

e^2 is the ellipsoid first eccentricity squared (= 0.006 694 380 022 901 for GRS-80)

Equation 2.2 Computing coordinates from GPS vector components

$$\boxed{X = X_b + \Delta X \qquad Y = Y_b + \Delta Y \qquad Z = Z_b + \Delta Z}$$

where X, Y, and Z are the ECEF coordinates to be determined

X_b, Y_b, and Z_b are the ECEF coordinates of the GPS base

ΔX, ΔY, and ΔZ are the delta ECEF components of the GPS vector

Equation 2.3 Converting ECEF coordinates to latitude, longitude, and height

$$\boxed{\begin{aligned} \varphi &= \tan^{-1}\left[\frac{Z}{\sqrt{X^2+Y^2}}\left(1+\frac{e^2 R_N \sin\varphi_0}{Z}\right)\right] \\ \lambda &= \tan^{-1}\left(\frac{Y}{X}\right) \\ h &= \frac{\sqrt{X^2+Y^2}}{\cos\varphi} - R_N \end{aligned}}$$ (Leick, 2004, pp. 371-372)

where φ_0 is a latitude that can be *initially* approximated as $\varphi_0 = \tan^{-1}\left(\dfrac{Z}{(1-e^2)\sqrt{X^2+Y^2}}\right)$.

This approximate latitude value is then substituted into the right side of the first line of Equation 2.3, and then the resulting value of φ is substituted as φ_0, and the process repeated until the change in φ is negligible.

Example computation

Given: A GPS base station located at midpoint between points ZAN A and ZAN B, with NAD 83 coordinates of φ = 61°13'49.09296" N, λ = 149°46'52.51783" W, and h = 220.072 sft. The following GPS vector components were determined from this base to point ZAN A:

$\Delta X = -89.636$ sft $\Delta Y = 387.005$ sft $\Delta Z = 63.554$ sft

Find: The NAD 83 coordinates of point ZAN A.

Computations:

Step 1. Convert GPS base latitude, longitude, and ellipsoidal height to ECEF coordinates.

The prime vertical radius of curvature for this station was computed in Exercise 1.3:

R_N = ____________________

Now compute the ECEF values for the GPS base:

$X_b = (\quad R_N \quad + \quad h \quad) \times \quad \cos\varphi \quad \times \quad \cos\lambda$

X_b = (______________ + _________) × cos(___________) × cos(___________)

= ______________

$Y_b = (\quad R_N \quad + \quad h \quad) \times \quad \cos\varphi \quad \times \quad \sin\lambda$

Y_b = (______________ + _________) × cos(___________) × sin(___________)

= ______________

$Z_b = [\quad R_N \quad \times (1- \quad e^2 \quad) + \quad h \quad] \times \quad \sin\varphi$

Z_b = [____________× (1−______________) + ________] × sin(___________)

= ______________

Step 2. Compute ECEF coordinates of new GPS station.

$X = X_b + \Delta X$ = ____________________ + ____________ = ____________________

$Y = Y_b + \Delta Y$ = ____________________ + ____________ = ____________________

$Z = Z_b + \Delta Z$ = ____________________ + ____________ = ____________________

Step 3. Convert ECEF coordinates of new station to latitude, longitude, and ellipsoid height.

Instead of using iterative Equation 2.3, perform this computation using the NGS Geodetic Toolkit, which gives

Latitude, φ = _____°____'____.__________" **N**

Longitude, λ = _____°____'____.__________" **W**

Ellipsoidal height, h = ___________ **sft**

These results can be verified using Equation 2.3

Solution:

Step 1. Convert GPS base latitude, longitude, and ellipsoidal height to ECEF coordinates.

The prime vertical radius of curvature for this station was computed in Exercise 1.3:

$R_N = \underline{20{,}979{,}630.421 \text{ sft}}$

Now compute the ECEF values for the GPS base:

$$X_b = (\quad R_N \quad + \quad h \quad) \times \quad \cos\varphi \quad \times \quad \cos\lambda$$

$$X_b = (20{,}979{,}630.421 + 220.072) \times \cos(61.2303036000°) \times \cos(-149.7812549528°)$$

$$= \underline{\mathbf{-8{,}725{,}261.854 \text{ sft}}}$$

$$Y_b = (\quad R_N \quad + \quad h \quad) \times \quad \cos\varphi \quad \times \quad \sin\lambda$$

$$Y_b = (20{,}979{,}630.421 + 220.072) \times \cos(61.2303036000°) \times \sin(-149.7812549528°)$$

$$= \underline{\mathbf{-5{,}082{,}045.935 \text{ sft}}}$$

$$Z_b = [\quad R_N \quad \times (1- \quad e^2 \quad) + \quad h \quad] \times \quad \sin\varphi$$

$$Z_b = [20{,}979{,}630.421 \times (1 - 0.006694380023) + 220.072] \times \sin(61.2303036000°)$$

$$= \underline{\mathbf{18{,}267{,}016.992 \text{ sft}}}$$

Step 2. Compute ECEF coordinates of new GPS station.

$$X = X_b + \Delta X = (-8{,}725{,}261.901 \text{ sft}) \quad + \quad (-89.636 \text{ sft}) \quad = \underline{\mathbf{-8{,}725{,}351.490 \text{ sft}}}$$

$$Y = Y_b + \Delta Y = (-5{,}082{,}045.922 \text{ sft}) \quad + \quad (387.005 \text{ sft}) \quad = \underline{\mathbf{-5{,}081{,}658.930 \text{ sft}}}$$

$$Z = Z_b + \Delta Z = (18{,}267{,}016.868 \text{ sft}) \quad + \quad (63.554 \text{ sft}) \quad = \underline{\mathbf{18{,}267{,}080.545 \text{ sft}}}$$

Step 3. Convert ECEF coordinates of new station to latitude, longitude, and ellipsoid height.

Equation 2.3 was used to compute the following results (compare to those computed using the NGS Geodetic Toolkit).

Latitude, φ = 61° 13’ 50.40678” N

Longitude, λ = 149° 47’ 00.27073” W

Ellipsoidal height, h = 219.317 sft

✓ *These results were computed using Equation 2.3 (required only 2 iterations in Excel for accuracy shown); small difference in longitude is due to rounding.*

Datums and datum transformations

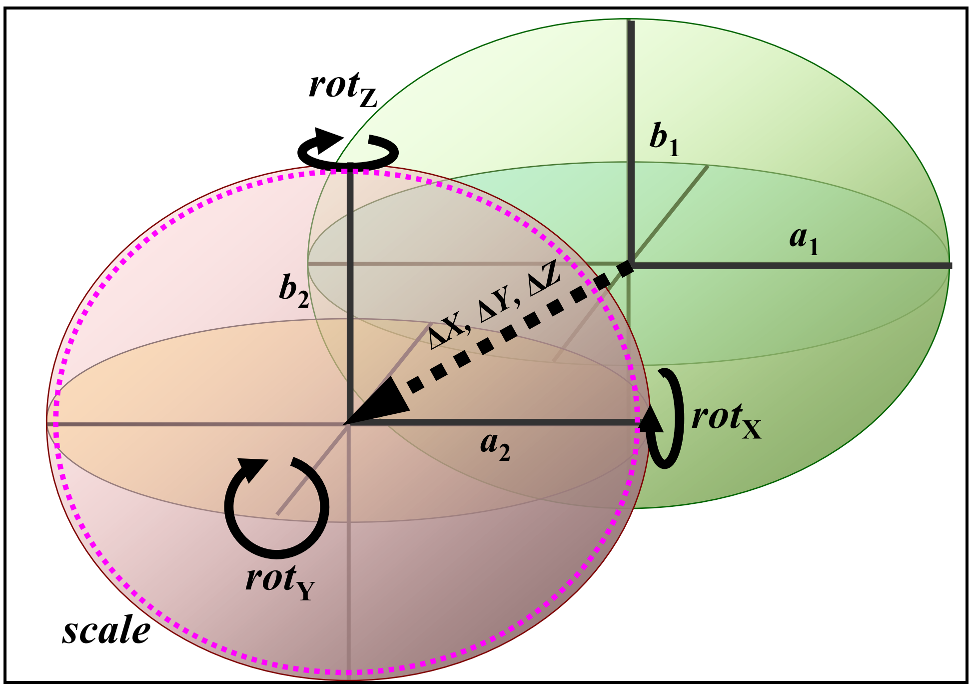
Datum. Any quantity or set of quantities used as a reference or basis for determining other quantities.

Ellipsoidal datum. An ellipsoidal coordinate system whose relation to an ECEF Cartesian XYZ reference frame can be determined through (at least) 8 parameters (3 origin translations, 3 axis rotations, and the adoption of an ellipsoidal semi-major axis and an ellipsoidal flattening), and used for determining latitude, longitude, and ellipsoidal height.

Vertical datum. Generally, some adopted surface of zero "elevation" to which all points in that datum refer. Often the realization of this datum is the publication of fundamental elevations at passive geodetic control marks.

Datum transformation. Mathematical method for converting one ellipsoidal or vertical datum to another (there are several types, and they vary widely in accuracy).

Ellipsoidal datum transformation



Typical ellipsoidal datum transformations. Note that the dimensions of the reference ellipsoid (*a* and *b* axes) may or may not change in the transformation.

3-parameter: 3-dimensional translation of origin as ΔX, ΔY, ΔZ (*just like a GPS vector*)

7-parameter: 3 translations *plus* 3 rotations (one about each of the axes) *plus* a scale

14-parameter: A 7-parameter where each parameter changes with time (each has a *velocity*)

Transformations are also used that model distortion, such as the NGS model NADCON

Vertical datum transformations. Can be a simple vertical shift or a complex operation that models distortion, such as the NGS model VERTCON.

Exercise 2.2: Geodetic azimuths

Forward and reverse grid azimuths differ by exactly 180°. Forward and reverse geodetic azimuths do not differ by 180° because of meridian convergence, as shown in the figure below.

Equation 2.4 Approximate forward geodetic azimuth (from point *A* to point *B*)

$$\boxed{\tilde{\alpha}_{AB} = \tan^{-1}\left(\frac{\lambda_B - \lambda_A}{\varphi_B - \varphi_A}\cos\varphi_B\right)}$$

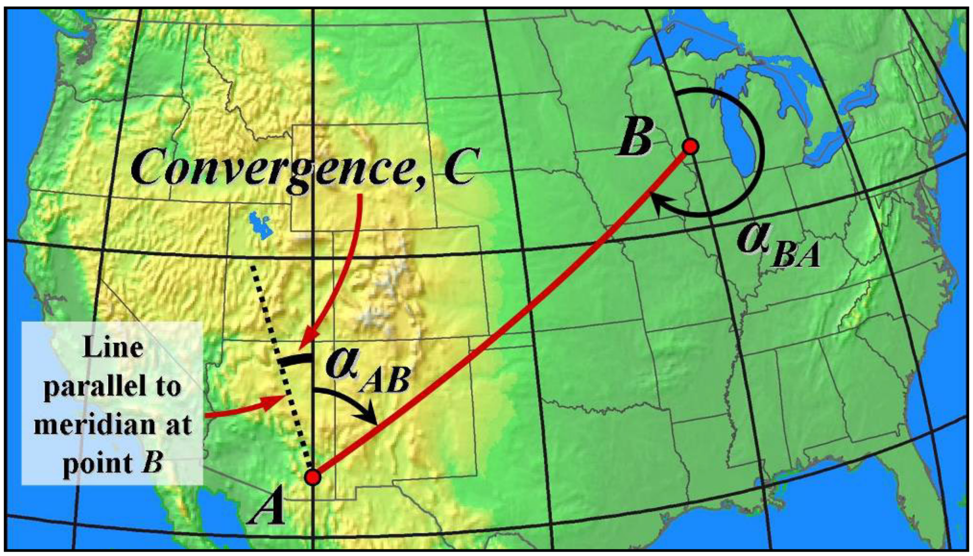
where $\tilde{\alpha}_{AB}$ is the *approximate* forward geodetic azimuths from point *A* to *B*

φ_A, φ_B are longitudes at azimuth end points *A* and *B*, respectively

λ_A, λ_B are longitudes at azimuth end points *A* and *B*, respectively

Equation 2.4 is accurate to within approximately ±0.5% for distances of less than about 100 miles.

Although forward and backward *grid* azimuths differ by exactly 180°, forward and backward geodetic azimuths generally do not due to meridian convergence, as shown in the figure below.



Rules of Thumb:

The convergence per mile east-west is about 01'35" in Anchorage and 01'50" in Fairbanks.

Equation 2.5 Difference between forward and back geodetic azimuths (meridian convergence)

$$\boxed{\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\overline{\varphi}}$$ (Stem, 1990, p. 51; Ewing and Mitchell, 1970, p. 44)

where α_{AB}, α_{BA} are the forward and back geodetic azimuths from point *A* to *B*, respectively

$\overline{\varphi}$ is the average latitude of the azimuth end points

Although Equation 2.5 is for a sphere, it is accurate to better than 0.2" for distances of less than about 100 miles.

Example computation

Given: A two points (ZAN A and ZAN B) with the following geodetic coordinates:

ZAN A: φ_A = 61°13'50.40678" N λ_A = 149°47'00.27075" W

ZAN B: φ_B = 61°13'47.77913" N λ_B = 149°46'44.76491" W

Find: The approximate geodetic azimuth from ZAN A to ZAN B and compute the difference between the forward and back geodetic azimuths (i.e., the convergence).

Computations:

To simplify the computations, the approximate geodetic azimuth can be computed using the coordinate differences in arc-seconds:

$$\tilde{\alpha}_{AB} \approx \tan^{-1}\left(\frac{\lambda_B-\lambda_A}{\varphi_B-\varphi_A}\cos\varphi_B\right) = \tan^{-1}\left(\frac{(\underline{\hspace{3cm}})-(\underline{\hspace{3cm}})}{(\underline{\hspace{3cm}})-(\underline{\hspace{3cm}})}\times\cos(\underline{\hspace{3cm}})\right)$$

$$= \tan^{-1}(\underline{\hspace{3cm}} \times \underline{\hspace{3cm}}) = \underline{\hspace{2cm}}° = \underline{\quad ° \quad ' \quad "} = \underline{\textbf{N} \quad ° \quad ' \quad " \textbf{ E}}$$

The difference between forward and back azimuths is

$$\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\bar{\varphi} \qquad \textit{(can use midpoint latitude from Exercises 1.3 or 2.1)}$$

$$= (\underline{\hspace{3cm}} + \underline{\hspace{3cm}}) \times \sin[(\underline{\hspace{3cm}} + \underline{\hspace{3cm}})/2]$$

$$= \underline{\hspace{3cm}} \times \sin[\underline{\hspace{4cm}}] = \underline{\hspace{3cm}}"$$

Solution:

The approximate geodetic azimuth can be computed as

$$\tilde{\alpha}_{AB} = \tan^{-1}\left(\frac{\lambda_B-\lambda_A}{\varphi_B-\varphi_A}\cos\varphi_B\right) = \tan^{-1}\left(\frac{(-44.76491")-(-60.27075")}{47.77913" - 50.40678"}\times\cos(61.2299386472°)\right)$$

$$= \tan^{-1}(-5.9010294 \times 0.481295714) = \underline{\mathbf{-70.60307°}} = \underline{\mathbf{109° \ 23' \ 49"}} = \underline{\mathbf{S\ 70° \ 36' \ 11"\ E}}$$

The difference between forward and back azimuths is

$$\alpha_{BA} - \alpha_{AB} - 180° \approx (\lambda_B - \lambda_A)\sin\bar{\varphi} \qquad \textit{(can use midpoint latitude from Exercises 1.3 or 2.1)}$$

= (−44.76491" + 60.27075") × sin(61.23030360°)

= 15.50584" × 0.87656136 = **+13.5918"**

Check using NGS Inverse tool:

Forward azimuth = 109° 22' 02.1577"

Error in approximate azimuth is +0° 01' 47" = +0.03% *(OK, but not very accurate)* ✓

Back azimuth = 289° 22' 15.7496"

Convergence = (289° 22' 15.7496") – (109° 22' 02.1577") –180° = +13.5919" ✓

(nearly identical!)

Exercise 2.3: An approximate method for computing ellipsoidal distance

This gives a method for computing an approximate ellipsoidal distance between two points with geodetic coordinates (latitude, longitude, and ellipsoidal height). For the GRS-80, WGS-84, Clarke 1866, and most other Earth ellipsoids, note the following:

Rules of Thumb

1 arc-second of latitude ≈ 101 ft (*short by about 0.5 ft in AK*)

1 arc-second of longitude ≈ 101 ft × cos(latitude) (*short by about 0.3 ft in AK*)

Based on these relationships, we can compute an approximate distance, to wit:

Equation 2.6 Approximate ellipsoidal distance between a pair of geodetic coordinates

$$\boxed{s \approx 101\sqrt{(\Delta\varphi'')^2 + (\Delta\lambda''\cos\overline{\varphi})^2}} \text{ feet}$$

This equation is accurate to within about ±1% everywhere on the Earth

where $\Delta\varphi''$ is change in latitude between two points in arc-seconds

$\Delta\lambda''$ is change in longitude between two points in arc-seconds

$\overline{\varphi}$ is average latitude of the two points

Example computation

Given: Points ZAN A and ZAN B from the previous example (in Exercise 2.2).

Find: The approximate ellipsoidal distance between the points ZAN A and ZAN B.

Computations:

From the previous examples, the average latitude of ZAN A and ZAN B is $\overline{\varphi} = 61.23030360°$

$$s \approx 101\sqrt{(\Delta\varphi'')^2 + (\Delta\lambda''\cos\overline{\varphi})^2}$$

$$= 101 \times \sqrt{(\underline{\hspace{2cm}} - \underline{\hspace{2cm}})^2 + [(\underline{\hspace{2cm}} - \underline{\hspace{2cm}}) \times \cos(61.23030360°)]^2}$$

$$= 101 \times \sqrt{(\underline{\hspace{2cm}})^2 + (\underline{\hspace{2cm}})^2} = \underline{\hspace{3cm}\textbf{ft}}$$

Solution:

$$= 101 \times \sqrt{(47.77913'' - 50.40678'')^2 + [(-44.76491'' + 60.27075'') \times \cos(61.23030360°)]^2}$$

$$= 101 \times \sqrt{(-2.62765)^2 + (7.46281)^2} = \underline{\textbf{799 ft}}$$

Check using NGS Inverse tool:

Actual ellipsoid distance (geodesic) = 245.2417 m = <u>804.597 sft</u>

Approximate geodetic inverse error = −5.6 ft = −0.7%

Exercise 2.4: A better method for computing ellipsoidal distance

Computation of accurate geodetic distances is difficult, but a good approximation over *short* distances can be computed using spherical angles based on an appropriate radius of curvature.

Equation 2.7 Central angle between two points on surface of a sphere

$$\boxed{\psi = \cos^{-1}\left(\cos\theta_A \ \cos\theta_B + \sin\theta_A \ \sin\theta_B \ \ \cos(\lambda_B - \lambda_A)\right)}$$

where θ_A, θ_B are the co-latitudes at points A and B, respectively, and $\theta = 90° - \phi$

λ_A, λ_B are longitudes at azimuth end points *A* and *B*, respectively

Note that co-latitude is defined as 90° minus the latitude, i.e., $\theta = 90° - \varphi$

Equation 2.8 Approximate geodetic inverse based on spherical angle

$$\boxed{s = R_\alpha \psi = \left(\frac{R_M \ R_N}{R_M \sin^2 \tilde{\alpha}_{AB} + R_N \cos^2 \tilde{\alpha}_{AB}}\right)\psi}$$

where all variables are as defined previously and radii of curvature are evaluated at the mean latitude of the two points.

The accuracy of the distances computed by Equation 2.8 vary with azimuth, and are generally shorter than actual by a maximum of 10 ppm for distances less of than about 10 miles (e.g., a one mile inverse is at most 0.05 ft shorter than actual).

A highly accurate method for computing geodetic distance and azimuth was published by Vincenty (1975), and is the one used in the NGS geodetic tool "Inverse".

Example computation

Given: Points ZAN A and ZAN B from the previous two examples (in Exercises 2.2 and 2.3).

Find: The approximate ellipsoidal distance between the points ZAN A and ZAN B.

Computations:

First compute the spherical angle,

$$\psi = \cos^{-1}\left(\cos\theta_A \ \cos\theta_B + \sin\theta_A \ \sin\theta_B \ \ \cos(\lambda_B - \lambda_A)\right)$$

$$\psi \ = \cos^{-1}\Big[\cos(90°- \underline{\hspace{6cm}}) \times \cos(90°- \underline{\hspace{6cm}})$$
$$+ \sin(90°- \underline{\hspace{6cm}}) \times \sin(90°- \underline{\hspace{6cm}})$$
$$\times \cos(\underline{\hspace{6cm}}) - (\underline{\hspace{6cm}})\Big]$$

$$= \cos^{-1}\Big[(\underline{\hspace{6cm}} \times \underline{\hspace{6cm}})$$
$$+ (\underline{\hspace{6cm}} \times \underline{\hspace{6cm}} \times \underline{\hspace{6cm}})\Big]$$

$$= \cos^{-1}\Big[\underline{\hspace{5cm}} + \underline{\hspace{5cm}}\Big] = \underline{\hspace{3cm}}° = \underline{\hspace{3cm}}''$$

Then compute the ellipsoid distance as (with ψ in radians) as

$$s = R_\alpha \psi = \left(\frac{R_M \ R_N}{R_M \sin^2 \tilde{\alpha}_{AB} + R_N \cos^2 \tilde{\alpha}_{AB}} \right) \psi \, .$$

From Exercise 1.3, the radius of curvature is R_α = 20,976,019 sft (rounded to the nearest foot) at the mean latitude $\overline{\varphi}$ = 61.230303600° of points ZAN A and ZAN B. This is at an (approximate) azimuth of $\tilde{\alpha}$ = 109.39693° from ZAN A and ZAN B (Exercises 1.3 and 2.2).

The spherical angle must be converted to radians for this computation, as follows:

$$s = \qquad R_\alpha \qquad \times \qquad \psi$$

$$s = \underline{\hspace{6cm}} \times \underline{\hspace{4cm}}° \times \frac{\pi}{180°} = \underline{\hspace{4cm}\textbf{sft}}$$

<u>Solution</u>:

$$\begin{aligned}\psi &= \cos^{-1}\big[\cos(90°-61.2306685500000°) \times \cos(90°-61.2299386472222°) \\ &\qquad + \sin(90°-61.2306685500000°) \times \sin(90°-61.2299386472222°) \\ &\qquad \times \cos(-149.779101363889° + 149.783408541667°)\big] \\ &= \cos^{-1}\big[(0.876564421423675 \times 0.876558290169227) \\ &\qquad + (0.481284546909806 \times 0.481295713606096 \times 0.999999997174402)\big] \\ &= \cos^{-1}\big[0.768359810466315 + 0.231640188798020\big] = \underline{0.0021977480°} = \underline{7.911893''}\end{aligned}$$

The spherical angle must be converted to radians for this computation, as follows:

$$s = 20{,}976{,}019 \text{ sft} \times 0.0021977480° \times \frac{3.14159265}{180°} = \underline{\textbf{804.597 sft}}$$

From Exercise 2.3, Vincenty inverse is the same to better than ±0.0005 ft, s = <u>804.597 sft</u> ✓

The results shown here were computed using Microsoft *Excel*™, which has a numerical precision of 15 digits. Note that most hand calculators have difficulty accurately performing these calculations due to lower numerical precision. Example computations using different numerical precisions are given below (these will vary depending on the calculator, sequence of computations, and number of digits entered):

14 digits of numerical precision ➔ s = 804.594 sft (−0.0004% error)

13 digits of numerical precision ➔ s = 804.616 sft (+0.0024% error)

12 digits of numerical precision ➔ s = 805.326 sft (+0.0906% error)

Exercise 2.5: Deflection of the vertical and the Laplace correction

In general, the plumbline (gravity vector) passing through the axis of an instrument is not parallel to a line perpendicular to the reference ellipsoid (the ellipsoid normal), and the angle between these two lines is called the *deflection of the vertical*. The deflection of the vertical is divided into north-south and east-west components, denoted as ξ and η, respectively. These can be obtained from the NGS model DEFLEC12A and USDOV2012 for any location in the US. DEFLEC12A was derived from the GEOID12A hybrid geoid model, and is the appropriate one to use for survey observations referenced to NAD 83 and NAVD 88. USDOV2012 was derived from the purely gravimetric geoid model USGG2012, which is referenced to IGS08.

If the deflection of the vertical is not zero, an instrument leveled to the local plumbline will not be "level" with respect to the ellipsoidal datum. When using terrestrial (optical) instruments, this affects determination of coordinates and azimuths using astronomic (or gyroscopic) methods; reductions of terrestrial observations to the ellipsoid; and change in ellipsoid height. In addition, since deflection of the vertical varies with location, it can cause horizontal and vertical errors in terrestrial surveys that are similar to the misclosure that occurs if a traverse is performed with an improperly leveled instrument.

The *Laplace correction* is the difference between astronomic and geodetic azimuth caused by deflection of the vertical. A simplified version of the Laplace correction is given on NGS datasheets, and *adding* this value to (clockwise) astronomic azimuths will give the geodetic azimuth for an approximately horizontal line of sight between stations.

Equation 2.9 The simplified (horizontal) Laplace correction (assumes approximately horizontal line of sight, a clockwise positive azimuth, and a positive east deflection of the vertical):

$$\boxed{L = \alpha - A = -\eta \tan\varphi}$$

where α and A are the geodetic and astronomic azimuths, respectively

η is the deflection of the vertical component in the east-west (prime vertical) direction

φ is the geodetic latitude

Rules of Thumb

Maximum deflection of the vertical in Alaska (on land) = ~90 arc-seconds

Maximum Laplace correction magnitude in Alaska (on land) = ~120 arc-seconds

Simplified Laplace correction error is less than approximately 10% for zenith angles within about 5° of horizontal

Example computation

Given: In Elbow Canyon of the Virgin Mountains of northwestern Arizona, GPS was used to locate the southwest corner and the west quarter corner of Section 16, T 39 N, R 15 W, Gila and Salt River Baseline and Meridian. The following NAD 83 coordinates were obtained:

Station	**Latitude**	**Longitude**	**Ellipsoid height**
SW Corner S16	36°46’31.61284”N	113°55’21.70113”W	3530.589 ift
W 1/4 Corner S16	36°46’57.75891”N	113°55’21.69613”W	3275.291 ift

The geodetic azimuth and horizontal ground distance from the southwest corner to the west quarter corner based on these coordinates is 0° 00’ 31.73” and 2644.715 ift.

Find: The astronomic quadrant bearing from the southwest corner to the west quarter corner of Section 16.

Computations:
For the southwest corner of Section 16, DEFLEC09 gives ξ = 9.18”, $\eta = -26.48$”, and L = 19.79”. Equation 2.9 can be rearranged to compute the astronomic azimuth:

$A = \alpha - L =$ ________________ − __________ = ________________

Astronomic quadrant bearing = _________________ (rounded to nearest arc-second)

Solution:
For the southwest corner of Section 16, DEFLEC09 gives ξ = 9.18”, $\eta = -26.48$”, and L = 19.79”. Equation 2.9 can be rearranged to compute the astronomic azimuth:

$A = \alpha - L =$ <u>0° 00’ 31.73”</u> − <u>19.18”</u> = <u>0° 00’ 12.55”</u>

Astronomic quadrant bearing = <u>**N 00° 00’ 13” E**</u> (rounded to nearest arc-second)

Check: $L = -\eta \tan\varphi$ = −(+26.48”) × tan(36°46’32”N) = 19.79”, as given by DEFLEC09.

How accurate is the simplified (horizontal) Laplace correction?

The complete Laplace correction is given by $L = -\eta \tan\varphi - (\xi \sin\alpha - \eta\cos\alpha)\cot\zeta$, where the first term is the same as Equation 2.9, and the second term is referred to as the *deflection correction*. The quantity ζ is the geodetic zenith angle, which can be estimated using the ellipsoid height difference and distance between the corners. Earth curvature increases the zenith angle, and can be accounted for by subtracting 0.0239×(distance in thousands of feet)^2 from the height difference (this correction is covered in more detail in Exercise 4.2):

$$\zeta = 90° - \tan^{-1}\left\{[(3275.291 - 3530.589) - 0.0239\times 2.644715^2] / 2644.715\right\} = 95.517°.$$

Thus the deflection correction is:

$$[9.18" \times \sin(0°00'31.73") - (-26.48") \times \cos(0°00'31.73")] \times \cot(95.517°) = \underline{-2.56"}$$

This gives a complete Laplace correction of L = 19.79” − (−2.56”) = 22.35”. Although this deflection correction is rather large, note that this is a worse-case scenario, because the deflection of the vertical value in this example is essentially the maximum for Arizona. In most cases, the deflection correction is smaller than can be resolved using optical methods, and the simplified Laplace correction will suffice. This helps tremendously, since the simplified Laplace correction does not depend on the azimuth or zenith angle between stations, and so a unique value can be specified at a point.

Section 3

GRID COORDINATE SYSTEMS AND COMPUTATIONS

How are the data displayed? How are the data used?

Examples of grid coordinate errors for Alaska

Table 3.1 Examples of various positioning error sources and their magnitudes for Alaska due to grid coordinate system and computation problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Alaska	Error magnitudes
Using SPCS 27 projection parameters for SPCS 83 projects (easting coordinate differences)	216 ***miles*** (Zones 2-9) 53.2 ***miles*** (Zone 10) *Zero* (Zone 1)
Determining State Plane coordinates in international feet when US survey feet are required	Up to 13 feet (horizontal)
Determining UTM coordinates in international feet when US survey feet are required	Up to 52 feet (horizontal)
Using linear coordinates from a geographic "projection" to compute distances	Up to ~3500 feet horizontal per mile (67% error)
Using SPCS grid distances when "ground" distances are required (example here is for point on projection axis)	Apprx −1.8 feet horizontal per mile at elevation of 1500 feet
Using UTM grid distances when "ground" distances are required (example here is for point on central meridian)	Apprx −3.4 feet horizontal per mile at elevation of 1500 feet
Using planar computation methods to transform geodetically-derived horizontal coordinates (example here is for converting from UTM to SPCS over a 10 mi × 10 mi area using planar scaling, rotation, and translation based on two common points)	Varies, but increases rapidly with size of area (~1 foot horizontal for this example)

Grid coordinate system information in NGS Datasheets and OPUS output

Both NGS Datasheets and OPUS output use the geodetic coordinates of the point to compute grid (map projection) coordinates in the State Plane and Universal Transverse Mercator coordinate systems. They also provide the convergence angle, grid point scale factor, and combined scale factor for both systems.

Portion of NGS Datasheet for station CE 314 U OF A RESET (TT2845)

```
 TT2845 ***********************************************************************
 TT2845  DESIGNATION -  CE 314 U OF A RESET
 TT2845  PID         -  TT2845
 TT2845  STATE/COUNTY-  AK/FAIRBANKS NORTH STAR
 TT2845  COUNTRY     -  US
 TT2845  USGS QUAD   -  FAIRBANKS D-2
 TT2845
 TT2845                         *CURRENT SURVEY CONTROL
 TT2845  ______________________________________________________________________
 TT2845* NAD 83(2011) POSITION- 64 51 21.30585(N) 147 49 08.66783(W)   ADJUSTED
 TT2845* NAD 83(2011) ELLIP HT-   168.829 (meters)        (06/27/12)   ADJUSTED
 TT2845* NAD 83(2011) EPOCH   -  2010.00
 TT2845* NAVD 88 ORTHO HEIGHT -   159.371  (meters)      522.87  (feet) ADJUSTED
 TT2845  ______________________________________________________________________
 TT2845  NAD 83(2011) X  - -2,300,080.306 (meters)                     COMP
 TT2845  NAD 83(2011) Y  - -1,447,368.715 (meters)                     COMP
 TT2845  NAD 83(2011) Z  -  5,751,055.481 (meters)                     COMP
.
.
                                                        = k           = γ
 TT2845;                    North         East     Units Scale Factor Converg.
 TT2845;SPC AK 3     - 1,210,477.976   413,740.131   MT  0.99999107   -1 38 48.5
 TT2845;UTM  06      - 7,192,648.384   461,168.110   MT  0.99961846   -0 44 29.3
 TT2845
 TT2845!             -  Elev Factor  x  Scale Factor =   Combined Factor   = δ + 1
 TT2845!SPC AK 3     -   0.99997359  x   0.99999107  =   0.99996466
 TT2845!UTM  06      -   0.99997359  x   0.99961846  =   0.99959206
```

Portion of OPUS output for station CLGO

```
                               NGS OPUS SOLUTION REPORT
                               ========================

 REF FRAME: NAD_83(2011)(EPOCH:2010.0000)              IGS08 (EPOCH:2014.1208)

         X:      -2299608.699(m)   0.013(m)          -2299609.729(m)   0.013(m)
         Y:      -1444754.305(m)   0.002(m)          -1444753.276(m)   0.002(m)
         Z:       5751925.446(m)   0.008(m)           5751925.756(m)   0.008(m)

       LAT:   64 52 25.58665       0.007(m)       64 52 25.58141       0.007(m)
     E LON:  212  8 22.34943       0.008(m)      212  8 22.24164       0.008(m)
     W LON:  147 51 37.65057       0.008(m)      147 51 37.75836       0.008(m)
    EL HGT:          195.736(m)    0.011(m)               196.155(m)   0.011(m)
 ORTHO HGT:          186.254(m)    0.020(m) [NAVD88 (Computed using GEOID12A)]

                        UTM COORDINATES    STATE PLANE COORDINATES
                         UTM (Zone 06)         SPC (5004 AK 4)
Northing (Y) [meters]     6791492.562           805057.180
Easting (X)  [meters]      350745.410           511804.561
Convergence  [degrees]    -2.43745613           0.19264553 = γ
Point Scale                0.99987294           0.99990171    = k
Combined Factor            0.99986045           0.99988921        = δ + 1
```

Map projection distortion

Map projection distortion is an *unavoidable* consequence of attempting to represent a curved surface on a flat surface. It can be thought of as a change in the "true" relationship between points located on the surface of the Earth and the *representation* of their relationship on a plane. Distortion cannot be eliminated — it is a ***F***act of ***L***ife. The best we can do is decrease certain effects (there is often a trade-off between one distortion and another and they cannot all be reduced to their absolute minimums simultaneously).

There are two general types of map projection distortion:

1. Linear distortion. Difference in distance between a pair of grid (map) coordinates when compared to the true horizontal ("ground") distance, denoted here by $\boldsymbol{\delta}$. There is no widely accepted definition of a horizontal ground distance. In this workbook, it is defined as the geodesic (ellipsoid) distance scaled to the mean topographic ellipsoid height of the endpoints using the geometric mean radius of curvature at the mean latitude of the endpoints.
   - Can express as a ratio of distortion length to ground length:
     - E.g., feet of distortion per mile; parts per million (= mm per km)
     - *Note:* 1 foot / mile = 189 ppm = 189 mm / km
   - Linear distortion can be positive or negative:
     - NEGATIVE distortion means the grid (map) length is SHORTER than the "true" horizontal (ground) length.
     - POSITIVE distortion means the grid (map) length is LONGER than the "true" horizontal (ground) length.
2. Angular distortion. For conformal projections (e.g., Transverse Mercator, Lambert Conformal Conic, Stereographic, Oblique Mercator), it equals the *convergence* (*mapping*) *angle*, $\boldsymbol{\gamma}$. This is the difference between grid (map) north and true (geodetic) north.
   - Convergence angle is zero on the projection central meridian, positive east of the central meridian, and negative west of the central meridian.
   - Magnitude of the convergence angle increases with distance from the central meridian (CM), and its rate of change increases with increasing latitude:

Latitude	**Convergence angle 1 mile from CM**	**Latitude**	**Convergence angle 1 mile from CM**
0°	0° 00' 00"	50°	±0° 01' 02"
10°	±0° 00' 09"	60°	±0° 01' 30"
20°	±0° 00' 19"	70°	±0° 02' 23"
30°	±0° 00' 30"	80°	±0° 04' 54"
40°	±0° 00' 44"	89°	±0° 49' 32"

   - Usually not as much of a concern as linear distortion, and for all conformal projections (other than the regular Mercator), it only be minimized be staying close to the central meridian (or near the Equator).

Total linear distortion of grid (map) coordinates is a combination of distortion due to Earth curvature and distortion due to ground height above the ellipsoid. In many places, distortion due to variation in ground height is greater than that due to curvature.

Table 3.2 Horizontal distortion of grid coordinates due to Earth curvature

Maximum zone width for secant projections (miles)	Maximum linear horizontal distortion, δ		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
16 miles	±1 ppm	±0.005 ft/mile	1 : 1,000,000
50 miles	±10 ppm	±0.05 ft/mile	1 : 100,000
71 miles	**±20 ppm**	**±0.1 ft/mile**	**1 : 50,000**
112 miles	±50 ppm	±0.3 ft/mile	1 : 20,000
159 miles (e.g., SPCS)*	±100 ppm	±0.5 ft/mile	1 : 10,000
318 miles (e.g., UTM)†	±400 ppm	±2.1 ft/mile	1 : 2500

*State Plane Coordinate System; zone width shown is valid between ~45° and 85° latitude
†Universal Transverse Mercator; zone width shown is valid between ~60° and 85° latitude

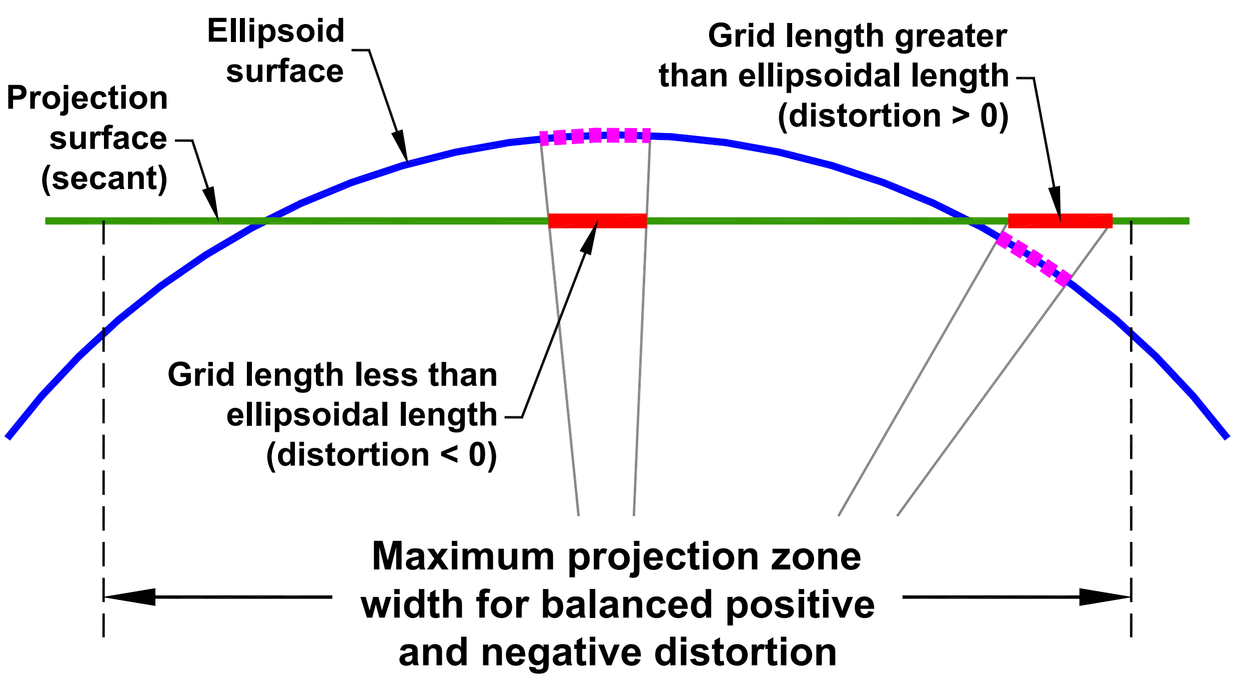
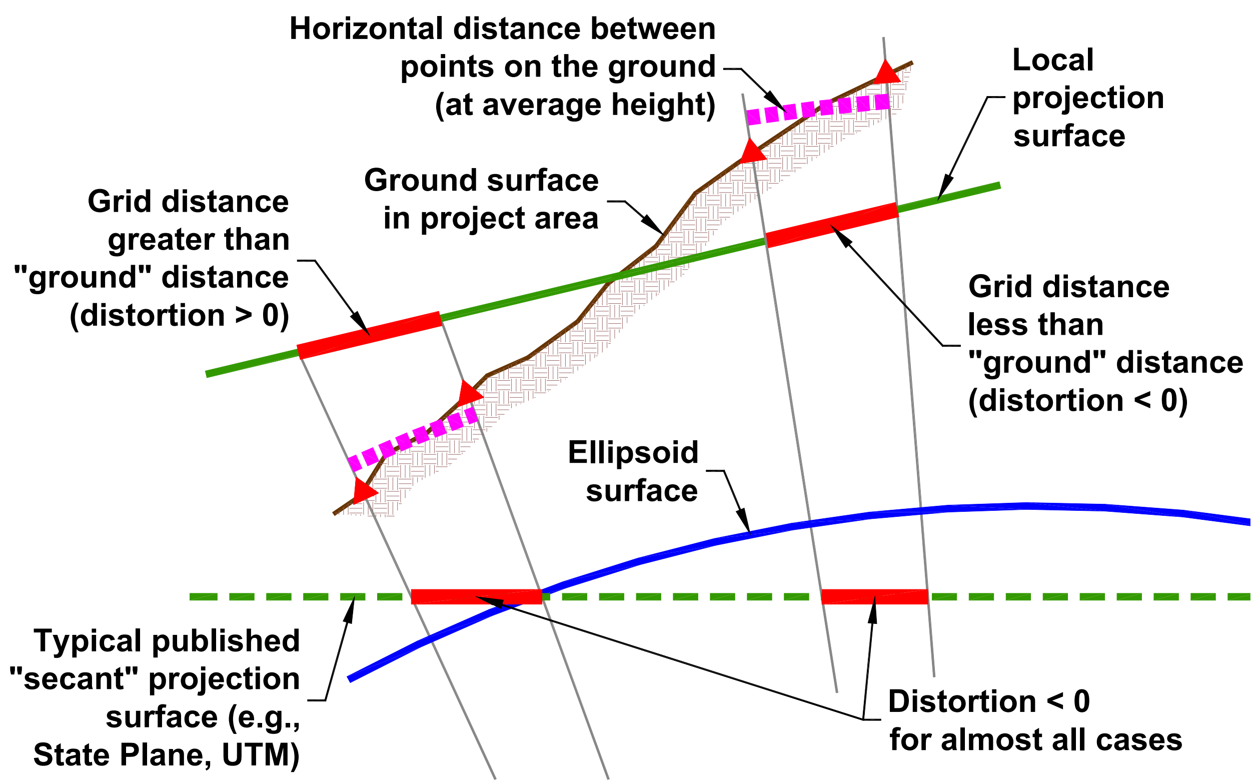


Table 3.3 Horizontal distortion of grid coordinates due to ground height above the ellipsoid

Height below (–) and above (+) projection surface	Maximum linear horizontal distortion, δ		
	Parts per million (mm/km)	Feet per mile	Ratio (absolute value)
–100 feet, +100 feet	±4.8 ppm	±0.03 ft/mile	~1 : 209,000
–400 feet, +400 feet	**±19 ppm**	**±0.1 ft/mile**	**~1 : 52,000**
–1000 feet, +1000 feet	±48 ppm	±0.3 ft/mile	~1 : 21,000
+1500 feet*	–72 ppm	–0.4 ft/mile	~1 : 14,000
+3000 feet	–143 ppm	–0.8 ft/mile	~1 : 7000
+10,000 feet	–477 ppm	–2.5 ft/mile	~1 : 2100
+20,000 feet†	–953 ppm	–5.0 ft/mile	~1 : 1000

* Approximate average topographic height in Alaska
† Approximate maximum topographic height in Alaska

Rule of Thumb:
A 100-ft change in height causes a 4.8 ppm change in distortion



Exercise 3.1: Distortion computations

Linear distortion is the ratio of grid distance to horizontal ground distance. One way to estimate distortion is to compute the distance between a pair of points based on the grid coordinates determined by the GPS software. This grid distance can then be divided by the ground distance between these points measured using a (properly calibrated) tape or EDM.

Equation 3.1 Approximating distortion at a point using measured grid and ground distances

$$\boxed{\delta \approx \left(\frac{\sqrt{\Delta N^2 + \Delta E^2}}{\text{measured horizontal ground distance}} \right) - 1}$$

Distortion can be computed more accurately (and conveniently) at a single point using the familiar "combined scale factor" approach:

Equation 3.2 Computing distortion at a point using Earth radius

$$\boxed{\delta = k\left(\frac{R_G}{R_G + h} \right) - 1}$$

Example computation

Given: Points ZAN A and ZAN B from the previous examples. The ellipsoid heights (h) of these points are listed below, along with the grid coordinates and grid point scale factors (k) derived from the adjusted geodetic coordinates (given in Exercise 2.2). A horizontal ground distance of 804.606 sft was carefully measured between these stations.

ZAN A: NAD 83 (NSRS2007) ellipsoid height, h = 219.317 sft

Coordinate system	Northing, N (sft)	Easting, E (sft)	Grid scale factor, k
SPCS 83, Alaska Zone 4 (5004)	2,641,792.282	1,678,582.475	0.999 901 657
UTM 83, Zone 6 North	22,282,314.286	1,150,199.349	0.999 873 540
Low Distortion Projection (LDP)	84,334.384	58,798.366	1.000 020 088

ZAN B: NAD 83 (NSRS2007) ellipsoid height, h = 220.826 sft

Coordinate system	Northing, N (sft)	Easting, E (sft)	Grid scale factor, k
SPCS 83, Alaska Zone 4 (5004)	2,641,528.002	1,679,342.347	0.999 901 724
UTM 83, Zone 6 North	22,282,015.425	1,150,946.272	0.999 872 707
Low Distortion Projection (LDP)	84,068.136	59,557.652	1.000 020 104

Find: The linear distortion (in parts per million) at the midpoint between points ZAN A and ZAN B in SPCS, UTM, and LDP coordinates using both Equations 3.1 and 3.2 (the geometric mean radius of curvature R_G = 20,963,274 sft was determined at the midpoint in Exercise 1.3, and the Low Distortion Projection parameters are from Example 3.2 in this workbook).

Computations: For midpoint, use the mean grid scale factor and mean ellipsoid height = 220 ft.

SPCS 83 AK 4

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\underline{\hspace{4cm}})^2 + (\underline{\hspace{2.5cm}} - \underline{\hspace{2.5cm}})^2}}{\underline{\hspace{2.5cm}}} \right) - 1 = \left(\frac{}{} \right) - 1$$

$$= \underline{\hspace{2.5cm}} - 1 \rightarrow \textit{in parts per million} \rightarrow \underline{\hspace{2.5cm}} \times 1{,}000{,}000 = \underline{\hspace{2cm}}$$

Using Equation 3.2:

$$\delta = \frac{\underline{\hspace{2cm}} + \underline{\hspace{2cm}}}{2} \left(\frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2cm}} + \underline{\hspace{1.5cm}}} \right) - 1 = \underline{\hspace{2cm}} - 1 \rightarrow \underline{\hspace{2cm}}$$

UTM 83 6N

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\underline{\hspace{4cm}})^2 + (\underline{\hspace{2.5cm}} - \underline{\hspace{2.5cm}})^2}}{\underline{\hspace{2.5cm}}} \right) - 1 = \left(\frac{}{} \right) - 1$$

$$= \underline{\hspace{2.5cm}} - 1 \rightarrow \textit{in parts per million} \rightarrow \underline{\hspace{2.5cm}} \times 1{,}000{,}000 = \underline{\hspace{2cm}}$$

Using Equation 3.2:

$$\delta = \frac{\underline{\hspace{2cm}} + \underline{\hspace{2cm}}}{2} \left(\frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2cm}} + \underline{\hspace{1.5cm}}} \right) - 1 = \underline{\hspace{2cm}} - 1 \rightarrow \underline{\hspace{2cm}}$$

LDP

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(\underline{\hspace{4cm}})^2 + (\underline{\hspace{2.5cm}} - \underline{\hspace{2.5cm}})^2}}{\underline{\hspace{2.5cm}}} \right) - 1 = \left(\frac{}{} \right) - 1$$

$$= \underline{\hspace{2.5cm}} - 1 \rightarrow \textit{in parts per million} \rightarrow \underline{\hspace{2.5cm}} \times 1{,}000{,}000 = \underline{\hspace{2cm}}$$

Using Equation 3.2:

$$\delta = \frac{\underline{\hspace{2cm}} + \underline{\hspace{2cm}}}{2} \left(\frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2cm}} + \underline{\hspace{1.5cm}}} \right) - 1 = \underline{\hspace{2cm}} - 1 \rightarrow \underline{\hspace{2cm}}$$

Solution: For midpoint, use the mean grid scale factor and mean ellipsoid height = 220 ft.

SPCS 83 AK 4

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(2{,}641{,}528.002 - 2{,}641{,}792.282)^2 + (1{,}679{,}342.347 - 1{,}678{,}582.475)^2}}{804.606} \right) - 1 = \left(\frac{804.518}{804.606} \right) - 1$$

= 0.9998906 – 1 ➔ *in parts per million* ➔ –0.0001094 × 1,000,000 = **–109.4 ppm**

Using Equation 3.2:

$$\delta = \frac{0.999901657 + 0.999901724}{2} \left(\frac{20{,}963{,}274}{20{,}963{,}274 + 220} \right) - 1 = 0.9998912 - 1$$ ➔ **–108.8 ppm**

(= –0.57 ft/mile)

UTM 83 6N

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(22{,}282{,}015.337 - 22{,}282{,}314.199)^2 + (1{,}150{,}946.234 - 1{,}150{,}199.311)^2}}{804.606} \right) - 1 = \left(\frac{804.495}{804.606} \right) - 1$$

= 0.9998620 – 1 ➔ *in parts per million* ➔ –0.0001380 × 1,000,000 = **–138.0 ppm**

Using Equation 3.2:

$$\delta = \frac{0.999873540 + 0.999872707}{2} \left(\frac{20{,}963{,}274}{20{,}963{,}274 + 220} \right) - 1 = 0.9998626 - 1$$ ➔ **–137.4 ppm**

(= –0.73 ft/mile)

LDP

Using Equation 3.1:

$$\delta \approx \left(\frac{\sqrt{(84{,}068.136 - 84{,}334.384)^2 + (59{,}557.652 - 58{,}798.366)^2}}{804.606} \right) - 1 = \left(\frac{804.614}{804.606} \right) - 1$$

= 1.0000099 – 1 ➔ *in parts per million* ➔ +0.0000099 × 1,000,000 = **+9.9 ppm**

Using Equation 3.2:

$$\delta = \frac{1.000020088 + 1.000020104}{2} \left(\frac{20{,}963{,}274}{20{,}963{,}274 + 220} \right) - 1 = 1.0000096 - 1$$ ➔ **+9.6 ppm**

(= +0.05 ft/mile)

Exercise 3.2: Six steps for designing a low-distortion grid coordinate system

1. **Define project area and performance objective, and determine *representative* ellipsoid height, h_0 (<u>not</u> elevation)**
   - The usual performance objective is to achieve minimum distortion over largest area possible. However, distortion typically increases as the size of the area increases, so it is an optimization problem. A common performance objective is to achieve ±20 ppm (±0.1 ft/mile) in "important" parts of the design area.
   - The *average* height of an area may not be appropriate (e.g., project near a mountain)
     - Usually no need to estimate height to an accuracy of better than about ±50 feet (in small areas of mild topographic relief ±20 feet is sufficient)
   - Note that as the size of the area increases, the effect of Earth curvature on distortion increases and it must be considered in addition to the effect of topographic height
     - E.g., for areas wider than about 35 miles (perpendicular to the projection axis), distortion due to curvature alone exceeds 5 parts per million (ppm)
2. **Place central meridian near centroid of project area**
   - For Transverse Mercator projection, in some cases it may be advantageous to offset the central meridian to account for east-west topographic slope in design area
3. **Scale central meridian of projection to representative ground height, h_0**

   Equation 3.3 Local map projection scaled to "ground"

   $$\boxed{k_0 = 1 + \frac{h_0}{R_G}}$$

   - Where R_G is geometric mean radius of curvature, $R_G = \dfrac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi}$ (Equation 1.7)
     - Alternatively, can initially approximate R_G as 20,960,000 feet for Anchorage (since k_0 will likely be refined in Step #4)
   - This procedure is for the Transverse Mercator projection
     - For Lambert Conical projection, use same equation for scale of standard parallel
4. **Check distortion at points distributed throughout project area**
   - Best approach is to compute distortion over entire area and generate distortion contours (this ensures optimal low-distortion coverage)
     - May require repeated evaluation using different k_0 values
   - Distortion computed at a point as $\delta = k\left(\dfrac{R_G}{R_G + h}\right) - 1$ (Equation 3.2)
     - Where k = projection grid scale factor at a point (with respect to ellipsoid; see Equations 3.3 and 3.4)
     - Multiply δ by 1,000,000 to get distortion in *parts per million* (ppm)

5. Keep the definition SIMPLE and CLEAN!

- Define k_0 to no more than SIX decimal places, e.g., 1.000206 (exact)
  - *Note:* A change of one unit in the sixth decimal place equals distortion caused by a 21-foot change in height
- Defining central meridian and latitude of grid origin to nearest whole arc-minute usually adequate (e.g., Central meridian = 111°48'00" W)
- Define grid origin using large whole values with as few digits as possible (e.g., False easting = 50,000; Max coordinate < 100,000)

6. *Explicitly* define linear unit and ellipsoidal datum

- E.g., Linear unit = international foot; Geometric reference system = North American Datum of 1983.
  - The international foot is shorter than the US survey foot by 2 ppm. Because coordinate systems typically use large values, it is critical that the type of foot used be identified (the values differ by 1 foot per 500,000 feet).
- *Note:* The reference system realization (i.e., "datum tag") should not be included in the coordinate system definition (just as it is not included in State Plane definitions). However, the datum tag *is* an essential component for defining the spatial data used within the coordinate system. For NAD 83, the NGS convention is to give the datum tag in parentheses after the datum name, usually as the year in which the datum was "realized" as part of a network adjustment. Common datum tags for horizontal control are listed below:
  - "2011" for the current NAD 83 (2011) epoch 2010.00 realization, which is referenced to the North America tectonic plate.
  - "2007" for the (superseded) NSRS2007 (National Spatial Reference System of 2007) realization. Superseded "CORS96" datum tag is referenced to an epoch date of 2003.00 for Alaska.
  - "1992" for the various Alaska "HARN" realization.

Example computation
Design a Low Distortion Projection (LDP) for Anchorage

1. Define project area and choose *representative* ellipsoid height, h_0 (<u>not</u> elevation)

From topo maps and benchmark information, a representative ellipsoid height is h_0 = **<u>420 ft</u>** (no need for greater accuracy than nearest ±10 feet)

2. Place central meridian near centroid of project area

Based on location and extent of Anchorage, a good, clean value is λ_0 = **<u>149° 50' 00" W</u>**

3. Scale central meridian of projection to representative ground height, h_0

First compute Earth radius at mid-latitude of Anchorage, φ = 61° 10' 00" N (no need for greater accuracy than nearest arc-minute of latitude):

$$R_G = \frac{a\sqrt{1-e^2}}{1-e^2\sin^2\varphi} = \frac{20{,}925{,}604.474 \times \sqrt{1-0.006694380023}}{1-0.006694380023\times\left[\sin(61.16666667°)\right]^2} = \underline{20{,}963{,}141 \text{ sft}}$$

Thus the central meridian scale factor scaled to the representative ellipsoid height is

$$k_0 = 1+\frac{h_0}{R_G} = 1+\frac{420}{20{,}963{,}141} = \underline{\mathbf{1.000\ 020}}$$

Based on these results, the following Transverse Mercator projection is defined (will refine definition if necessary based on results of Step #4):

Latitude of grid origin,	φ_0 = 61° 00' 00" N
Longitude of central meridian,	λ_0 = 149° 50' 00" W
False northing,	N_0 = 0.000 sft
False easting,	E_0 = 50,000.000 sft
Central meridian scale factor,	k_0 = 1.000 02 (exact)

4. **Check distortion at points distributed throughout project area**

Distortion can be computed at various points throughout the project area. These can be survey control points or even artificial points taken from topo maps.

To illustrate, we can compute the distortion at station ZAN A (computed at the point rather than at midpoint as in the previous example, but with same results):

$$\delta = k\left(\frac{R_G}{R_G+h}\right)-1 = 1.00002009\times\left(\frac{20{,}963{,}274}{20{,}963{,}274+219}\right)-1 = 1.000\ 009\ 6 - 1 = \underline{\mathbf{+9.6\ ppm}}$$

For NGS station UNSTAMPED 4 (PID DH4689), in the southeastern high elevation part of Anchorage) we have:

$$\delta = k\left(\frac{R_G}{R_G+h}\right)-1 = 1.00002028\times\left(\frac{20{,}963{,}050}{20{,}963{,}050+876}\right)-1 = 0.999\ 978\ 5 - 1 = \underline{\mathbf{-21.5\ ppm}}$$

This computation can be performed at discrete points throughout the project area, but best approach is to compute distortion over entire area (for example on a 3-arc-second grid) and generate distortion contours to ensure optimal low-distortion coverage.

The ability to achieve low distortion is limited by change in elevation (height) within the project area. A reasonable goal might be to limit distortion to ±0.1 ft per mile, which is about ±20 ppm and corresponds to a height change of about ±400 ft.

5. **Keep the definition SIMPLE and CLEAN!**

All of the projection parameters were initially defined in Step #3, but trial-and-error may be necessary to refine definition.

- Note k_0 is defined to *exactly* SIX decimal places: $\boldsymbol{k_0}$ **= 1.000 02 (exact)**

- Both latitude of grid origin and central meridian are defined to nearest whole arc-minute:

$\boldsymbol{\varphi_0}$ = 61° 00' 00" N and **$\boldsymbol{\lambda_0}$ = 149° 50' 00" W**

φ_0 was selected far enough south to ensure positive northings, but far enough north to keep northings less than 100,000 sft.

- Grid origin is defined using clean whole values with as few digits as possible:

$$\boldsymbol{N_0 = 0.000 \text{ sft}} \quad \text{and} \quad \boldsymbol{E_0 = 50{,}000.000 \text{ sft}}$$

These values were selected to keep grid coordinates positive but less than 100,000 sft within the Anchorage area (it is conventional to set N_0 to zero at φ_0 but is not required).

6. *Explicitly* define linear unit and ellipsoidal datum

Linear unit is **US Survey Foot**, and ellipsoidal datum is **NAD 83**

Final Low Distortion Projection definition for this example:

Linear unit: US Survey Foot
Ellipsoidal datum: North American Datum of 1983
System: Alaska LDP
Zone: Anchorage
Projection: Transverse Mercator
Latitude of grid origin: 61° 00' 00" N
Longitude of central meridian: 149° 50' 00" W
Northing at grid origin: 0.000 ft
Easting at central meridian: 50,000.000 sft
Scale factor on central meridian: 1.000 02 (exact)

Note that this coordinate system definition only deals with horizontal coordinates (no vertical datum is specified).

Methods for creating low-distortion grid coordinate systems

1. **Design a Low Distortion Projection (LDP) for a specific project geographic area.**

   Use a conformal projection referenced to the existing geodetic datum.

   Described in detail previously in this document.

2. **Scale the reference ellipsoid "to ground".**

   A map projection referenced to this new "datum" is then designed for the project area.

   Problems:

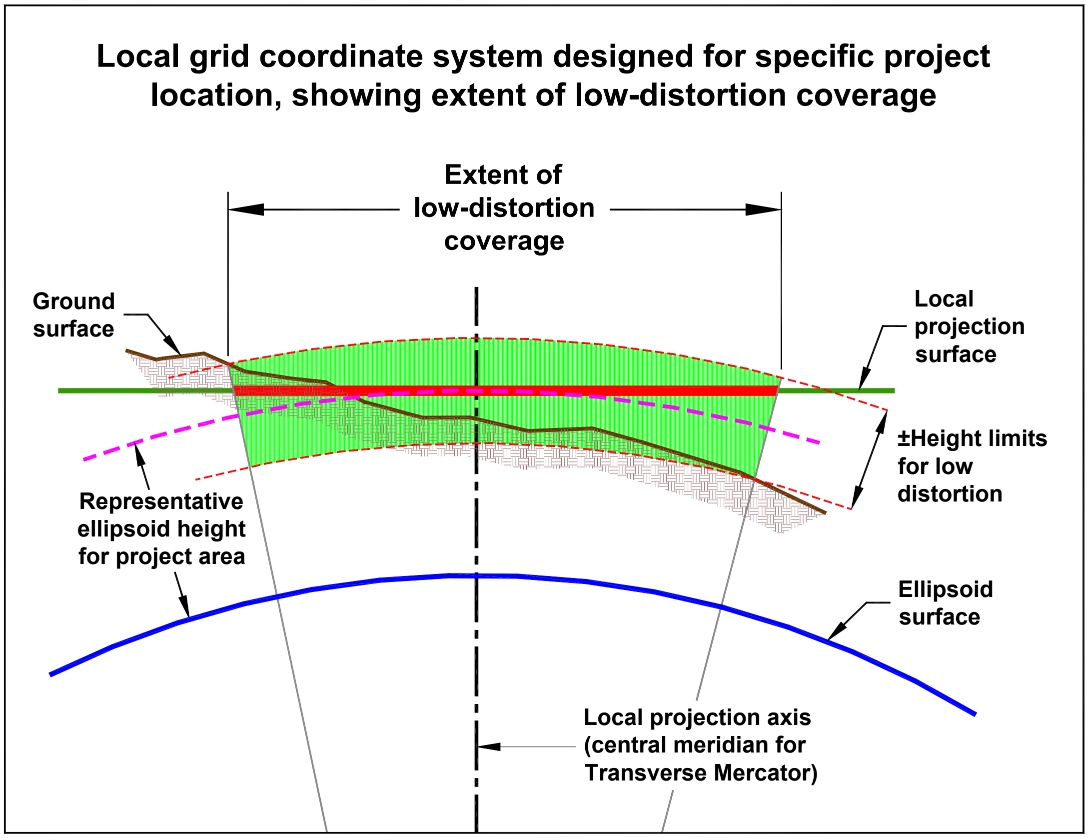
   - Requires a new ellipsoid (datum) for every coordinate system, which makes it more difficult to implement than an LDP.
   - New datum makes it more complex than an LDP, yet it does not perform any better.
   - *Generates new set of latitudes that can be substantially different from original latitudes.*
     - Change in latitude can exceed 3 feet per 1000 ft of topographic height, depending on method used for scaling the ellipsoid (this case is for scaling with constant flattening).
     - Can lead to confusion over which latitude values are correct.

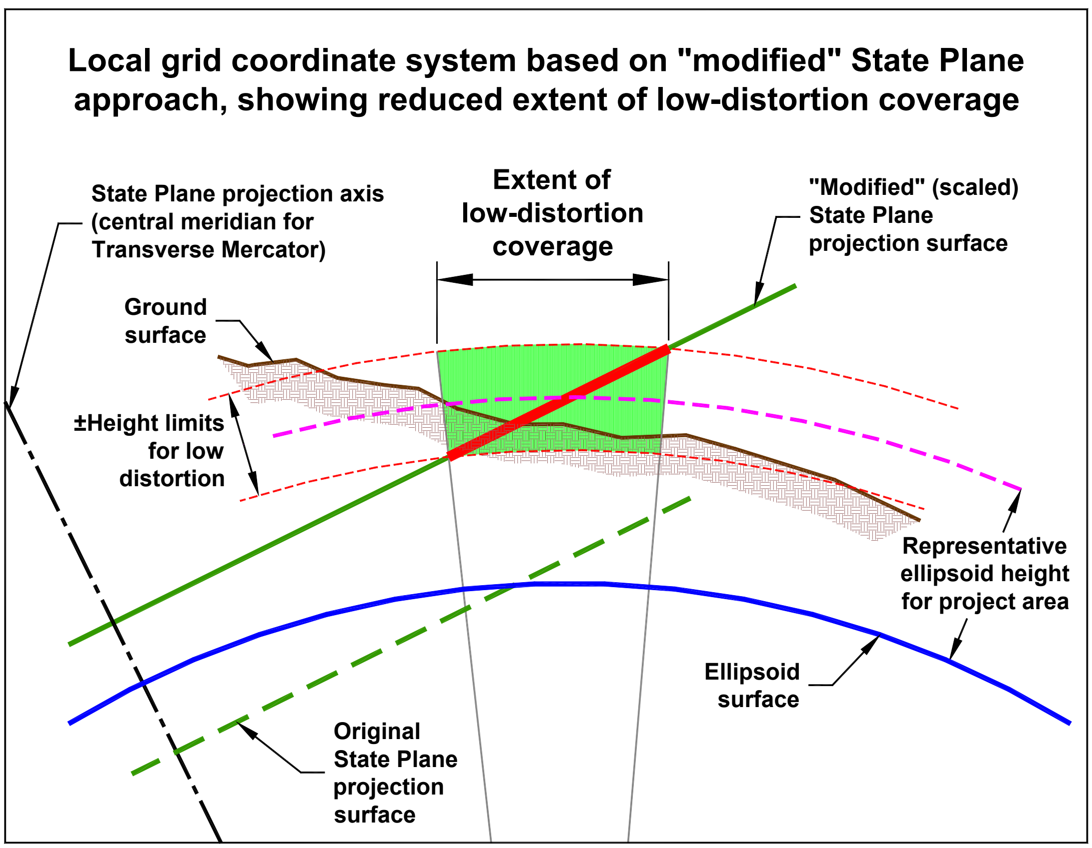
3. **Scale an existing published map projection "to ground".**

   Referred to as "modified" State Plane when an existing SPCS projection definition is used.

   Problems:

   - Generates coordinates with values similar to "true" State Plane (can cause confusion).
     - Can eliminate this problem by translating grid coordinates to get smaller values.
   - Often yields "messy" parameters when a projection definition is back-calculated from the scaled coordinates (in order to import the data into a GIS).
     - More difficult to implement in a GIS, and may cause problems due to rounding or truncating of "messy" projection parameters (especially for large coordinate values).
     - Can reduce this problem through judicious selection of "scaling" parameters.
   - Does ***not*** reduce the convergence angle (it is same as that of original SPCS definition).
     - In addition, the *arc-to-chord correction* may be significant; it can reach ½ arc-second for a 1-mile line located 75 miles from the projection axis (this correction is used along with the convergence angle for converting grid azimuths to geodetic azimuths).
   - ***MOST IMPORTANT:* Usually does not minimize distortion over as large an area as the other two methods.**
     - Extent of low-distortion coverage generally *decreases* as distance *increases* from projection axis (i.e., central meridian for TM and central parallel for LCC projection).
     - State Plane axis usually does NOT pass through the project area.
     - *Sketches illustrating this problem with "modified" SPCS are shown on the next page.*


Local grid coordinate system designed for specific project location, showing extent of low-distortion coverage
Extent of low-distortion coverage
Ground surface
Local projection surface
±Height limits for low distortion
Representative ellipsoid height for project area
Ellipsoid surface
Local projection axis (central meridian for Transverse Mercator)


Local grid coordinate system based on "modified" State Plane approach, showing reduced extent of low-distortion coverage
State Plane projection axis (central meridian for Transverse Mercator)
Extent of low-distortion coverage
"Modified" (scaled) State Plane projection surface
Ground surface
±Height limits for low distortion
Representative ellipsoid height for project area
Ellipsoid surface
Original State Plane projection surface

Exercise 3.3: Computing horizontal ground distance using geodetic coordinates

Equation 3.4 Approximate geodetic "ground" distance based on ellipsoid distance (geodesic)

$$\boxed{D_{grnd} = s\left(1 + \frac{\bar{h}}{\bar{R}_G}\right)}$$

where s is the ellipsoid distance (geodesic)

$\bar{h}$ is the average ellipsoid height of the two points

$\bar{R}_G$ is the average geometric radius of curvature at the two points

Example computation

Given: ZAN A and ZAN B from the previous example (Exercises 3.1).

Find: The ground distance between these points.

Computations:

From Exercise 2.3 and 2.4, ellipsoid distance (geodesic) is s = 804.597 sft

From Exercises 1.3 and 3.1, R_G = 20,963,274 sft at midpoint between ZAN A and ZAN B (which is the same as the average R_G for the two points)

From the ellipsoid heights in Exercise 3.1, the average ellipsoid height is

$$\bar{h} = (\underline{\hspace{3cm}} + \underline{\hspace{3cm}})\ /\ 2 = \underline{\hspace{3cm}\text{sft}}$$

So ground distance is

$$D_{grnd} = s\left(1 + \frac{\bar{h}}{\bar{R}_G}\right) = \underline{\hspace{3cm}} \times \left(1 + \frac{\hspace{3cm}}{\hspace{3cm}}\right) = \underline{\hspace{2.5cm}\textbf{sft}}$$

Solution:

From Exercise 2.4, ellipsoid distance (geodesic) is s = 804.597 sft

From Exercises 1.3 and 3.1, R_G = 20,963,274 sft at point midpoint between ZAN A and ZAN B (which is the same as the average R_G for the two points)

From the ellipsoid heights in Exercise 3.1, the average ellipsoid height is

$$\bar{h} = (219.317 + 220.826)\ /\ 2 = 220.072 \text{ sft}$$

So ground distance is

$$D_{grnd} = 804.597 \times \left(1 + \frac{220.072}{20{,}963{,}274}\right) = 804.597 \times 1.000010498 = \underline{\textbf{804.605 sft}}$$

Exercise 3.4: Grid versus geodetic bearings

Illustrates misclosure problem with geodetic azimuths, and shows how to convert grid azimuths to geodetic azimuths.

Equation 3.5 Relationship between grid and forward geodetic azimuth from point *A* to *B*

$$\boxed{\alpha_{AB} = t_{AB} + \gamma_A - (t - T)_{AB}}$$

where α_{AB} and t_{AB} = geodetic and grid azimuths from point *A* to *B*, respectively

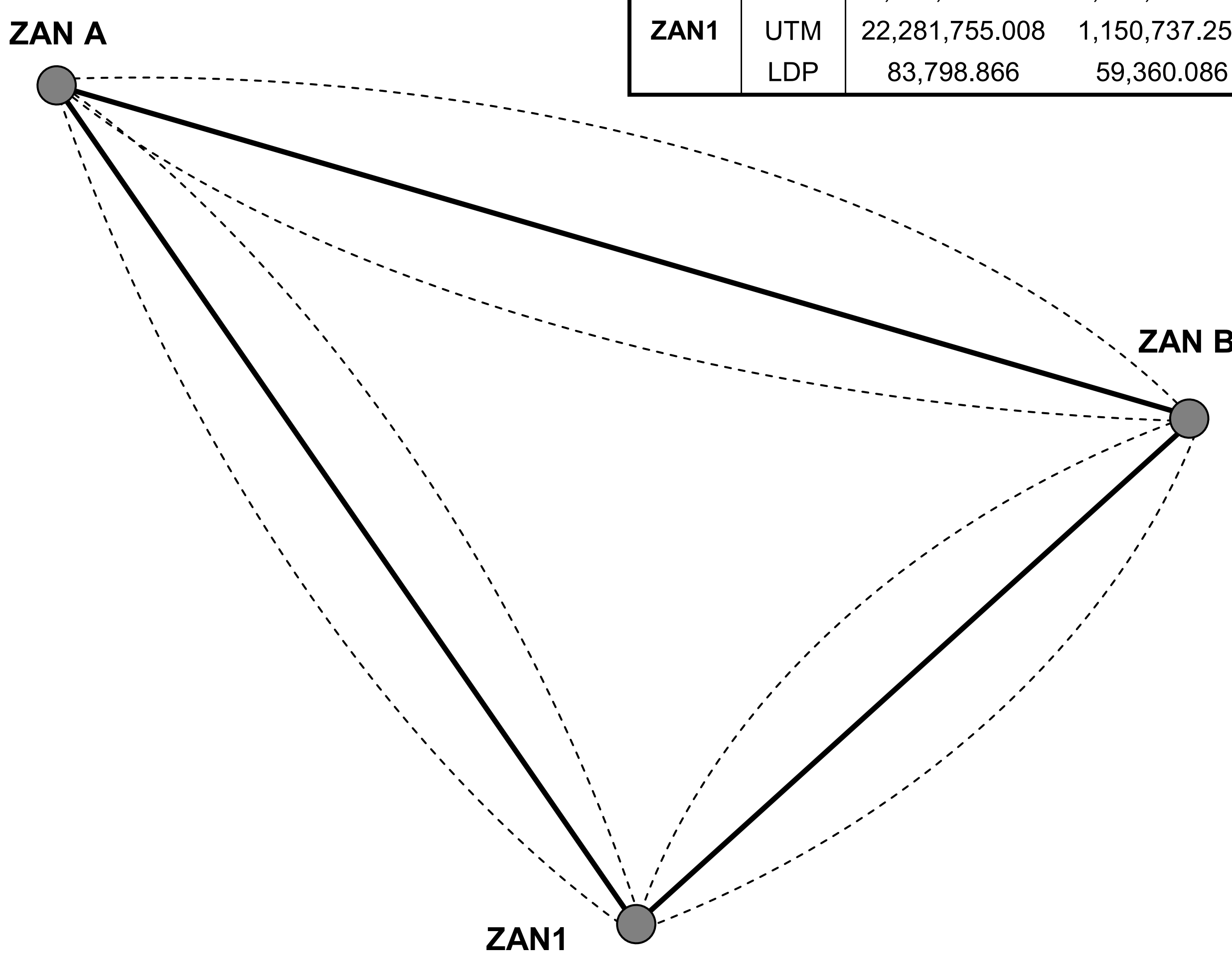
γ_A = map projection convergence angle at point *A*

$(t - T)_{AB}$ = Arc-to-chord ("second term") correction from *A* to *B*, usually negligible

Example using State Plane, UTM, and Low Distortion Projection (LDP) coordinates

Consider closed polygon below from points ZAN A to ZAN B to ZAN1 to ZAN A (not to scale). Label the figure with distances, grid azimuths, and geodetic forward and back azimuths.

Grid coords		Northing (sft)	Easting (sft)
ZAN A	SPCS	2,641,792.282	1,678,582.475
	UTM	22,282,314.286	1,150,199.349
	LDP	84,334.384	58,798.366
ZAN B	SPCS	2,641,528.002	1,679,342.347
	UTM	22,282,015.425	1,150,946.272
	LDP	84,068.136	59,557.652
ZAN1	SPCS	2,641,258.261	1,679,145.492
	UTM	22,281,755.008	1,150,737.253
	LDP	83,798.866	59,360.086

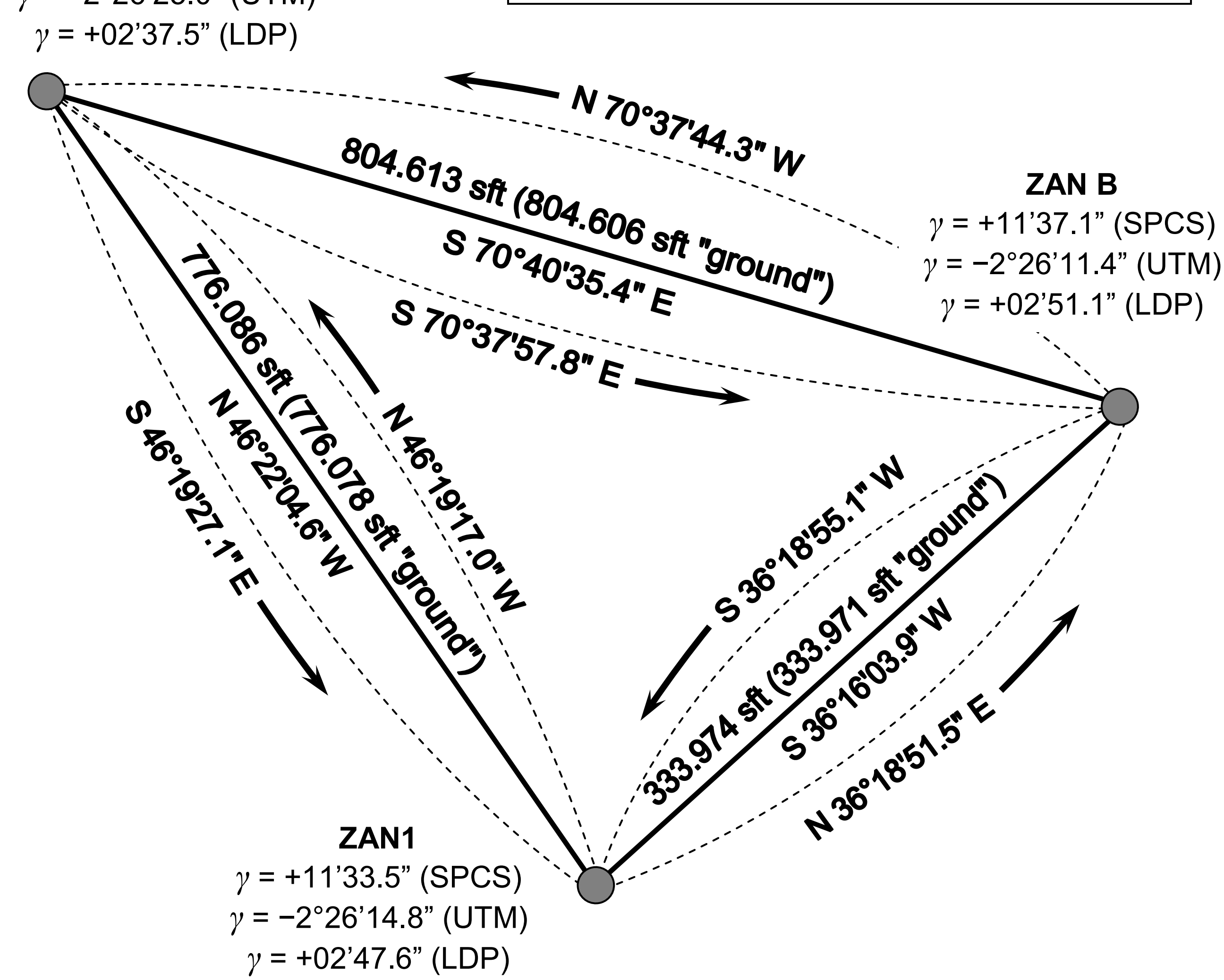


Example solution: Computed using SPCS 83 AK 4 coordinates

MISCLOSURES (computed using SPCS 83 AK 4 coordinates)	
Grid bearings and grid distances (misclosure due to rounding)	0.0005 ft
Grid bearings and "ground" distances	0.0008 ft
Forward geodetic bearings and grid distances	0.0415 ft
Forward geodetic bearings and "ground" distances	0.0409 ft
Back geodetic bearings and grid distances	0.0505 ft
Back geodetic bearings and "ground" distances	0.0506 ft
Mean forward & back geodetic bearings and grid distances	0.0115 ft
Mean forward & back geodetic bearings and "ground" distances	0.0103 ft

Notes

1) Misclosures the same for all grid coordinate systems
2) Maximum magnitude of arc-to-chord correction ($t - T$):
   a) 00.0011" for LDP coordinates
   b) 00.0048" for SPCS 83 AK 4 coordinates
   c) 00.0641" for UTM 83 6N coordinates



Section 4

VERTICAL DATUMS AND HEIGHT SYSTEMS

How high is it? How deep is it? Where will water go?

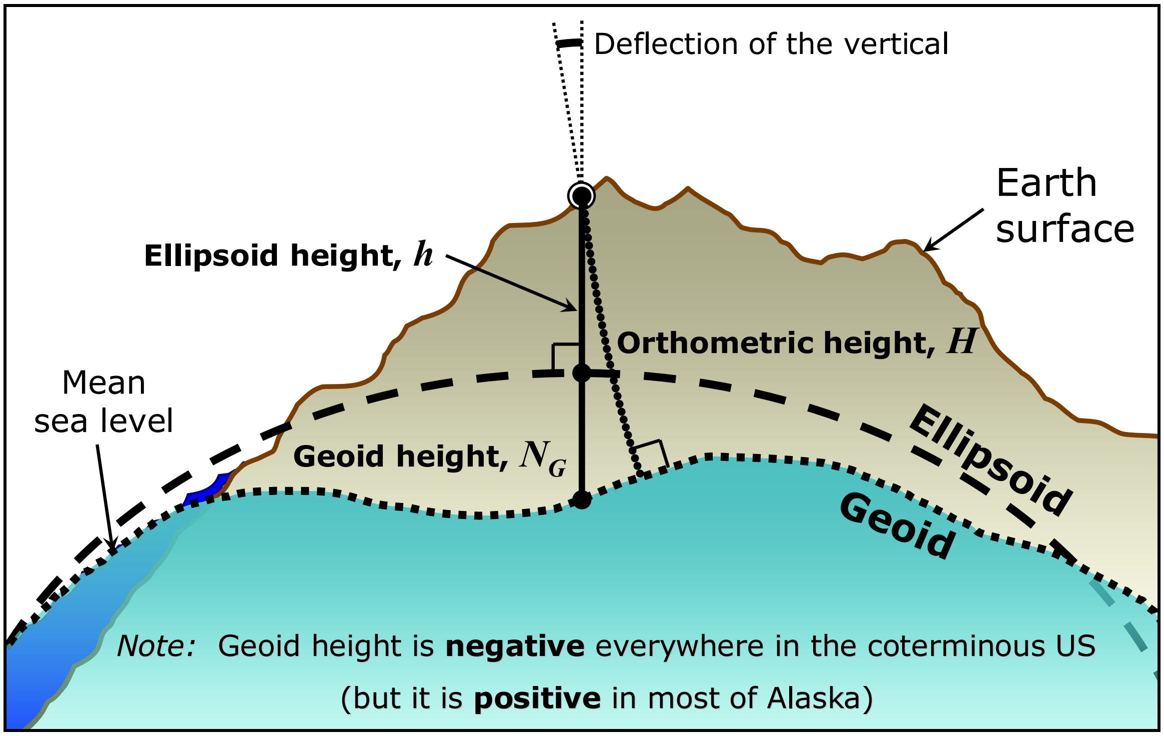
Examples of height determination errors for Alaska

Table 4.1 Examples of various positioning error sources and their magnitudes for Alaska due to vertical datum and height system problems (abbreviations and technical terms are defined in the Glossary).

Positioning error examples for Alaska	Error magnitudes
Using NGVD 29 when NAVD 88 required	~5.7 to 6.8 feet too low in Anchorage vicinity (vertical)
Using ellipsoid heights for elevations	Varies from −20 feet to +69 feet (vertical)
Using GEOID99 when GEOID06 is required	Varies from −10.8 ft to −2.4 ft (~ −5.7 ft in Fairbanks)
Using GEOID06 when GEOID09 is required	Varies from −13.7 ft to +14.3 ft (~−0.1 ft in Fairbanks)
Using GEOID09 when GEOID12A is required	Varies from −3.5 ft to +1.9 ft (~ +0.1 ft in Fairbanks)
Neglecting geoid slope when transferring elevations with GPS	Up to 1.7 feet vertical per mile horizontal (up to 0.7 ft/mi in Anchorage area)
Using leveling without orthometric corrections to "correct" GPS-derived elevations	Can exceed 0.05 foot vertical per mile horizontal

Exercise 4.1: Ellipsoid, orthometric, and geoid heights

The relationship between ellipsoidal, orthometric, and geoid heights is shown in the figure below. Note that everywhere in the coterminous US, the geoid height is negative (i.e., the geoid is below the ellipsoid). But in most of Alaska, the geoid height is positive.



Equation 4.1 Relationship between ellipsoidal, orthometric, and geoid heights

$$\boxed{h = H + N_G}$$

where h, H, and N_G are the ellipsoidal, orthometric, and geoid heights, respectively.

Although Equation 4.1 is approximate due to deflection of the vertical, this effect is at the sub-millimeter level, so Equation 4.1 can be considered exact for all practical purposes.

The following ***approximate*** Alaska geoid accuracy estimates are based on comparisons to GPS benchmarks covering only part of the eastern half of the state (and western Canada):

Based on ellipsoid heights	***Approximate* geoid model accuracy with respect to NAVD 88 NGS leveled benchmarks (at the 95% confidence level)**				
	GEOID96	**GEOID99**	**GEOID06**	**GEOID09**	**GEOID12A**
NAD 83(1992)	11 ft (3.5 m)	11 ft (3.5 m)	0.4 ft (0.1 m)	0.6 ft (0.2 m)	
NAD 83(2007)	15 ft (4.6 m)	12 ft (3.7 m)	0.7 ft (0.2 m)	0.1 ft (0.03 m)	
NAD 83(2011)					0.1 ft (0.03 m)

GEOID06, GEOID09, and GEOID12A are "hybrid" models that have been modified to best match NAVD 88 orthometric heights derived from GPS on benchmarks. In contrast, GEOID99 and GEOID96 are purely gravimetric and are referenced to a geocentric ellipsoid (which is why they don't match NAVD 88 benchmarks as well). It is important to also note that in many areas of Alaska the hybrid models are extrapolated beyond the area of benchmark coverage, and in those areas it is likely not as accurate with respect to NAVD 88 or tide gauge estimates of mean sea level. Note also that GEOID06 was created using NAD 83(1992) ellipsoid heights, GEOID09 used NAD 83(NSRS2007) ellipsoid heights, and GEOID12A used NAD 83(2011) epoch 2010.00 ellipsoid heights. Consequently, GEOID12A gives a better estimate of NAVD 88 heights when NAD 83(2011) epoch 2010.00 ellipsoid heights are used. The relative accuracies of these models should be within about 5 parts per million in most areas of Alaska.

Example computation

Given: An NGS Datasheet for conventional NGS control station AIRCOM (below):

```
TT0843 **********************************************************************
TT0843  DESIGNATION -  AIRCOM
TT0843  PID         -  TT0843
TT0843  STATE/COUNTY-  AK/ANCHORAGE BOROUGH
TT0843  COUNTRY     -  US
TT0843  USGS QUAD   -  ANCHORAGE B-8
TT0843
TT0843                         *CURRENT SURVEY CONTROL
TT0843  ______________________________________________________________________
TT0843* NAD 83(1986) POSITION- 61 15 10.31677(N) 149 43 35.26923(W)   ADJUSTED
TT0843* NAVD 88 ORTHO HEIGHT -    82.735  (meters)     271.44  (feet) ADJUSTED   = H
TT0843  ______________________________________________________________________
TT0843  LAPLACE CORR     -         -5.16  (seconds)                   DEFLEC12A
TT0843  GEOID HEIGHT     -          7.45  (meters)                    GEOID12A   = N_G
TT0843  DYNAMIC HEIGHT   -         82.845 (meters)     271.80  (feet) COMP
TT0843  MODELED GRAVITY  -    981,926.9   (mgal)                      NAVD 88
TT0843
TT0843  HORZ ORDER       -  FIRST
TT0843  VERT ORDER       -  FIRST     CLASS I
```

Find: The ellipsoidal height of AIRCOM in feet.

Computations:

Sometimes the only horizontal control station available for a GPS survey was determined using conventional methods. These do not have an ellipsoidal height, but there is enough information to compute an estimate of it if an accurate NAVD 88 orthometric height is available. From the Datasheet we have:

$$h = \quad H \quad + \quad N_G$$

$$h = \underline{\hspace{3cm}} + \underline{\hspace{3cm}} = \underline{\hspace{3cm}}\text{ m} = \underline{\hspace{3cm}}\text{ ift} = \underline{\hspace{3cm}}\text{ ft}$$

Solution:

Using GEOID12A:

h = 82.735 m + 7.45 m = 90.185 m = 295.88 ft

***h* = 295.9 ft (± ~0.1 ft at 95% confidence for NAD 83(2011) ellipsoid height)**

Note that this ellipsoid height (and its estimated accuracies) is ***ONLY*** for the case where the AK GEOID12A model is used. Note also that GEOID12A is intended for use with NAD 83(2011) epoch 2010.00 ellipsoid heights.

In the following calculation, the GEOID09 model is used instead. It was derived using NAD 83(NSRS2007) ellipsoid heights (which is reflected in the estimated accuracy):

Using GEOID09:

h = 82.735 m + 7.38 m = 90.115 m = 295.65 ft

***h* = 295.7 ft (± ~0.1 ft at 95% confidence for NAD 83(2007) ellipsoid height)**

vs. (± ~0.6 ft at 95% confidence for NAD 83(1992) ellipsoid height)

In the following calculation, the GEOID06 model is used instead. It was derived using NAD 83(1992) ellipsoid heights (which is reflected in the estimated accuracy):

Using GEOID06:

h = 82.735 m + 7.42 m = 90.155 m = 295.78 ft

***h* = 295.8 ft (± ~0.4 ft at 95% confidence for NAD 83(1992) ellipsoid height)**

vs. (± ~0.7 ft at 95% confidence for NAD 83(2007) ellipsoid height)

If the GEOID99 or GEOID96 models are used, significantly different ellipsoid heights will be obtained, and they will typically be of much lower accuracy since they were not "corrected" to best match the NAVD 88 datum (the estimated accuracies are for NSRS2007 ellipsoid heights):

GEOID99, N_G = 9.56 m → h = 82.735 m + 9.56 m = 92.295 m = 302.81 ft

***h* = 303 ft (± ~12 ft at 95% confidence)**

GEOID96, N_G = 10.08 m → h = 82.735 m + 10.08 m = 92.815 m = 304.51 ft

***h* = 305 ft (± ~15 ft at 95% confidence)**

Exercise 4.2: Dynamic heights and geopotential numbers

In addition to orthometric heights, H ("elevations"), NGS Datasheets also give *dynamic heights*, H^D. A dynamic "height" is actually not a height in the geometric sense of a distance above a reference surface. Rather, it is a *geopotential number*, C, that has been divided (scaled) by a constant value of gravity, which gives H^D units of length. Both C and H^D represent the gravity potential energy at a point, and changes in H^D are the only "height" differences that give true change in hydraulic head. That is, unconfined water will not flow from one point to another if the water surface at both points has the same H^D, even though the points will generally ***not*** have the same "elevation", H (i.e., $\Delta H^D \neq \Delta H$, although the difference is often small). This is why NGS states that orthometric heights *usually* show the direction of water flow, and that dynamic heights *always* show the direction of water flow.

Equation 4.2 Relationship between dynamic height and geopotential number

$$\boxed{H^D = \frac{C}{\gamma_0}} \qquad \boxed{H^D = \frac{C}{9.806199}} \text{ [meters]} \qquad \boxed{H^D = \frac{C}{32.172569}} \text{ [ift]}$$

where C = geopotential number (units of m^2/s^2 or ft^2/s^2)

γ_0 = 9.806199 m/s^2 = normal gravity on the GRS 80 ellipsoid at 45° latitude (given on NGS Datasheets as 980.6199 gals, where 1 m/s^2 = 100 gals)

Both the dynamic and orthometric heights shown on NGS Datasheets were originally computed from the same set of adjusted geopotential numbers. The relationship between these two types of heights is given below.

Equation 4.3 Relationship between NAVD 88 dynamic and Helmert orthometric heights

$$\boxed{H^D = \frac{H}{\gamma_0}\bar{g} = \frac{H}{\gamma_0}\left(g + \frac{H}{K}\right) = \frac{H}{\gamma_0}\left(g + \frac{H}{2{,}358{,}000}\right)}$$

(modified from Zilkoski et al., 1992)

where $\bar{g}$ = Mean gravity along the plumbline

g = "Observed" (modeled) NAVD 88 surface gravity (given on NGS Datasheets in milligals, where 1 m/s^2 = 100,000 mgals)

K = 2,358,000 s^2 = 1 / (4.24 × 10^{-7} s^{-2}) is a constant factor for computing NAVD 88 mean gravity (assumes constant topographic density of 2670 kg/m^3)

Equations 4.4 and 4.5 show that orthometric heights can also be computed from geopotential numbers, as $H = C / \bar{g}$.

Example computation

Given: The NGS Datasheet for NGS station PEND (on the next page):

Find: The geopotential number of PEND from both the dynamic and orthometric height (in ift).

Computations:

Using the published NAVD 88 dynamic height:

$$C = \gamma_0 \times H^D$$

$$C = \underline{\hspace{3cm}} \times \frac{\underline{\hspace{3cm}}\ \text{m}}{0.3048\ \text{m}/\text{ift}} = \underline{\hspace{3cm}}\ \text{ift}^2/\text{s}^2$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(g + \frac{H}{K} \right) \times H$$

$$C = \left(\underline{\hspace{2.5cm}} + \frac{\underline{\hspace{2.5cm}}}{\underline{\hspace{2.5cm}}} \right) \times \underline{\hspace{3cm}} = \frac{\underline{\hspace{3cm}}\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} =$$

$$= \underline{\hspace{3cm}}\ \text{ift}^2/\text{s}^2$$

```
FQ0306 ***********************************************************************
FQ0306  DESIGNATION -  PEND
FQ0306  PID         -  FQ0306
FQ0306  STATE/COUNTY-  AZ/COCONINO
FQ0306  USGS QUAD   -  FLAGSTAFF WEST (1983)
FQ0306
FQ0306                         *CURRENT SURVEY CONTROL
FQ0306  ________________________________________________________________________
FQ0306* NAD 83(1992)-  35 11 18.46326(N)     111 41 28.38215(W)     ADJUSTED
FQ0306* NAVD 88     -      2160.187  (meters)     7087.21    (feet)  ADJUSTED     = H
FQ0306  ________________________________________________________________________
FQ0306  LAPLACE CORR-            -2.48   (seconds)                   DEFLEC12A
FQ0306  GEOID HEIGHT-           -23.12   (meters)                    GEOID12A
FQ0306  DYNAMIC HT  -        2157.097 (meters)     7077.08    (feet)  COMP        = H^D
FQ0306  MODELED GRAV-      979,125.4    (mgal)                        NAVD 88     = g
FQ0306
FQ0306  HORZ ORDER  -  SECOND
FQ0306  VERT ORDER  -  FIRST     CLASS II
  :
FQ0306.The dynamic height is computed by dividing the NAVD 88
FQ0306.geopotential number by the normal gravity value computed on the
FQ0306.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
FQ0306.degrees latitude (g = 980.6199 gals.).                                  = γ0
FQ0306
FQ0306.The modeled gravity was interpolated from observed gravity values.
```

Solution:

Using the published NAVD 88 dynamic height:

$$C = 32.172569\ \text{ift/s}^2 \times \frac{2157.097\ \text{m}}{0.3048\ \text{m}/\text{ift}} = \underline{\mathbf{227{,}688}}\ \mathbf{ift^2/s^2}$$

Using the published NAVD 88 Helmert orthometric height:

$$C = \left(9.791254\ \text{m}/\text{s}^2 + \frac{2160.187\ \text{m}}{2{,}358{,}000\ \text{s}^2} \right) \times 2160.187\ \text{m} = \frac{21{,}152.92\ \text{m}^2/\text{s}^2}{(0.3048)^2\ \text{m}^2/\text{ift}^2} = \underline{\mathbf{227{,}688}}\ \mathbf{ift^2/s^2}$$

Exercise 4.3: Computing orthometric and dynamic heights from leveling

Leveling, by itself, does not yield true change in orthometric or dynamic heights. But when leveling is combined with surface gravity, the change in geopotential numbers can be computed (within certain approximations). If the geopotential number is known for at least one point in a leveling network, then it can be computed at all points in the network. The geopotential numbers can then be converted to orthometric and dynamic heights using the relationships from the previous section, where orthometric height is $H = C / \overline{g}$, and dynamic height is $H^D = C / \gamma_0$.

Equation 4.4 Determining change in geopotential from leveled height differences

$$\boxed{C_B \approx C_A + \left(\frac{g_A + g_B}{2} \right) \Delta n_{AB}}$$

where g_A and g_B = surface gravity at adjacent stations A and B (in m/s^2 or ft/s^2)

Δn_{AB} = leveled height difference from station A and B (in same linear units as gravity)

Alternatively, leveled height differences can be converted to orthometric heights and dynamic heights by adding an orthometric correction (*OC*) or dynamic correction (*DC*) to observed leveled height differences between adjacent stations.

Equation 4.5 The NAVD 88 Helmert orthometric correction for leveled height differences

$$\boxed{OC_{AB} \approx \frac{\left[K\left(g_A - g_B\right) - 2\,\Delta n_{AB} \right]\left[2H_A + \Delta n_{AB} \right]}{2\left(K\,g_B + H_A + \Delta n_{AB} \right)}}$$ (modified from Hwang and Hsiao, 2003)

where all variables are as defined previously, and the orthometric correction is added to the observed leveled height difference, i.e., $H_B \approx H_A + \Delta n_{AB} + OC_{AB}$.

Equation 4.6 The dynamic correction for leveled height differences

$$\boxed{DC_{AB} \approx \left(\frac{g_A + g_B}{2\gamma_0} - 1 \right) \Delta n_{AB}}$$ (modified from Hofmann-Wellenhof and Moritz, 2005)

where all variables are as defined previously, and the dynamic correction is added to the observed leveled height difference, i.e., $H_B^D \approx H_A^D + \Delta n_{AB} + DC_{AB}$.

“Approximately equal” symbols were used for equations 4.6 – 4.8 because the surface gravity varies continuously along the leveling route. These equations will be exactly true only when the gravity varies linearly between stations. For best results they should be applied to every turning point on a leveling route. However, in most cases, Equation 4.7 (orthometric corrections) should work well for stations less than about 2 km apart. Equations 4.6 and 4.8 (geopotential numbers and dynamic corrections) are more sensitive to variation in surface gravity, and may not give good results even for stations less than 2 km apart, especially in mountainous areas.

Example computation

Given: A leveled height difference of +50.387 ft measured from NGS stations M 504 (PID FQ0543) to L 504 (PID FQ0544). The following data apply to these stations:

	M 504 (*station A*)	**L 504 (*station B*)**
Orthometric height	6104.396 ift	?
Dynamic height	6095.991 ift	?
Surface gravity	32.125673 ift/s^2	32.125305 ift/s^2

Find: The orthometric and dynamic heights of L 504 (in ift). The stations are 6450 ft apart.

Computations: The stations are (slightly) less than about 2 km apart, so using gravity values only at the stations themselves should be adequate (rather than at every leveling turning point).

Alternative 1: Solve using geopotential numbers.

$$C_B = C_A + \left(\frac{g_A + g_B}{2}\right)\Delta n_{AB} \quad = \quad \gamma_0 \, H_A^D + \left(\frac{g_A + g_B}{2}\right)\Delta n_{AB}$$

$$C_B = \underline{\qquad\qquad} \times \underline{\qquad\qquad} + \left(\frac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{2}\right) \times \underline{\qquad\qquad\qquad}$$

$$C_B = \underline{\qquad\qquad\qquad\qquad} \text{ ift}^2/\text{s}^2$$

Orthometric height:

$$H_B = \frac{C_B}{\bar{g}_B} = \frac{C_B}{g_B + \dfrac{H_A + \Delta n_{AB}}{K}} = \frac{\underline{\qquad\qquad\qquad}}{\underline{\qquad\qquad} + \dfrac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{\underline{\qquad\qquad\qquad}}}$$

$$H_B = \underline{\qquad\qquad\qquad} \ \textbf{ift}$$

Dynamic height:

$$H_B^D = \frac{C_B}{\gamma_0} = \frac{\underline{\qquad\qquad\qquad}}{\underline{\qquad\qquad\qquad}} = \underline{\qquad\qquad\qquad} \ \textbf{ift}$$

Alternative 2: Solve using dynamic and orthometric corrections.

$$OC_{AB} = \frac{\left[K \times (g_A - g_B) - 2 \times \Delta n_{AB}\right] \times \left[2 \times H_A + \Delta n_{AB}\right]}{2 \times (K \times g_B + H_A + \Delta n_{AB})}$$

$$OC_{AB} = \frac{\left[\underline{\qquad\qquad} \times (\underline{\qquad\qquad} - \underline{\qquad\qquad}) - 2 \times \underline{\qquad\qquad}\right] \times \left[2 \times \underline{\qquad\qquad} + \underline{\qquad\qquad}\right]}{2 \times (\underline{\qquad\qquad} \times \underline{\qquad\qquad} + \underline{\qquad\qquad} + \underline{\qquad\qquad})}$$

$$OC_{AB} = \underline{\qquad\qquad\qquad} \text{ft}$$

$$DC_{AB} = \left(\frac{g_A + g_B}{2 \times \gamma_0} - 1\right) \times \Delta n_{AB}$$

$$DC_{AB} = \left(\frac{\underline{\qquad\qquad} + \underline{\qquad\qquad}}{2 \times \underline{\qquad\qquad\qquad}} - 1\right) \times \underline{\qquad\qquad\qquad} = \underline{\qquad\qquad\qquad} \text{ft}$$

Orthometric height:

$H_B \approx H_A + \Delta n_{AB} + OC_{AB} =$ ______________ + __________ + _______ = ______________ **ift**

Dynamic height:

$H_B^D \approx H_A^D + \Delta n_{AB} + DC_{AB} =$ ______________ + __________ + _______ = ______________ **ift**

Solution:

Alternative 1: Solve using geopotential numbers.

$$C_B = 32.172569 \times 6095.991 + \left(\frac{32.125673 + 32.125303}{2} \right) \times 50.387$$

$$C_B = 196{,}123.7 + 32.125489 \times 50.387 = \underline{197{,}742.4} \text{ ift}^2/\text{s}^2$$

Orthometric height: $$H_B = \frac{C_B}{\bar{g}_B} = \frac{197{,}742.4}{32.125305 + \dfrac{6104.396 + 50.387}{2{,}358{,}000}} = \underline{\mathbf{6154.847}} \textbf{ ift}$$

Dynamic height: $$H_B^D = \frac{C_B}{\gamma_0} = \frac{197{,}742.4}{32.172569} = \underline{\mathbf{6146.304}} \textbf{ ift}$$

Alternative 2: Solve using orthometric and dynamic corrections.

$$OC_{AB} = \frac{\left[2{,}358{,}000 \times (32.125673 - 32.125305) - 2 \times 50.387 \right] \times \left[2 \times 6104.396 + 50.387 \right]}{2 \times (2{,}358{,}000 \times 32.125305 + 6104.396 + 50.387)}$$

$$OC_{AB} = \frac{\left[766.970 \right] \times \left[12{,}259.179 \right]}{151{,}515{,}248} = \underline{+0.062} \text{ ft}$$

$$DC_{AB} = \left(\frac{32.125673 + 32.125305}{2 \times 32.172569} - 1 \right) \times 50.387 = (-0.001463) \times 50.387 = \underline{-0.074} \text{ ft}$$

Orthometric height:

$H_B = H_A + \Delta n_{AB} + OC_{AB} = 6104.396 + 50.387 + 0.062 =$ $\underline{\mathbf{6154.845}}$ **ift**

Dynamic height:

$H_B^D = H_A^D + \Delta n_{AB} + DC_{AB} = 6095.991 + 50.387 + (-0.074) =$ $\underline{\mathbf{6146.304}}$ **ift**

Check: The NGS Datasheet for station L 504 gives:

H_B = 1875.997 m = <u>6154.846 ift</u> and H_B^D = 1873.393 m = <u>6146.302 ift</u> ✓

These results are essentially within the displayed precision of the NGS Datasheet values (±0.0005 m = ±0.0016 ft). However, part of the difference is likely also due to non-linear variation in gravity between the stations, which are 6450 ft (1.95 km) apart

Note that ΔH = 50.449 ft does *not* equal ΔH^D = 50.311 ft, and that only ΔH^D gives true change in hydraulic head (even though it is not really a change in "height", at least in the geometric sense).

Section 5

DOCUMENTATION AND ACCURACY REPORTING

Is it in the right place? By how much? How do you know?

Examples of documentation and accuracy reporting errors

Table 5.1 Examples of various positioning error sources and their magnitudes due to documentation and accuracy reporting problems (abbreviations and technical terms are defined in the Glossary).

Documentation error examples	Problem
Documenting ellipsoidal datum as “WGS-84” when data actually referenced to NAD 83	Perpetuates confusion about “equivalence” of WGS-84 and NAD 83
Listing grid coordinates (such as SPCS) as “NAD 83”	NAD 83 is a ellipsoidal datum, not a grid coordinate system
Documenting ellipsoidal datum as “GRS-80”	GRS-80 is a reference ellipsoid, not a datum
Documenting vertical datum as “Mean Sea Level” (MSL)	There is no nationwide MSL datum in the US (name changed to NGVD 29 in 1976)
Using precision as an accuracy estimate with data containing systematic errors (e.g., incorrect reference coordinates)	Accuracy estimate is meaningless
Reporting horizontal error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 39% confidence level
Reporting vertical error using unscaled standard deviation, rather than at the 95% confidence level (as specified by the FGDC)	Gives error estimates at 68% confidence level
Using radial and circular estimates for horizontal error rather than semi-major axis of horizontal error ellipse	Typically makes errors appear less than actual
Using trivial vectors in GPS network adjustments	Varies, but always makes errors appear less than actual
Relying on precision computed by baseline processor for a single GPS vector as an indicator of accuracy	Varies, but precision value usually very optimistic and will not reveal systematic errors

Exercise 5.1: Computing accuracy estimates from standard deviations

Accuracies for GNSS stations are given on the NGS Datasheet as linear values for the horizontal and ellipsoid height components (in centimeters) scaled to the 95% confidence level. The horizontal accuracy of a station is computed from the standard deviations in the north and east components, along with the horizontal correlation coefficient. The height accuracy is computed from the ellipsoid height standard deviation. The standard deviation and horizontal correlation values were computed in the constrained least-squares adjustments performed for determining the published coordinates. A hyperlink on the Datasheet immediately below the published accuracies opens an Accuracy Datasheet that given the standard deviations and horizontal correlations, along with other information.

For passive GNSS control, both "network" and "local" accuracies are given. The network accuracy represents the accuracy of the station with respect to the NSRS. Local accuracy is the accuracy of the station with respect to another station that was processed simultaneously. It represents the error of the adjusted GNSS observations between the two stations and is a property of the station pair, not of one station or the other. The median values of all local accuracies are given on the Datasheet, along with the median distance between local accuracy station pairs. The complete list of all local accuracies and distances are given on the Accuracy Datasheet. Local accuracies are not given for CORS, because the method used for determining CORS coordinates is not amenable to that approach.

The following approach is used for computing accuracies on the NGS Datasheet.

Equation 5.1 Ellipsoid height accuracy on the NGS Datasheet (at the 95% confidence level)

$$\boxed{E_{95}^{h} = 1.9600 \times \sigma_h}$$

where E_{95}^{h} is the ellipsoid height error ("accuracy") at 95% confidence

σ_h is the ellipsoid height standard deviation

1.9600 is the *univariate* (one-dimensional) scalar for a confidence level of 95%. See Table 5.2 below for this and other scalars at various confidence levels.

Equation 5.2 Horizontal accuracy on the NGS Datasheet (at the 95% confidence level)

$$\boxed{E_{95}^{Horz} = a\,(1.960790 + 0.004071\ C + 0.114276\ C^2 + 0.371625\ C^3)} \quad \text{(Leenhouts, 1985)}$$

where E_{95}^{Horz} is the radius of error circle ("horizontal accuracy") at 95% confidence

$C = b / a$

a and *b* are the error ellipse semi-major and semi-minor axes, computed as follows

Equation 5.3 Horizontal error ellipse axes computed from standard deviations and covariance

$$\boxed{a,\ b = \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2 \pm \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}}$$

The "±" operator in Equation 5.3 allows computation of both *a* and *b* with this one equation, and *a* is always greater than *b*. The horizontal covariance is computed as follows.

Equation 5.4 Horizontal covariance computed from the horizontal correlation coefficient

$$\boxed{\sigma_{NE} = \rho\, \sigma_N\, \sigma_E}$$

where σ_{NE} is the horizontal covariance

ρ is the horizontal correlation coefficient

The orientation (rotation) of a horizontal error ellipse can be computed from the standard deviations and covariance.

Equation 5.5 Horizontal error ellipse rotation computed from standard deviation and covariance

$$\boxed{\theta = \frac{1}{2}\tan^{-1}\left(\frac{2\sigma_{NE}}{\sigma_E^2 - \sigma_N^2}\right)}$$

where θ is the rotation angle of the semi-major axis, with respect to the east direction (positive counterclockwise). If $\sigma_N > \sigma_E$, rotation is with respect to the *positive* east axis. If, rotation is $\sigma_N < \sigma_E$,with respect to the *negative* east axis. If $\sigma_N = \sigma_E$, then $\theta = \pm 45°$, where the sign of the rotation is determined by the sign of σ_{NE}.

Table 5.2 Values used to scale standard errors (accuracies) to various confidence levels. The *univariate scalar* is used for single error components, such as vertical error. The *bivariate scalar* is used for dual (two-dimensional) error components, such as horizontal error, and can be used to scale an error ellipse to a desired confidence level. The *trivariate scalar* is rarely used but is provided here for the sake of completeness. It is for three-dimensional error components and can be used for scaling an error ellipsoid to a desired confidence level. In all cases, these scalars are based on the normal probability distribution of random variables, and the multivariate scalars are for jointly distributed random variables.

Univariate scalars		**Bivariate scalars**		**Trivariate scalars**	
Scalar, c_X^1	**Confidence level,** X	**Scalar,** c_X^2	**Confidence level,** X	**Scalar,** c_X^3	**Confidence level,** X
0.6745	50.00%	1.0000	39.35%	1.0000	19.87%
1.0000	68.27%	1.1774	50.00%	1.5382	50.00%
1.6449	90.00%	2.0000	86.47%	2.0000	73.85%
1.9600	95.00%	2.1460	90.00%	2.5003	90.00%
2.0000	95.45%	2.4477	95.00%	2.7955	95.00%
2.5758	99.00%	3.0000	98.89%	3.0000	97.07%
3.0000	99.73%	3.0349	99.00%	3.3682	99.00%
3.2905	99.90%	3.7169	99.90%	4.0331	99.90%

In contrast to the preceding definition of horizontal accuracy, and for the sake of completeness, a horizontal (circular) accuracy can be computed that consistent with the approach used by the National Standard for Spatial Data Accuracy (NSSDA) as developed by the Federal Geographic Data Committee (1998, Part 3).

Equation 5.6 Horizontal (circular) accuracy computed at the 95% confidence level per NSSDA

$$\boxed{CEP_{95} = 2.4477 \frac{\sigma_N + \sigma_E}{2}}$$

where CEP_{95} is the estimated Circular Error Probable (horizontal accuracy) at 95% confidence (note that *CEP* is typically computed at the 50% confidence level)

σ_N and σ_E are the north and east standard deviations, respectively

2.4477 is the *bivariate* (two-dimensional) scalar for 95% confidence, per Table 5.2

Again, for the sake of completeness, note that the trivariate scalar can be used to scale the estimated Spherical Error Probable (*SEP*) to a desired confidence level. As with *CEP*, typically *SEP* is computed at 50% confidence.

Equation 5.7 Three-dimensional (spherical) accuracy computed at the 95% confidence level

$$\boxed{SEP_{95} = 2.7955 \frac{\sigma_N + \sigma_E + \sigma_h}{3}}$$

Example computation

Given: The NGS Datasheet and Accuracies Datasheet (on the next page) for station J 99 (QE0722):

```
1         National Geodetic Survey,   Retrieval Date = MARCH  9, 2013
 QE0722 ***********************************************************************
 QE0722  DESIGNATION -  J 99
 QE0722  PID         -  QE0722
 QE0722  STATE/COUNTY-  OR/POLK
 QE0722  COUNTRY     -  US
 QE0722  USGS QUAD   -  MONMOUTH (1986)
 QE0722
 QE0722                        *CURRENT SURVEY CONTROL
 QE0722  ______________________________________________________________________
 QE0722* NAD 83(2011) POSITION- 44 48 11.97228(N) 123 13 20.88312(W)   ADJUSTED
 QE0722* NAD 83(2011) ELLIP HT-     53.898 (meters)        (06/27/12)   ADJUSTED
 QE0722* NAD 83(2011) EPOCH   -  2010.00
 QE0722* NAVD 88 ORTHO HEIGHT -     76.442  (meters)      250.79  (feet) ADJUSTED
 QE0722  ______________________________________________________________________
 QE0722  NAD 83(2011) X  - -2,483,622.713 (meters)                       COMP
 QE0722  NAD 83(2011) Y  - -3,792,127.021 (meters)                       COMP
 QE0722  NAD 83(2011) Z  -  4,471,905.090 (meters)                       COMP
 QE0722  LAPLACE CORR    -          -8.54  (seconds)                     DEFLEC12A
 QE0722  GEOID HEIGHT    -         -22.54  (meters)                      GEOID12A
 QE0722  DYNAMIC HEIGHT  -          76.439 (meters)      250.78  (feet) COMP
 QE0722  MODELED GRAVITY -     980,572.9    (mgal)                       NAVD 88
 QE0722
 QE0722  VERT ORDER      -  SECOND    CLASS 0
 QE0722
 QE0722  FGDC Geospatial Positioning Accuracy Standards (95% confidence, cm)
 QE0722  Type                                       Horiz  Ellip  Dist(km)
 QE0722  -------------------------------------------------------------------
 QE0722  NETWORK                                     0.81   1.45
 QE0722  -------------------------------------------------------------------
 QE0722  MEDIAN LOCAL ACCURACY AND DIST (012 points)   1.17   1.83      9.15
 QE0722  -------------------------------------------------------------------
 QE0722  NOTE: Click here for information on individual local accuracy
 QE0722  values and other accuracy information.
```

E_{95}^{Horz} E_{95}^{h}

Find: The horizontal and ellipsoid height network accuracy at the 50%, 95%, and 99% confidence levels using standard deviations and horizontal correlation coefficient values on the Accuracy Datasheet for this station. Also compute the circular error probable (*CEP*), spherical error probable (*SEP*), and the horizontal error ellipse axes and rotation angle, at 95% confidence. Give the final results in feet.

Computations: The ellipsoid height network accuracy (error) is one-dimensional, so the univariate scalars from Table 5.2 should be used to scale the ellipsoid height standard deviation from the Accuracy Datasheet to the required confidence levels:

$$E_X^h = c_X^1 \times \sigma_h$$

where c_X^1 is the univariate scalar at the X% confidence level.

NGS Accuracy Datasheet

```
Program lna_ret Version 2.3 Date November 27, 2012
National Geodetic Survey, Retrieval Date = MARCH  9, 2013
 QE0722 ********************************************************************
 QE0722  ACCURACIES  -  Complete network and local accuracy information.
 QE0722  DESIGNATION -  J 99
 QE0722  PID         -  QE0722
 QE0722
 QE0722  Statistical Information, in cm, for point QE0722 follows.
 QE0722
 QE0722  Note that Horz and Ellip values are the official 95%
 QE0722  FGDC accuracy standards.  The values of StdN, StdE and Stdh are the
 QE0722  standard deviations (one sigma) of the coordinates (NETWORK) or
 QE0722  of the difference in the coordinates (LOCAL) in Latitude, Longitude
 QE0722  and Ellipsoid Height.  The value CorrNE is the correlation
 QE0722  coefficient between the latitude and longitude components of either
 QE0722  the coordinate (NETWORK) or coordinate difference (LOCAL).
 QE0722
 QE0722  Type/PID  Horz  Ellip  Dist(km)   StdN   StdE   Stdh         CorrNE
 QE0722
 QE0722  NETWORK   0.81   1.45      0.00   0.37   0.28   0.74    +0.13256717
 QE0722  ---------------------------------------------------------------------
 QE0722  LOCAL:
 QE0722  DE5636    2.36   2.29      6.01   1.06   0.85   1.17    +0.03633879
 QE0722  DH7240    0.50   1.72      7.33   0.23   0.17   0.88    +0.05108340
 QE0722  DE5653    3.15   2.74      7.52   1.50   0.94   1.40    -0.10227759
 QE0722  DE5634    0.52   1.61      7.72   0.24   0.18   0.82    +0.04851202
 QE0722  DE5640    1.78   1.82      7.96   0.82   0.60   0.93    -0.06364293
 QE0722  DE5633    2.09   2.29      8.53   0.95   0.73   1.17    -0.04015267
 QE0722  QE1942    0.97   1.65      9.77   0.44   0.34   0.84    +0.05167620
 QE0722  DE5641    1.65   1.84     11.66   0.74   0.59   0.94    -0.03528859
 QE0722  QE0693    0.55   2.06     11.97   0.25   0.19   1.05    +0.06308021
 QE0722  DE5652    0.81   1.43     15.30   0.37   0.28   0.73    +0.12679848
 QE0722  QE0742    1.37   2.06     26.38   0.61   0.50   1.05    +0.06791869
 QE0722  AJ6959    0.77   1.45     96.42   0.35   0.26   0.74    +0.16460721
 QE0722
 QE0722  MEDIAN    1.17   1.83      9.15
 QE0722  ---------------------------------------------------------------------
```

E_{95}^{Horz} E_{95}^{h} σ_N σ_E σ_h ρ

$$E_{50}^{h} = 0.6745 \times 0.74 \quad = \quad 0.50 \text{ cm} \quad = \quad \underline{\textbf{0.016 ft (at 50\% confidence)}}$$

$$E_{95}^{h} = 1.9600 \times 0.74 \quad = \quad 1.45 \text{ cm} \quad = \quad \underline{\textbf{0.048 ft (at 95\% confidence)}}$$

$$E_{99}^{h} = 2.5758 \times 0.74 \quad = \quad 1.91 \text{ cm} \quad = \quad \underline{\textbf{0.063 ft (at 99\% confidence)}}$$

Computing the horizontal network accuracy first requires computing the error ellipse semi-major and semi-minor axes from the north and east standard deviations and horizontal correlation coefficient on the Accuracy Datasheet. First the horizontal covariance is computed:

$$\sigma_{NE} = \rho\, \sigma_N\, \sigma_E = +0.13256717 \times 0.37 \text{ cm} \times 0.28 \text{ cm} = \underline{+0.01373396 \text{ cm}^2}.$$

The standard error ellipse axes can now be computed using Equation 5.3. Note that there is a "±" symbol in the equation — *a* is computed for the case where "±" is "+", and *b* is computed for the case where "±" is "–":

$$a, b = \sqrt{\frac{1}{2}\left[\sigma_N^2 + \sigma_E^2 \;\pm\; \sqrt{\left(\sigma_N^2 - \sigma_E^2\right)^2 + 4\sigma_{NE}^2}\right]}$$

$$= \sqrt{\frac{1}{2}\left[0.37^2 + 0.28^2 \;\pm\; \sqrt{\left(0.37^2 - 0.28^2\right)^2 + 4 \times 0.01373396^2}\right]} = \begin{cases} \underline{a = 0.374 \text{ cm} = 0.012 \text{ ft}} \\ \\ \underline{b = 0.274 \text{ cm} = 0.009 \text{ ft}} \end{cases}$$

Since the *a* and *b* dimensions are for the standard error ellipse, the scalar is 1, which corresponds to a confidence level of 39.35% (as shown in Table 5.2) for the bivariate (2-D) case. The coefficients in Equation 2 for horizontal accuracy are already scaled to 95% confidence. First compute $C = 0.274 / 0.374 = 0.73262$, which gives:

$$E_{95}^{Horz} = 0.374 \times (1.960790 + 0.004071 \times 0.73262 + 0.114276 \times 0.73262^2 + 0.371625 \times 0.73262^3)$$

E_{95}^{Horz} = 0.81 cm = **<u>0.027 ft (at 95% confidence)</u>**

To get the accuracy at different confidence levels, simply take multiply the 95% confidence accuracy by the ratio of the bivariate scalar of the desired confidence level to the 95% scalar of 2.4477:

E_{50}^{Horz} = (1.1774 / 2.4477) × 0.81 = 0.39 cm = **<u>0.013 ft (at 50% confidence)</u>**

E_{99}^{Horz} = (3.0349 / 2.4477) × 0.81 = 1.00 cm = **<u>0.033 ft (at 99% confidence)</u>**

The circular error probable and spherical error probable at 95% confidence are given by equations 5.6 and 5.7:

CEP_{95} = 2.4477 × (0.37 + 0.28) / 2 = 0.80 cm = **<u>0.026 ft (at 95% confidence)</u>**

and

SEP_{95} = 2.7955 × (0.37 + 0.28 + 0.74) / 3 = 1.30 cm = **<u>0.042 ft (at 95% confidence)</u>**

Surveying & mapping spatial data requirements & recommendations
These should be explicitly specified in surveying and mapping projects

1. Completely define the coordinate system

a. Linear unit (e.g., international foot, U.S. survey foot, meter)
   i. Use same linear unit for horizontal and vertical coordinates
b. Geodetic datum (recommend North American Datum of 1983)
   i. Should include "datum tag", e.g., 1986, 1991, 1998, 2007, 2011, as necessary, as well as epoch date for modern high-accuracy positions, e.g., 2010.00
   ii. WGS 84, ITRF/IGS, and NAD 27 are **NOT** recommended
c. Vertical datum (e.g., North American Vertical Datum of 1988)
   i. If GPS used for elevations, recommend using a modern geoid model (e.g., GEOID12A)
   ii. Recommend using NAVD 88 rather than NGVD 29 when possible
d. Map projection type and parameters (e.g., Transverse Mercator, Lambert Conformal Conic)
   i. Special attention required for low-distortion grid (a.k.a. "ground") coordinate systems
      1) Avoid scaling of existing coordinate systems (e.g., "modified" State Plane)

2. Require *direct* referencing of the NSRS (National Spatial Reference System)

a. Ties to published control strongly recommended (e.g., National Geodetic Survey control)
   i. Relevant component of control must have greater accuracy than positioning method used
      1) E.g., network accuracies that meet project needs, 2nd order (or better) for vertical control
b. NGS Continuously Operating Reference Stations (CORS) can be used to reference the NSRS
   i. Free Internet GPS post-processing service: OPUS (Online Positioning User Service)

3. Specify *accuracy* requirements (*not* precision)

a. Use objective, defensible, and robust methods (published ones are recommended)
   i. Mapping and surveying: National Standard for Spatial Data Accuracy (NSSDA)
      1) Require occupations ("check shots") of known high-quality control stations
   ii. Surveys performed for establishing control or determining property boundaries:
      1) Appropriately constrained and over-determined least-squares adjusted control network
      2) Beware of "cheating" (e.g., using "trivial" GPS vectors in network adjustment)

4. Documentation is *essential* (metadata!)

a. Require a report detailing methods, procedures, and results for developing final deliverables
   i. This must include any and all post-survey coordinate transformations
      1) E.g., published datum transformations, computed correction surfaces, "rubber sheeting"
b. Documentation should be complete enough that someone else can reproduce the product
c. For GIS data, recommend that accuracy and coordinate system information be included as feature attributes (not just as separate, easy-to-lose and easy-to-ignore metadata files)

Example of surveying and mapping documentation (*metadata*)
Basis of Bearings and Coordinates

Linear unit: International foot (ift)

Geodetic datum: North American Datum of 1983 (2011) epoch 2010.00

Vertical datum: North American Vertical Datum of 1988 (see below)

System: Oregon Coordinate Reference System

Zone: Salem

Projection: Transverse Mercator

- Latitude of grid origin: 44° 20' 00" N
- Longitude of central meridian: 123° 05' 00" W
- Northing at grid origin: 0.000 m
- Easting at central meridian: 50,000.000 m (164,041.995 ift)
- Scale factor on central meridian: 1.000 01 (exact)

All distances and bearings shown hereon are projected (grid) values based on the preceding projection definition. The projection was defined to minimize the difference between projected (grid) distances and horizontal ("ground") distances at the topographic surface within the design area of this coordinate system.

The basis of bearings is geodetic north. Note that the grid bearings shown hereon (or implied by grid coordinates) do not equal geodetic bearings due to meridian convergence.

Orthometric heights (elevations) were transferred to the site from NGS control station "J 99" (PID QE0722) using GNSS with NGS geoid model "GEOID12A" referenced to the current published NAVD 88 height of this station (76.442 m).

The survey was conducted using GNSS referenced to the National Spatial Reference System. A partial list of point coordinates is given below (additional coordinates are available upon request). Accuracy estimates are at the 95% confidence level and are based on an appropriately constrained and weighted least-squares adjustment of redundant observations.

Point #1, CGS brass cap, J 99 (PID QE0722), constrained control		Estimated accuracy
Latitude = 44° 48' 11.97228" N	Northing = 171,381.417 ift	Horiz = ±0.027 ift
Longitude = 123° 13' 20.88312" W	Easting = 127,926.947 ift	Elpsd ht = ±0.048 ift
Ellipsoid height = 176.831 ift	Elevation = 250.794 ift	Elevation FIXED

Point #1002, 1/2" rebar with aluminum cap, derived coordinates		Estimated accuracy
Latitude = 44° 48' 49.06162" N	Northing = 175,130.264 ift	Horiz = ±0.034 ift
Longitude = 123° 12' 16.36945" W	Easting = 132,584.160 ift	Elpsd ht = ±0.056 ift
Ellipsoid height = 227.621 ift	Elevation = 301.752 ift	Elev = ±0.074 ift

Point #1006, 1/2" rebar with plastic cap, derived coordinates		Estimated accuracy
Latitude = 44° 48' 37.68144" N	Northing = 173,977.764 ift	Horiz = ±0.047 ift
Longitude = 123° 12' 16.83756" W	Easting = 132,548.693 ift	Elpsd ht = ±0.068 ift
Ellipsoid height = 214.925 ift	Elevation = 289.071 ift	Elev = ±0.082 ift

GLOSSARY

Below is a list of the abbreviations and terms used in this workbook. In the interest of brevity, the definitions are highly general and simplified. Please note also that this list gives only a portion of the terms and abbreviations frequently encountered in GPS positioning and geodesy. Terms in *italics* within the definitions are also defined in this glossary. Cited references are listed at the end of the workbook.

Autonomous position. A *GPS* position obtained with a single receiver using only the ranging capability of the *GPS* code (i.e., with no *differential correction*).

Cartesian coordinates. Coordinates based on a system of two or three mutually perpendicular axes. *Map projection* and *ECEF* coordinates are examples two- and three-dimensional Cartesian coordinates, respectively.

Confidence interval or level. A computed probability that the "true" value will fall within a specified region (e.g., 95% confidence level). Applies only to randomly distributed errors.

CORS (Continuously Operating Reference Stations). A nation-wide system of permanently mounted *GPS* antennas and receivers that collect *GPS* data continuously. The CORS network is extremely accurate and constitutes the primary survey control for the US. CORS data can be used to correct *GPS* survey and mapping results, and the data are freely available over the Internet.

Datum transformation. Mathematical method for converting one *ellipsoidal* or *vertical datum* to another (there are several types, and they vary widely in accuracy).

Differential correction. A method for removing much of the error in an autonomous *GPS* position. Typically requires at least two simultaneously operating *GPS* receivers, with one of the two at a location of known geodetic coordinates.

ECEF (Earth-Centered, Earth-Fixed). Refers to a global three-dimensional (X, Y, Z) *Cartesian coordinate* system with its origin at the Earth's center of mass, and "fixed" so that it rotates with the solid Earth. The Z-axis corresponds to the Earth's conventional spin axis, and the X- and Y-axes lie in the equatorial plane. Widely used for geodetic and *GPS* computations.

Ellipsoid height. Straight-line height above and perpendicular to the *ellipsoid*. This is the type of height determined by *GPS*, and it does not equal elevation. Can be converted to orthometric heights ("elevations") using a *geoid* model.

Ellipsoid normal. A line perpendicular to the reference *ellipsoid* along which *ellipsoid heights* are measured.

Ellipsoid. A simple mathematical model of the Earth, historically corresponding to mean sea level or (the *geoid*) and used as part of an *ellipsoidal datum* definition. Constructed by rotating an ellipse about its semi-minor axis. Less frequently referred to as a "spheroid".

Ellipsoidal datum. Reference system for computing geodetic coordinates (latitude, longitude, and ellipsoid height) of a point. A datum always refers to a particular *ellipsoid* and a specific adjustment (e.g. the 2011 adjustment of *NAD 83* for the most recent NGS national adjustment).

FBN (Federal Base Network). Nationwide network of passive *GPS* control stations observed using *GPS* and adjusted by the *NGS*. A nation-wide readjustment of the FBN is scheduled for 2007.

FGDC (Federal Geographic Data Committee). Develops and promulgates information on spatial data formats, accuracy, specifications, and standards. Widely referenced by other organizations. Includes the Federal Geodetic Control Subcommittee (FGCS) and the *NSSDA*.

Geodesic. The shortest distance between two points on the surface of an *ellipsoid*. Analogous to the great circle for the shortest distance between two points on a sphere.

Geographic "projection". This is not a true *map projection* in the sense that it does not transform geodetic coordinates (latitude and longitude) into linear units. However, it is a projection in the sense that it represents geodetic coordinates on a regular flat grid, such that the difference in angular units (e.g., decimal degrees) is equal in all directions. Because of meridian convergence, this results in an extremely distorted coordinate system, especially at high latitudes, and the distortion varies greatly with direction.

Geoid. Surface of constant gravity equipotential (a level surface) that best corresponds to global mean sea level. Often used as a reference surface for *vertical datums*.

GPS (Global Positioning System). A constellation of satellites used for navigation, mapping, surveying, and timing. Microwave signals transmitted by the satellites are observed by GPS receivers to determine a three-dimensional position. Accuracy varies greatly depending on the type of receiver and methods used.

Grid distance. The horizontal distance between two points on a flat plane. This is the type of distance obtained from *map projections*.

Ground distance. The horizontal distance between two points as measured on the curved Earth surface. There is no widely accepted definition of a "horizontal ground distance". In this workbook, it is defined as the *geodesic* (*ellipsoid*) distance scaled to the mean topographic ellipsoid height of the endpoints using the geometric mean radius of curvature at the mean latitude of the endpoint.

GRS-80 (Geodetic Reference System of 1980). The reference *ellipsoid* currently used for many *ellipsoidal datums* throughout the world, including *NAD 83* and *ITRF* (as used by the *NGS*).

HARN (High Accuracy Reference Network). Network of *GPS* stations adjusted by the *NGS* on a state-by-state basis. The Oregon HARN was adjusted in 1991 and 1998. In some states it is referred to as a High Precision *GPS* (or Geodetic) Network (HPGN).

International Foot. Linear unit adopted by the US in 1959, and defined such that one foot equals exactly 0.3048 meter. Shorter than the *US Survey Foot* by 2 *parts per million* (ppm).

ITRF (International Terrestrial Reference Frame). Global geodetic reference system that takes into account plate tectonics (continental drift) and is used mainly in scientific studies. A new ITRF "epoch" is computed periodically and is referenced to a specific time (e.g., ITRF 2000 1997.0). Each epoch is a realization of the International Terrestrial Reference System (ITRS). See Soler (2007), and Soler and Snay (2004) for information on its relationship to *NAD 83* and *WGS 84*.

Local geodetic horizon. A "northing", "easting", and "up" planar coordinate system defined at a point such that the northing-easting plane is perpendicular to the *ellipsoid normal*, north corresponds to true geodetic north, and "up" is in the direction of the *ellipsoid normal* at that point.

Map projection. A functional (one-to-one) mathematical relationship between geodetic coordinates (latitude, longitude) on the curved *ellipsoid* surface, and grid coordinates (northings, eastings) on a planar (flat) map surface. All projections are distorted, in that the relationship between projected coordinates differs from that between their respective geodetic coordinates. See Snyder (1987) for details.

NAD 27 (North American Datum of 1927). *Ellipsoidal datum* of the US prior to *NAD 83*, and superseded by *NAD 83* in 1986. This is the datum of *SPCS* 27 and *UTM* 27.

NAD 83 (North American Datum of 1983). Current official *ellipsoidal* (historically called "horizontal") *datum* of the US. Replaced *NAD 27* in 1986, which is the year of the initial NAD 83 realization. This is the datum of *SPCS* 83 and *UTM* 83. See Schwarz (1986) for details.

NADCON. Mapping-quality d*atum transformation* computer program developed by the NGS for transforming coordinates between *NAD 27* and *NAD 83*, and also between the *NAD 83* 1986 adjustment and the various *HARN* adjustments. See Dewhurst (1990) for details.

NAVD 88 (North American Vertical Datum of 1988). Current official vertical datum of the US. Replaced *NGVD 29* in 1991. See Zilkoski et al. (1992) for details.

NDGPS (National Differential GPS). A nation-wide system of "beacons" (permanently mounted *GPS* receivers and radio transmission equipment) that transmits real-time *differential corrections* which can be used by *GPS* receivers equipped with the appropriate radio receivers. Operated and maintained by the US Coast Guard. See US Coast Guard (2004) for details.

NGS (National Geodetic Survey). Federal agency within the Department of Commerce responsible for defining, maintaining, and providing access to the *NSRS* within the US and its territories.

NGVD 29 (National Geodetic Vertical Datum of 1929). Previous *vertical datum* of the US, superseded by *NAVD 88* in 1991. Not referenced to the *geoid* and called "Mean Sea Level" (MSL) datum prior to 1976.

NSRS (National Spatial Reference System). The framework for latitude, longitude, height, scale, gravity, orientation and shoreline throughout the US. Consists of geodetic control point coordinates and sets of models describing relevant geophysical characteristics of the Earth, such as the *geoid* and surface gravity. Defined and maintained by the *NGS* (see Doyle, 1994, for details).

NSSDA (National Standard for Spatial Data Accuracy). *FGDC* methodology for determining the positional accuracy of spatial data (see Federal Geographic Data Committee, 1998).

OPUS (Online Positioning User Service). A free *NGS* service that computes *NSRS* and *ITRF* coordinates with respect to the *CORS* using raw *GPS* data submitted via the Internet.

Orthometric correction. A correction applied to leveled height differences which reduces systematic errors due to variation in gravitational potential. See Dennis (2004) for details.

Parts per million (ppm). A method for conveniently expressing small numbers, accomplished by multiplying the number by 1 million (e.g., 0.00001 = 10 ppm). Exactly analogous to percent, which is "parts per hundred".

SPCS (State Plane Coordinate System). A system of standardized *map projections* covering each state with one or more zones such that a specific distortion criterion is met (usually 1:10,000). Projection parameters (including units of length) are independently established by the legislature of each state. Can be referenced to either the *NAD 83* or *NAD 27* datums (SPCS 83 and SPCS 27, respectively). See Stem (1989) for details.

Triangulation. A method for determining positions from angles measured between points (requires at least one distance to provide scale).

Trilateration. A method for determining positions from measured distances only.

Trivial vector. A *GPS* vector (computed line connecting two *GPS* stations) that is not statistically independent from other *GPS* vectors observed at the same time.

US Survey Foot. Linear unit of the US prior to 1959, and defined such that one foot equals exactly 1200 / 3937 meter. Longer than the *International Foot* by 2 *parts per million* (ppm).

UTM (Universal Transverse Mercator). A grid coordinate system based on the Transverse Mercator *map projection* which divides the Earth (minus the polar regions) into 120 zones in order to keep map scale error within 1:2500. Can be referenced to either the *NAD 83* or *NAD 27* datums (UTM 83 and UTM 27, respectively). See Hager et al. (1989) for details.

Vertical datum. Reference system for determining "elevations", typically through optical leveling. Modern vertical datums typically use the *geoid* as a reference surface and allow elevation determination using *GPS* when combined with a *geoid* model.

WAAS (Wide Area Augmentation System). A system of geosynchronous satellites and ground *GPS* reference stations developed and managed by the Federal Aviation Administration and used to provide free real-time *differential corrections*. See Federal Aviation Administration (2003) for details.

WGS 84 (World Geodetic System of 1984). Reference *ellipsoid* and *ellipsoidal datum* of *GPS*, defined and maintained by the US Department of Defense. Current realizations of WGS 84 are considered identical to *ITRF* 2000 at the 2 cm level. See National Imagery and Mapping Agency (1997) for details, and Merrigan et al. (2002) for information on the most recent realization.

SELECTED GPS AND GEODESY REFERENCES

Primary resource: The National Geodetic Survey **(http://www.geodesy.noaa.gov/)**

Some NGS web pages of particular interest

Control station datasheets: http://www.ngs.noaa.gov/cgi-bin/datasheet.prl

The Geodetic Tool Kit: http://www.ngs.noaa.gov/TOOLS/

Online Positioning User Service (OPUS): http://www.ngs.noaa.gov/OPUS/

Continuously Operating Reference Stations (CORS): http://www.ngs.noaa.gov/CORS/

The Geoid Page: http://www.ngs.noaa.gov/GEOID/

NGS State Geodetic Advisors: http://www.ngs.noaa.gov/ADVISORS/AdvisorsIndex.shtml

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