NOAA Technical Report NOS 92 NGS 22

Results of Leveling Refraction Tests by the National Geodetic Survey

Rockville, Md.
November 1981

U. S. DEPARTMENT OF COMMERCE
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U. S. DEPARTMENT OF COMMERCE
Malcolm Baldrige, Secretary

National Oceanic and Atmospheric Administration
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RESULTS OF LEVELING REFRACTION TESTS BY THE NATIONAL GEODETIC SURVEY

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ABSTRACT. The National Geodetic Survey (NGS), a component of the National Ocean Survey, NOAA, conducted leveling refraction tests at a site near Gaithersburg, Md., during July-September 1979, and at a site near Tucson, Ariz., in April 1980. The tests were designed to measure refraction errors between bench marks at sight distances of 30, 45, 50, and 60 m by comparing nominal 2-meter refraction-free elevation differences to observed elevation differences containing refraction errors. The refraction-free elevation differences were determined using sight distances of 13 m or less.

The effectiveness of the refraction correction equations developed by T. J. Kukkamaki and B. Garfinkel was evaluated using temperature differences computed from air temperatures observed at 0.5, 1.5, and 2.5 m above the ground surfaces at the test sites. S. R. Holdahl's modeled temperature differences were also evaluated. Single-sight refraction equations by Garfinkel and Kukkamaki, when used with differences computed from daytime-observed air temperatures and with Holdahl's modeled temperature differences, reduced the sum of the differences between observed and standard (O-S) nominal 2-meter elevation differences for Gaithersburg and Tucson by at least 85 percent. Holdahl's modeled air temperature differences, based on meteorological data obtained from NOAA weather stations and on wind and sun codes recorded during the NGS vertical control surveys, performed as well as differences computed from air temperatures observed during the tests. The excellent test results indicate that leveling observations from past surveys (where temperature differences cannot be computed from observed air temperatures) can be greatly improved by applying refraction corrections based on Holdahl's modeled air temperature differences. Application of the Garfinkel and Kukkamaki equations to differences computed from observed air temperatures reduced the sum of the (O-S) values for nighttime observations by 70 and 89 percent, respectively.

INTRODUCTION

One of the largest errors in leveling observations is caused by atmospheric refraction (Kukkamaki 1938, 1939; Hytonen 1967; Remmer 1980; Angus-Leppan 1979, 1980; Brunner 1980; Holdahl 1980a,b, 1981; Strange 1980, 1981; Whalen 1980). The National Geodetic Survey, a component of the National Ocean Survey, NOAA, has conducted refraction tests to compare leveling observations containing refraction errors to refraction-free "standard" elevation differences. Test observations were made at a site on the grounds of the National Bureau of Standards (NBS) in Gaithersburg, Md., during July-September 1979. Procedures for computing and removing refraction errors (Kukkamaki 1938, 1939; Garfinkel 1978, 1979, 1980) and for estimating temperature differences near the surface (Holdahl 1980, 1981) were evaluated. Results of the Gaithersburg test were very encouraging and led to a second test at a site near Tucson, Ariz., in April 1980. The Tucson test was made to determine if refraction correction procedures which were effective at the Gaithersburg test site would also be effective in a semiarid climate with greater solar radiation.

A report has been published documenting preliminary daytime test results for 60-meter sight distances at Gaithersburg and Tucson (Whalen 1980). The present report differs from the earlier report as follows: Results are presented for 30- and 50-meter sight distances at Gaithersburg and for 30- and 45-meter sight distances at Tucson in addition to the 60-meter sight distances. A calibration of each leveling rod graduation is used instead of the 4-point calibration. The coefficient of thermal expansion of the invar band has been determined from NBS observations for each rod and is used for correcting observations. Leveling rod readings are corrected for the difference between a linear slope between the instrument station and bench mark and the mean ground slope. Kukkamaki's single-sight refraction equation is used instead of his balanced-sight equation. A modified Garfinkel single-sight refraction equation is used. These single-sight equations can be used for unbalanced-sight distances and for different ground slopes from the instrument station to each rod of a pair. Unbalanced-sight distance results are included as well as results of night observations at the Tucson test site.

TEST LOCATIONS

Figures 1 and 2 illustrate the Gaithersburg, Md., and Tucson, Ariz., refraction test sites. Bench marks consisted of steel rods driven to a depth of 2.7 m at the Gaithersburg test site, and sleeved "quality A" stainless steel rod marks (Floyd 1978) driven to a depth of 8 m at the Tucson site.

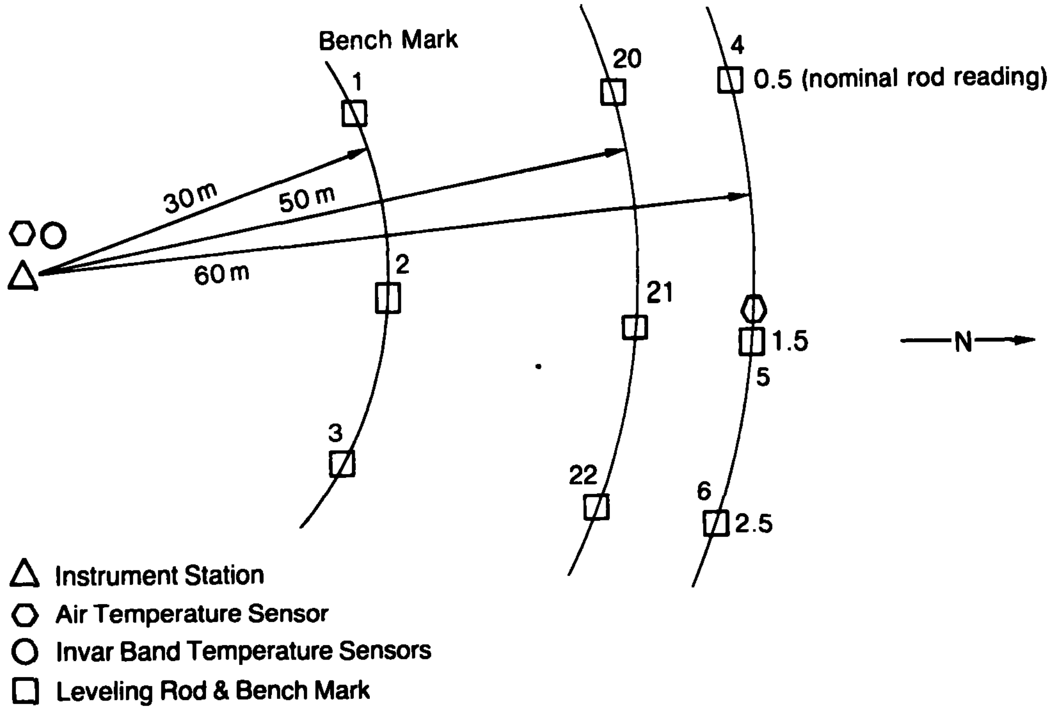

Figure 1.—Sketch of the Gaithersburg test site.

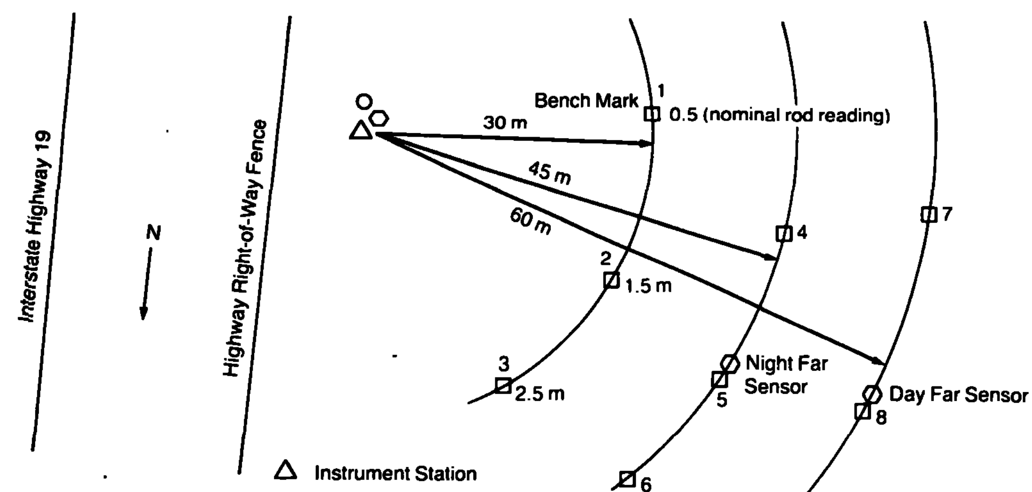

Figure 2.—Sketch of the Tucson test site.

At the Gaithersburg test site (fig. 1) leveling rods were observed at nominal heights above the ground: 0.5 m on bench marks 1, 20, and 4; 1.5 m on bench marks 2, 21, and 5; and 2.5 m at bench marks 3, 22, and 6. Aspirated air temperature sensors were located at the instrument station and at bench mark 5, and invar band temperature sensors were located at the instrument station. Air and invar band temperatures were observed at 0.5, 1.5, and 2.5 m above the ground. Both backsights and foresights were observed to leveling rods at bench marks 2, 21, and 5. Computed differences (backsight minus foresight) provided an estimate of observing error (free of refraction, rod calibration, and temperature errors) associated with each sight distance.

At the Tucson test site (fig. 2) leveling rods were also observed at nominal heights above the ground: 0.5 m on bench marks 1, 4, and 7; 1.5 m on bench marks 2, 5, and 8; and 2.5 m on bench marks 3, 6, and 9. Air temperature sensors were located at the instrument station, and at bench mark 8 during the day. The sensor was moved to bench mark 5 at night because poor visibility at 60 m precluded observations to bench mark 8. The invar band temperature sensor was located at the instrument station for day and night observations. Temperatures were observed at 0.5, 1.5, and 2.5 m above the ground at each sensor location. Backsights and foresights were observed on leveling rods located on bench marks 2, 5, and 8 to estimate random observing errors associated with each sight distance.

Figure 3 is a sketch of three leveling rods on bench marks at an equal sight distance from the instrument stations at the test sites. The ground line slopes downward from left to right and the resulting elevation difference between lower and upper bench marks is a nominal 2 m. The horizon intercepts the rods, from left to right, at nominal heights (above the ground) of 0.5, 1.5, and 2.5 m.

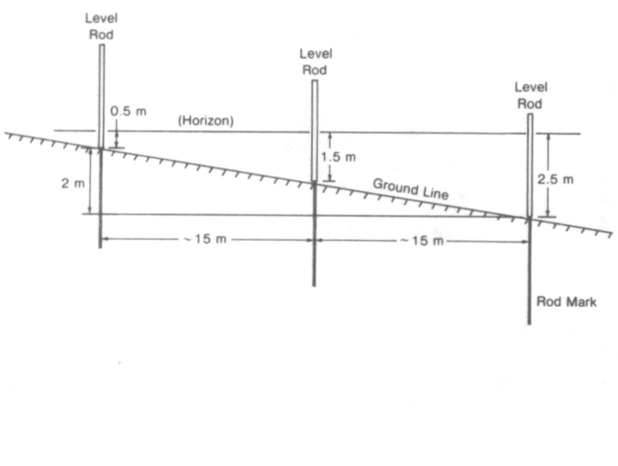

Figure 3.—Sketch of three leveling rods on bench marks.

TEST EQUIPMENT

Figure 4 provides an eastward view of the instrument station and the recorder's table at the Gaithersburg test site. A Ni 002 reversible compensator leveling instrument and 1/2-cm Kern leveling rods were used for the observations. Figure 5 shows details of the footplate, a metal plate set in concrete with drill holes which allowed the instrument height to be varied. Figure 6 shows the aspirated air temperature sensors (left) and a leveling rod with attached invar band temperature sensors (right), located near the instrument station. The temperature sensors and leveling rods were each guyed

Figure 4.—Leveling team in operation.

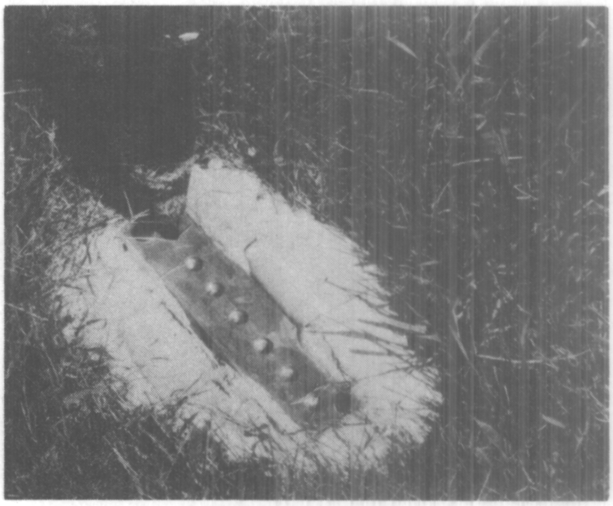

Figure 5.—Metal footplate.

Figure 6.—Air and invar band temperature sensors.

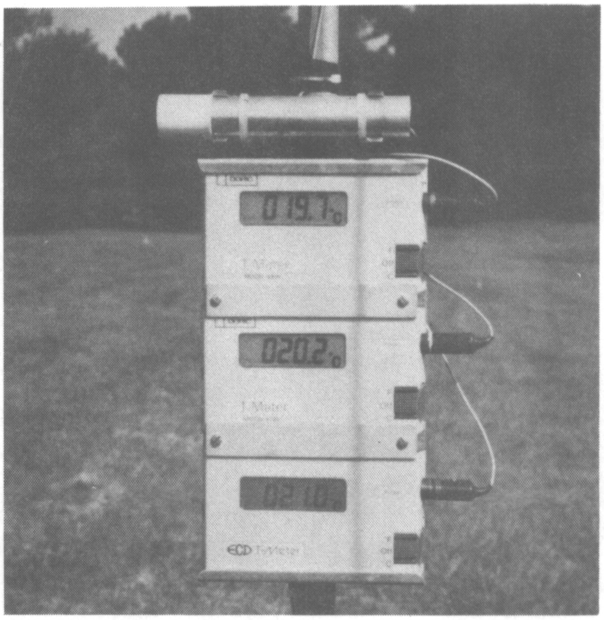

Figure 7.—T-Meters and air temperature sensor.

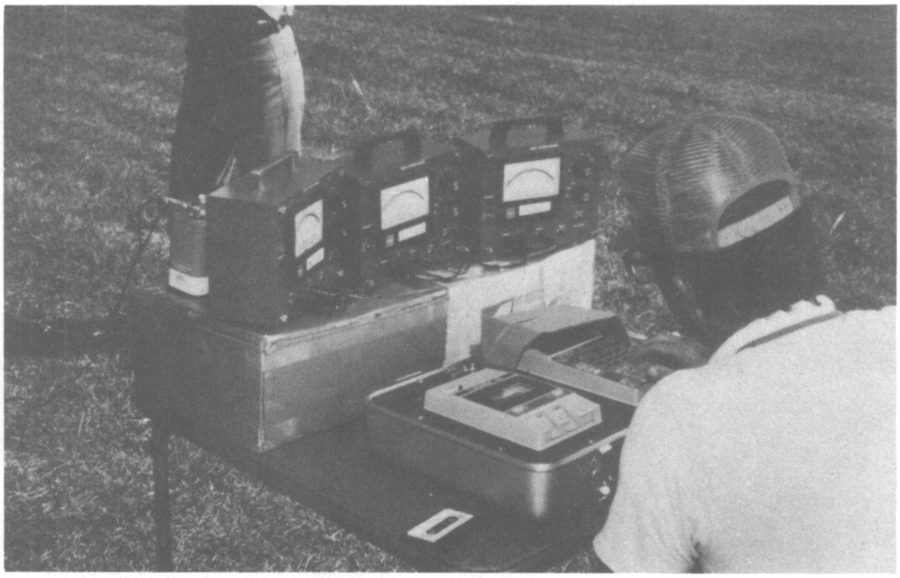

Figure 8.—Recording test data.

with three wires for stability. A close-up of the digital temperature readout devices (T-Meters) and an aspirated air temperature sensor are shown in figure 7. Temperatures were read through the leveling instrument for the readout device located at the far center bench mark. Figure 8 shows the recorder's table with three Yellowsprings readout devices for the invar band temperature sensors, plus the programmable calculator and cassette recorder used to record the test data.

ELEVATION STANDARDS

Figure 9 is a schematic of the third Gaithersburg test site survey configuration used to provide refraction-free "standard" elevation differences between bench marks. The diagonals were not observed during the first two Gaithersburg test site surveys. The survey configuration in figure 9 was also used at the Tucson test site. Forward and backward elevation differences appear as arrows in the illustration. The leveling instrument was set up midway between the bench marks to observe each elevation difference. Air temperatures at 0.5, 1.5, and 2.5 m above the ground were observed and recorded at the instrument station for each observed elevation difference. Sight distances were generally 7 m or less, except for the diagonals which were 13 m or less. The short sight distances kept refraction errors generally smaller than 0.01 mm. At each test site, the same leveling rods remained at each bench mark throughout the surveys to determine both the standard and "refraction" observations. Table 1 lists the rods and the bench marks on which the rods were placed. In the fall of 1979, three surveys were made at the Gaithersburg test site to determine standards. The first survey was not used because a 0.5 mm change occurred in the elevation difference between bench marks 6 and 4 during the time which elapsed between the first and second surveys. (See appendix A, table A-3.) At the Tucson test site, 10 surveys were performed to determine standards.

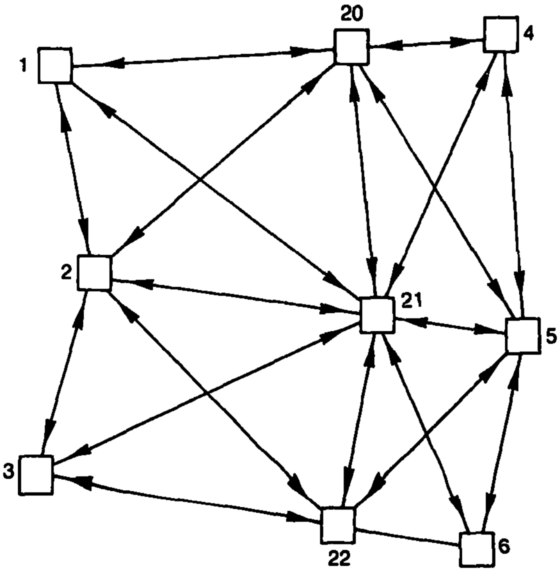

Figure 9.—A typical "standard" survey configuration.

Leveling observations were corrected for systematic errors (see "Corrections to Observations" that follows) before being combined in least-squares adjustments to determine elevations for each survey. (See appendixes A and B.)

ELEVATION DIFFERENCES CONTAINING REFRACTION ERRORS

Each set of observations on three leveling rods at each sight distance consisted of the following items: nine temperature measurements (invar and air near the instrument, and air at the far center bench mark with both sets taken at 0.5, 1.5, and 2.5 m above the ground); center rod observation, left rod observation, right rod observation, and center rod observation; and a repetition of the nine temperature observations. Each rod

Table 1.—Slope corrections

	Gaithersburg			*Tucson*		
Kern rod No.	*Bench mark No.*	*Distance (m)*	*Correction (m)*	*Bench mark No.*	*Distance (m)*	*Correction (m)*
269718	1	30.0	−0.05	1	30.5	−0.04
269721	2	30.0	−0.06	2	30.1	−0.08
269723	3	30.0	0.03	3	30.1	−0.04
269720	4	60.0	−0.16	4	45.2	−0.13
269719	5	60.0	−0.26	5	45.0	−0.03
270714	6	60.0	−0.18	6	45.1	−0.12
270711	20	50.0	−0.18	7	59.6	−0.41
270719	21	50.0	−0.21	8	60.4	−0.24
270718	22	50.0	−0.06	9	59.7	−0.26

observation consisted of an observation of the lower scale in compensator position one, and an observation of the upper scale in compensator position two. Rods were observed sequentially at sight distances of 30, 50, and 60 m at the Gaithersburg test site and at 30, 45, and 60 m at the Tucson test site. The observations containing refraction errors made between the surveys for the first and second standards at the Gaithersburg test site were not used because of the change noted previously in the elevation difference between bench marks 6 and 4.

Observations were made on one night at the Gaithersburg test site and on three nights at the Tucson site. These night observations were made to determine the effectiveness of applying refraction corrections to observations performed after sundown because observing units occasionally must level across bridges at night to avoid impeding traffic.

CORRECTIONS TO OBSERVATIONS

Details on the corrections applied by NGS to the leveling observations have been discussed by Balazs and Young (1981). The 1/2-cm Kern leveling rods used for the test were calibrated by the National Bureau of Standards. During the calibration process, the meter value from the center of the footplate to each graduation on the low and high scales of each rod was determined using a laser calibration system, along with a corresponding invar band temperature. The root-mean-square calibration error of each rod graduation is of the order of 5 micrometers. The rods were then placed in a controlled-temperature chamber where temperatures were varied and a manual calibration was performed between the first and last graduation of each rod at four different temperatures. These measurements were used to determine the coefficient of thermal expansion for each rod.

The coefficients of thermal expansion were used to correct the meter values for the rod graduations to a reference temperature of 25.0°C. The tables of meter values for the rods were stored in a "rod and instrument" computer file, which included the reference calibration temperatures and coefficients of thermal expansion.

Each rod reading from the refraction test was converted to a value in meters by selecting the meter value from the rod and instrument file corresponding to the rod graduation portion of the reading; then the meter value for the micrometer portion of the reading was added to the converted value. The resulting readings (in meters) were corrected for the difference between the observation and reference calibration temperatures using the proper coefficients of thermal expansion.

Meter values for the refraction test rod readings were corrected for curvature so that the effects of refraction could be examined using unbalanced sight distances. The curvature correction (C_c) was computed by

$$C_c = s^2/2r \tag{1}$$

in meters, where s is the sight distance in meters and r is the mean radius of the Earth, 6,363,000 m.

Mean temperatures were computed for 0.5, 1.5, and 2.5 m above the ground for each set of observations at each sight distance. The mean temperature for each height above the ground was based on four observed temperatures: two readings taken before the set of observations (one near the instrument station and one near bench mark 5 at Gaithersburg and bench mark 8 at Tucson) and two more temperature readings after the set of observations, at the same locations. Interpolated temperatures were determined for bench marks 2 and 21 at Gaithersburg (2 and 5 at Tucson) before the temperatures were meaned, using the following equation:

$$t_i = t_0 + (t_j - t_0)\, s_i/s_j \tag{2}$$

where t is the air temperature at a given height above the ground, s is the distance in meters from the instrument station, the subscript i refers to intermediate bench mark i; o refers to the instrument station, and j refers to the temperature sensor located at far bench mark j. For example, if the air temperatures at 0.5 m above the ground are 25.0°C at the instrument station, and 25.5°C at the 60-meter center bench mark, the interpolated air temperature at 0.5 m above the ground at the center 30-meter bench mark is

$$t_{30} = 25.0 + (25.5 - 25.0)\ 30/60 = 25.25\ °C.$$

Thus, for each set of leveling observations, observed or interpolated temperatures at both ends of the sight path between the instrument station and center rods were meaned to obtain representative temperatures at heights of 0.5, 1.5, and 2.5 m above the ground.

The refraction equations assume a uniform ground slope between the instrument station and rod supports. Ground profiles between the instrument station and rod supports are shown for Gaithersburg in figure 10 and for Tucson in figure 11. Supporting data are tabulated in tables A-1 (appendix A) and B-1 (appendix B). Table 1 rod-reading corrections are computed by

$$C_r = [\Delta h/s - \textstyle\sum(h_i - h_0)/\sum s_i]s \tag{3}$$

in meters, where Δh is the elevation difference between the instrument station and bench mark, s is the sight distance from the instrument station to the bench mark, h_i is the elevation of the ground along the profile between the instrument station and bench mark at point i, h_0 is the elevation of the ground at the instrument station,

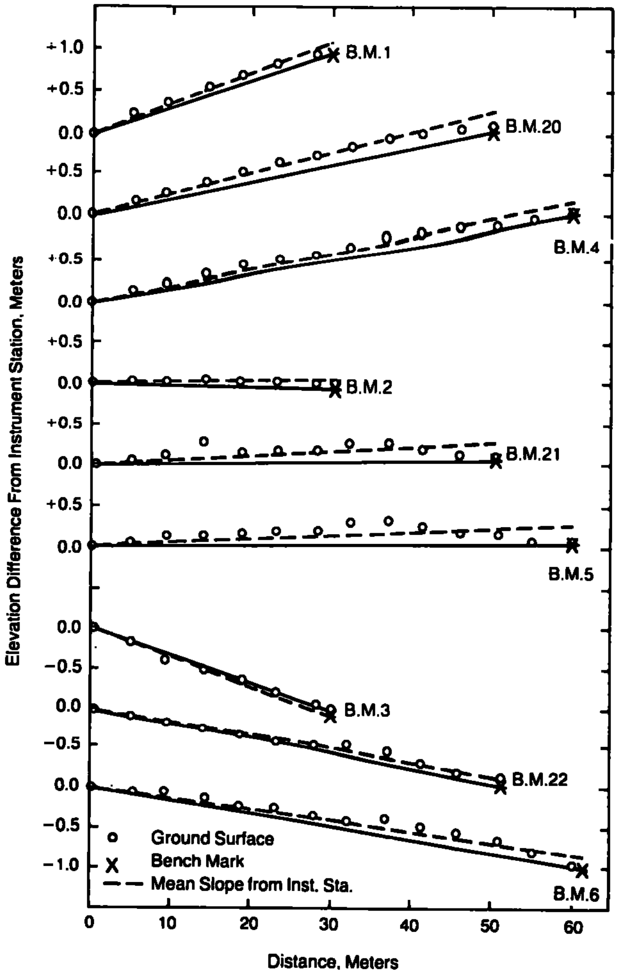

Figure 10.—Ground profiles, Gaithersburg test site.

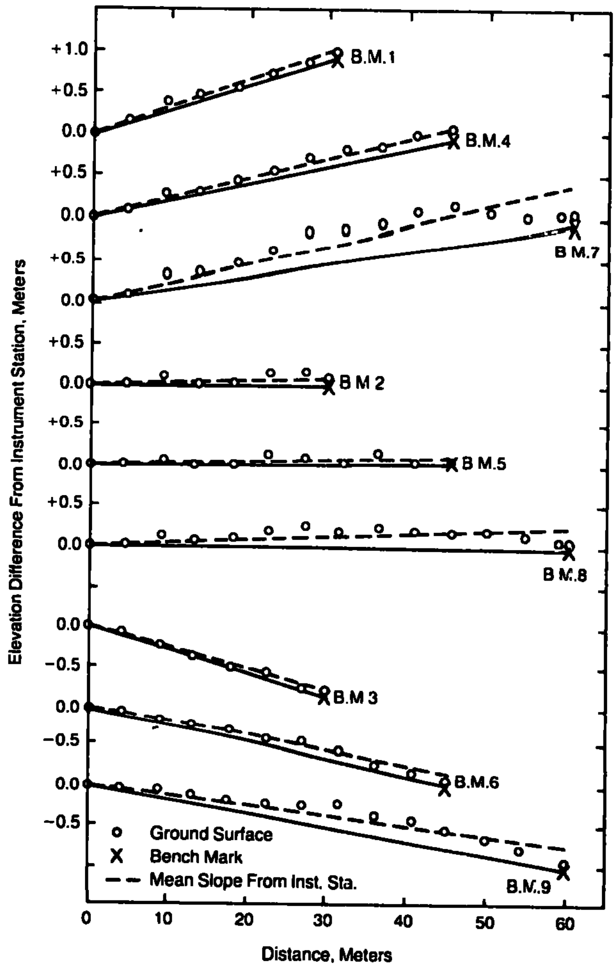

Figure 11.—Ground profiles, Tucson test site.

and s_i is the distance from the instrument station to point i, with all values in meters. The summations ($\sum$) are made over the number of ground points along the profile. For example, using the data from table A-1, the correction for the rod readings on bench mark 1 is

$$C_r = [0.99/29.9 - 4.54/130.40]29.9 = -0.05\,m.$$

The rod-reading corrections were applied in computing elevation differences for use with the refraction equations. Rod-reading corrections were not applied when computing differences between observed and standard elevation differences (O-S). Corrections for nonuniformity of ground slopes between instrument stations and leveling rod supports are important for the refraction tests where they cause systematic errors due to a fixed set of observing conditions. Such errors are random and insignificant for actual vertical control surveys.

REFRACTION EQUATIONS

Kukkamaki's Single-Sight Equation

One representation of the refraction error for a single observation between the instrument and rod is given in Kukkamaki's (1938) equation:

$$R_i = \mathrm{Cot}^2\theta\, d\frac{\Delta t}{z_2^c - z_1^c}\left\{\frac{1}{c+1}Z_i^{c+1} - Z_o^c Z_i + \frac{c}{c+1}Z_o^{c+1}\right\} \quad (4)$$

in meters, where θ is the ground slope from the instrument station towards the leveling rod, Δt (°C) is the temperature difference between heights z_2 and z_1, Z_0 is the height of the instrument, and Z_i is the rod reading, with all heights and rod readings in meters. c is the exponent in the temperature equation:

$$t = a + bz^c \quad (5)$$

where z is the height in meters above the surface to which t corresponds. a, b, and c are constants, and

$$d = -10^{-6}[0.933 - 0.0064(t_m - 20)]P \quad (6)$$

with mean temperature (°C):

$$t_m = (t_{2.5} + t_{0.5})/2 \quad (7)$$

with the subscripts showing the temperature sensor height in meters, and P is the air pressure, in atmospheres, approximated by

$$P = [1 - bH/T_0]^{g/Rb} \quad (8)$$

where b is the lapse rate (0.0065 K m^{-1}), H is the height above sea level in meters, g is the mean value of the Earth's gravity (9.81 m sec^{-2}), R is the gas constant at the point considered (287 m^2 s^{-2} K^{-1}), and T_0 is the air temperature in degrees Kelvin at mean sea level. In terms of the mean air temperature at the instrument (t_m, see eq. 7)

$$T_0 = t_m + 0.0065H + 273. \quad (9)$$

The cotangent of the slope angle is computed by

$$Cot\theta = s/\Delta h \quad (10)$$

where s is the sight distance in meters and Δh is the elevation difference

$$\Delta h = Z_0 - Z_i \quad (11)$$

in meters, with Z_0 and Z_i, as described previously.

A c-value of $-1/3$ is used with eq. (4) in this report with both observed temperatures and Holdahl's (1981) predicted temperatures.

Garfinkel's Single-Sight Equation

Garfinkel (1980) provides the following single-sight equation to compute refraction errors for rod readings with unbalanced-sight distances:

$$R_i = KWs^2 \left\{ \Delta h \left[1 + \frac{33}{160} \frac{\Delta h^2}{a^2} \eta^2 \right] \frac{\Delta t''}{a^2} - 2.83 \frac{(1 + 1/2\,\eta^2)}{(1 + 2/3\,\eta^2)} \left[1 + \frac{\Delta h^2}{12a^2} \eta^2 \right] \frac{\Delta t'}{a} \right\} \quad (12)$$

in meters, where

$$K = 1.81 \times 10^{-7}/a^2(1+r) \quad (13)$$

and

$$r = \eta^2(c-1)(c-3)/12 \quad (14)$$

where a is the air temperature sensor separation (1 m for the tests), $\eta = a/Z_0$. c, from eq. (5), is approximated by $-1/3$, and

$$W = P/T^2. \quad (15)$$

P is the air pressure in atmospheres from eq. (8), T is the air temperature in K/273°, s is the sight distance in meters, and

$$\Delta t' = (t_{2.5} - t_{0.5})/2 \quad (16)$$

in degrees Celsius, with the height of the temperature sensors above the surface shown by the t subscript, and

$$\Delta t'' = (t_{2.5} - t_{1.5}) - (t_{1.5} - t_{0.5}) \quad (17)$$

in degrees Celsius, and Δh is calculated using eq. (11).

TEMPERATURES

Observed Temperatures

Differential thermometers have been used in Finland since 1938 to measure the Δt term for refraction computations. Details of the Finnish differential thermometer are given by Hytonen (1967). On a few specialized surveys, NGS has used a differential thermometer similar to the one described by Hytonen. Because differential thermometers require frequent calibrations, NGS has changed to thermistors with digital T-Meter (fig. 7) and Yellowsprings display devices (fig. 8) for the refraction tests. Temperatures are read from T-Meter digital displays to 0.1°; the Yellowsprings meters have a similar reading precision. The thermistor-display combinations were tested before the refraction tests against standardized thermometers at the NGS Instrument and Equipment Branch. Agreement with temperatures from the standardized thermometers was within the reading precision of the meters. The aspirated air temperature sensor consists of a thermistor mounted within a double-sleeved shield and equipped with a small fan to draw air past the thermistor. The surface of the outer shield is polished to reflect sunlight.

Holdahl's Modeled Temperatures

Holdahl (1981) provides equations to estimate temperature differences in degrees Celsius between heights z_2 and z_1 above the surface:

$$\Delta t = 3 \left\{ \frac{H^2 T}{(C_P \rho) g} \right\}^{1/3} (z_2^c - z_1^c) \quad (18)$$

where H is the upward sensible heat flux in Watts/meter, T is the air temperature at instrument height in degrees

Kelvin, C_p is the specific heat of air at constant pressure, ρ is the air density ($C_p\rho$ = 1200), g is the acceleration due to gravity in m sec^{-2}, z_1=50 cm, z_2=250 cm, and c from eq. (5) is assumed to be a constant of −1/3.

During 2-hour intervals near sunup and sundown the atmosphere is in a "neutral" or transitional stage (Kukkamaki 1939), and Δt should be calculated with the equation:

$$\Delta t = -\frac{H}{C_P \rho\, k\, u_*} \ell n(z_2/z_1) \qquad (19)$$

in degrees Celsius, where k is the von Karman constant (k=0.4), u_* is the wind friction velocity (m/sec), and the other terms are the same as described previously.

Holdahl's modeled temperature differences were computed with eq. (18) or (19) and used in eq. (4) to compute refraction corrections for the tests at Gaithersburg and Tucson. The predicted temperature differences were multiplied by a sun correction factor based on the sun codes recorded by NGS leveling teams. The sun codes are

0=cloudy (less than 25 percent of setups under sunny conditions),

1=partly cloudy (from 25 to 75 percent of setups under sunny conditions),

2=clear (more than 75 percent of setups under sunny conditions).

The u_* term in eq. (19) is based on wind codes also recorded by NGS leveling teams. The wind codes are

0=calm (wind speed less than 10 km/hour),
1=moderate (wind speed 10 to 25 km/hour),
2=strong (wind speed greater than 25 km/hour).

Ordinarily, leveling observations are not made when the wind speed exceeds 25 km/hour. The Δt correction factors based on the sun code, and the friction velocity (u_*) factors (to be used in eq. 19) based on the wind code are shown in table 2.

Table 2.—Δt sun correction factors and friction velocity factors

Sun code	*Sun correction factor*	*Wind code*	*Friction velocity u_**
0	0.4	0	0.356
1	1.0	1	0.356
2	1.0	2	0.524

Holdahl's temperature model takes into account "historical records of solar radiation, sky cover, precipitation, and ground albedo from many locations in the conterminous United States" (Holdahl 1981). His predicted temperature differences were on the average 0.12°C less than the mean temperature difference (−0.56°C) computed from observed air temperatures at Gaithersburg and 0.22°C less than the mean temperature difference (−1.03°C) computed from observed air temperatures at Tucson.

TEST RESULTS

Balanced-Sight Distances, Daytime Observations

Balanced-sight results from the refraction tests are shown without slope corrections in figures 12 through 17 and in table 3. Estimated standard errors for (O-S) sums are given in table 4. The standard error of the (O-S) sums are estimated by:

$$\sigma_{\Sigma(o-s)} = \left[n_o\sigma_o^2 + \sigma_s^2 \Sigma n_i^2 \right]^{1/2} \qquad (20)$$

where subscripts o and s refer to observed and standard values, respectively, n_o is the number of observed elevation differences, n is the number of times the i-th standard was used, and the n_i^2 term is summed for k standards. σ_o^2 is the variance of the differences between backsight and foresight observations on the center rod, for each sight distance. (See table 4.) σ_s^2 is the pooled variance estimate for a standard elevation difference based on between-set variance. (See table A-3 in appendix A and table B-3 in appendix B.) The signal-to-noise

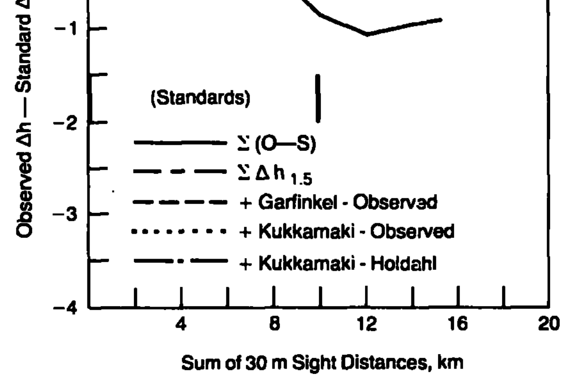

Figure 12.—Gaithersburg test site, 30-meter sight distances.

ratio (S/N) was obtained by dividing the summed (O-S) values by their standard errors. In figures 12 through 17, the sum of the differences between observed and standard (nominally 2 m) elevation differences between the lower and upper bench marks is plotted in centimeters against the sum of the sight distances in kilometers for each sight distance. The sum is also shown with the different refraction corrections applied. The heavy vertical lines in the figures indicate the surveys which were made for elevation standards. The "$\sum \Delta h_{1.5}$" curve shows accumulated instrument and observer errors, independent of refraction error, computed from repeat observations to the center rods which were observed at the nominal height of the instrument.

Figure 12 shows that the refraction corrections tended to overcorrect at 30-meter sight distances for the Gaithersburg test. In table 4 the signal-to-noise ratio for the Gaithersburg 30-meter sight distance was only 0.7, indicating the test results were not very conclusive at the 30-meter sight distance.

Figure 13 shows that the refraction equations gave good corrections for the Gaithersburg 50-meter sight distances.

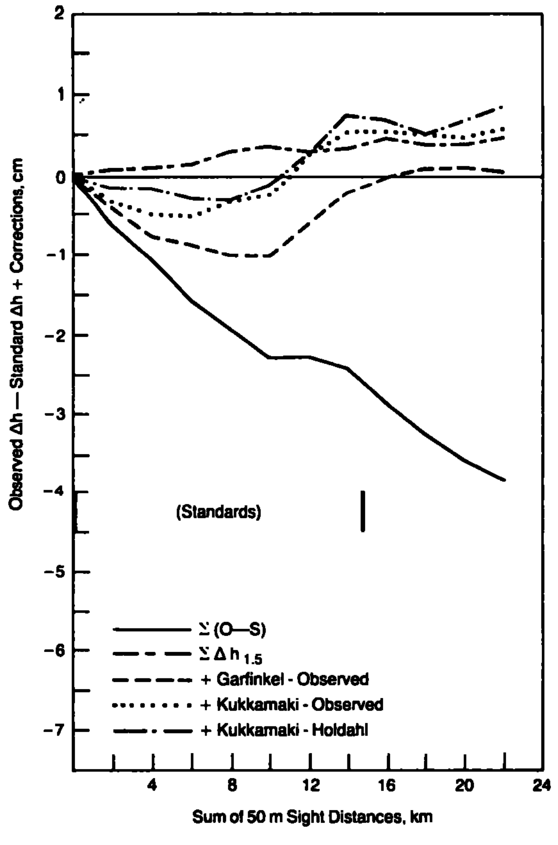

Figure 13.—Gaithersburg test site, 50-meter sight distances.

Figure 14 shows that all the refraction equations undercorrected the sum of the (O-S) values for the 60-meter sight distance at Gaithersburg, with the systematic

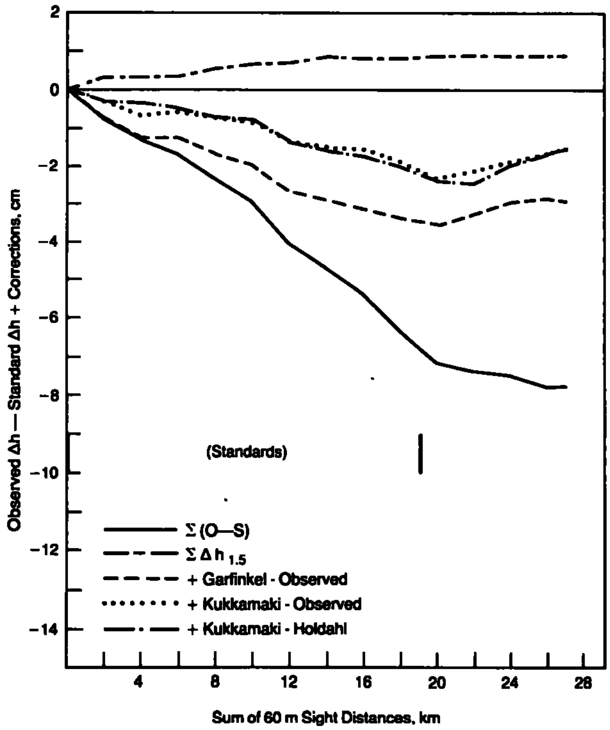

Figure 14.—Gaithersburg test site, 60-meter sight distances.

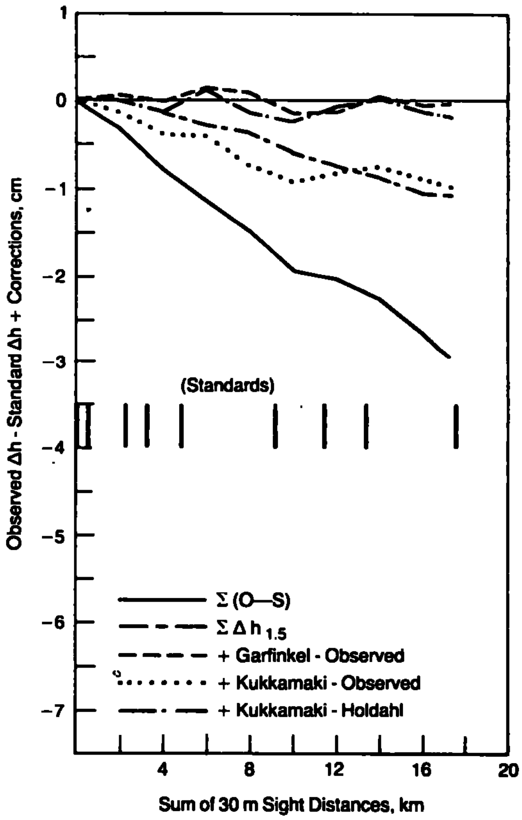

Figure 15.—Tucson test site, 30-meter sight distances.

instrument-observer error accumulating to approximately 1 cm. Note the change of scale for the (O-S) sums in figure 14 versus figures 12 and 13.

The reductions in table 3 for the sum of the (O-S) terms for the Gaithersburg test were 83, 96, and 100 percent using the Garfinkel, Kukkamaki, and Kukkamaki-Holdahl corrections, respectively.

Surveys are normally made with a variety of sight distances. The sum of the (O-S) terms provides a rough indication of what would happen on a 64-kilometer survey with sight distances varying from 30 to 60 m with 2-meter elevation differences observed at each instrument station, under similar weather and ground surface conditions. The accumulated refraction errors shown by the sums of the (O-S) values are the worst case for these sight distances in that setup elevation differences would probably never average 2 m on an actual survey.

The plot in figure 15 indicates that all of the corrections reduced the sum of the (O-S) values to within the sum of the instrument-observer errors on the center rod ($\sum \Delta h_{1.5}$). The signal-to-noise ratio, taken from table 4, is 5.1.

Figure 16 shows an overcorrection by the end of the survey, when the Garfinkel and Kukkamaki-Holdahl corrections were used for Tucson 45-meter sight distances, and an almost perfect correction when the Kukkamaki correction was applied.

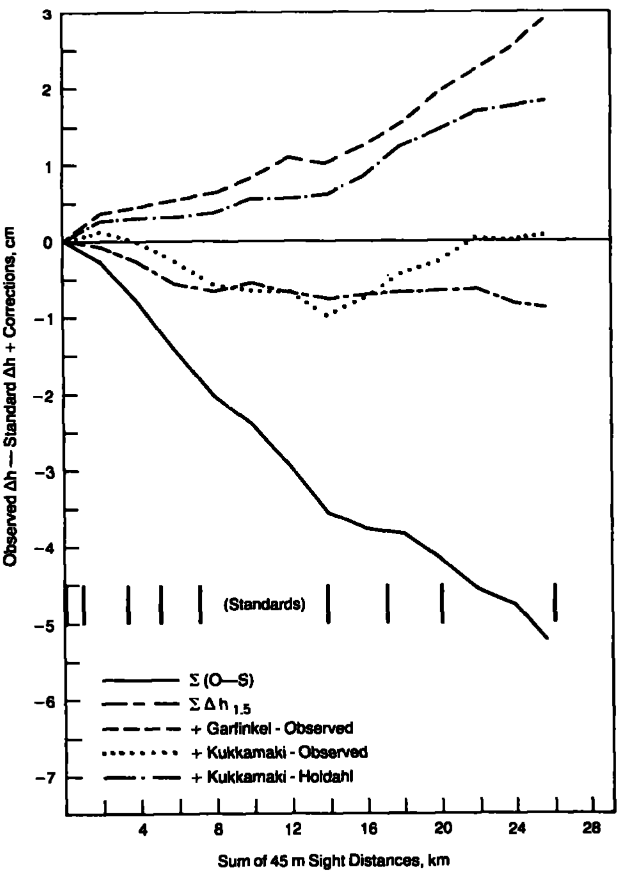

Figure 16.—Tucson test site, 45-meter sight distances.

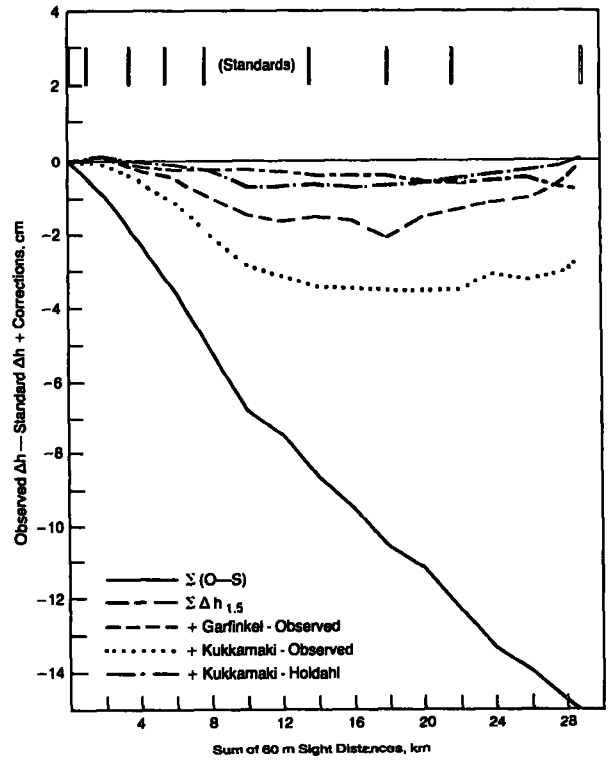

Figure 17.—Tucson test site, 60-meter sight distances.

Figure 17 depicts the 60-meter sight-distance results from the Tucson test. The (O-S) values accumulated to −15 cm over an accumulated sight distance of 29 km. There is a small undercorrection with the Kukkamaki equation. The signal-to-noise ratio (table 4) of 26.5 for the Tucson 60-meter (O-S) sum indicates this to be the most reliable balanced-sight result of the tests.

The reductions shown in table 3 for the sum of the (O-S) values for Tucson were 89, 84, and 93 percent using Garfinkel, Kukkamaki, and Kukkamaki-Holdahl corrections, respectively.

The percent reductions in the sum of the sums of the (O-S) values from Gaithersburg and Tucson for daytime observations were 99 for Garfinkel, 88 for Kukkamaki, and 95 for the Kukkamaki-Holdahl corrections. The sum of the sums represents results from a 135-kilometer survey with 1,499 instrument stations, with sight distances varying between 30 and 60 m, 2-meter elevation differences observed at each instrument station, and with the Gaithersburg and Tucson mixture of ground surfaces and climates. The accumulated refraction errors shown by the −356 mm sum of the (O-S) sums are the "worst case" for these sight distances because setup elevation differences would normally average less than 2 m on an actual survey. Such large refraction-induced errors would not be uncommon in older NGS surveys along railroads where grades were controlled and longer sight distances were permitted.

Table 3.—Balanced-sight results

Sight distance (m)	*Number of sets*	*Sum of distance (km)*	*Sum of error (mm)*	*Sum of (O-S)*[1] *(mm)*	*Sum (O-S) plus Garfinkel's correction (mm)*	*Sum (O-S) plus Kukkamaki's correction (mm)*	*Sum (O-S) plus Holdahl's correction (mm)*
(Gaithersburg, day)							
30	243	14.6	0.3	−9.1	5.8	5.0	6.4
50	221	22.1	4.4	−38.4	0.7	5.6	8.5
60	225	27.0	9.2	−77.5	−27.9	−15.6	−15.2
Sums	689	63.7	13.9	−125.0	−21.4	−5.0	−0.3
Percent reduction in sum (O-S)					83	96	100
(Tucson, day)							
30	287	17.3	−10.8	−29.4	−0.4	−10.0	−1.9
45	284	25.6	−8.7	−52.1	28.8	0.5	18.1
60	239	28.6	−7.4	−149.7	−2.6	−28.0	0.2
Sums	810	71.5	−26.9	−231.2	25.8	−37.5	16.4
Percent reduction in sum (O-S)					89	84	93
Sum of Sums	1,499	135.2	−13.0	−356.2	4.4	−42.5	16.1
Percent reduction in sum of sums (O-S)					99	88	94
(Tucson, night)							
30	181	10.9	−0.3	16.2	1.0	1.7	——
45	165	14.9	−5.5	40.0	15.9	4.7	——
Sums	346	25.8	−5.8	56.2	16.9	6.4	——
Percent reduction in sum (O-S)					70	89	——

[1](O-S) = observed elevation difference minus standard (adjusted) elevation difference between bench marks.

Table 4.—Balanced sights, error estimates for (O-S)[1] sums

Sight distance (m)	*Number of observations*	σ_o *(mm)*	*Number of standards*	σ_s *(mm)*	*Sum* n_i^2	σ *Sum (O-S) (mm)*	*Sum (O-S) (mm)*	*Sum (O-S)/*σ *(signal/noise)*
(Gaithersburg, day, nominal elevation difference = 2 m)								
30	243	0.078	2	0.075	34604	14.00	−9.1	0.7
50	221	0.160	2	0.075	27400	12.64	−38.4	3.0
60	225	0.212	2	0.075	29464	13.26	−77.5	5.8
(Tucson, day, nominal elevation difference = 2 m)								
30	287	0.106	9	0.047	13592	5.77	−29.4	5.1
45	284	0.160	9	0.047	13390	6.07	−52.1	8.6
60	239	0.224	9	0.047	9026	5.65	−149.7	26.5
(Tucson, night, nominal elevation difference = 2 m)								
30	182	0.094	7	0.047	10402	4.96	16.2	3.3
45	165	0.176	5	0.047	8686	4.93	40.0	8.1

[1](O-S) = observed elevation difference minus standard (adjusted) elevation difference between bench marks.

Balanced-Sight Distances, Nighttime Observations

Nighttime observations are sometimes made on the deserts of the southwestern United States by survey organizations because daytime observations are almost impossible during the summer due to "shimmer." The NGS occasionally surveys across bridges at night when traffic would be impeded by daytime observations.

Observations were made on three nights at the Tucson site to test the suitability of the refraction equations for night observations. The rods were illuminated by a spotlight mounted atop the Ni 002 instrument. Successful observations were made at 30- and 45-meter sight distances, but observations were discontinued at 60 m because the light was not sufficiently bright.

The results of the test indicate that the Garfinkel refraction equation gives corrections with the wrong algebraic sign for night observations. The $\Delta t''$ term in Garfinkel's equation remained positive both day and night. The corrections appear to be of the correct magnitude, but of the wrong algebraic sign. Use of $c = -1/3$ for Kukkamaki's eq. (2) for both day and night observations resulted in a change in sign of the refraction correction for night observations. This led to a computer test run with the signs of the corrections based on Garfinkel's equation reversed when $\Delta t'$ was positive. Figures 18 and 19 as well as table 3 show the results of the test for 30- and 45-meter sight distances. The percent reductions in the sum of the (O-S) values from Garfinkel's single-sight equation, when applied with the sign reversed, and from the Kukkamaki single-sight equation, were 70 and 89, respectively.

Reversal of the sign of the refraction corrections, based on Garfinkel's equation, for positive $\Delta t'$ values provided an empirical solution to the problem, but the inadequacies of the model for night observations remain to be resolved.

Unbalanced-Sight Distances, Daytime Observations

Long uphill and short downhill sight distances increase refraction errors and improve the test signal-to-noise ratio. Table 5 provides the unbalanced-sight results. The columns show the backsight (from) and foresight (to) bench marks and sight distances, sight imbalances (backsight − foresight), sum of the sight distances, number of observations, sum of the (O-S) values, and (O-S) sums after applying corrections based on the Garfinkel and Kukkamki single-sight equations used with temperature differences computed from observed air temperatures. (See also table 1 and figs. 1 and 2.)

Error estimates for unbalanced-sight (O-S) sums are cited in table 6, based on eq. (20). The σ_s^2 variance estimates are the squares of the pooled standard error estimates (pooled sigma x) based on between-set variance from tables A-4 (appendix A) and B-4 (appendix B). The σ_o estimates shown in table 6 are calculated by:

$$\sigma_o = \left[(\sigma_{BS}^2 + \sigma_{FS}^2)/2 \right]^{1/2} \qquad (21)$$

in millimeters, where σ_{BS} and σ_{FS} are the σ_o estimates for the appropriate backsight and foresight distances from table 4. For example, the estimate for an observed elevation difference between bench marks 3 (s = 30 m) and 20 (s = 50 m) at Gaithersburg, independent of refraction errors, is

$$\sigma_o = [(0.078^2 + 0.160^2)/2]^{1/2} = 0.126\,mm$$

The improvement in the signal-to-noise ratio for unbalanced versus balanced sight distances can be seen by

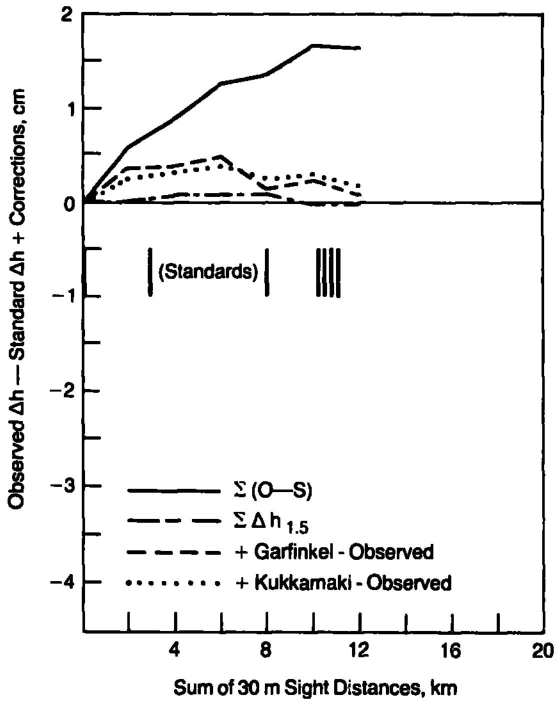

Figure 18.—Tucson test site, 30-meter sight distances, night.

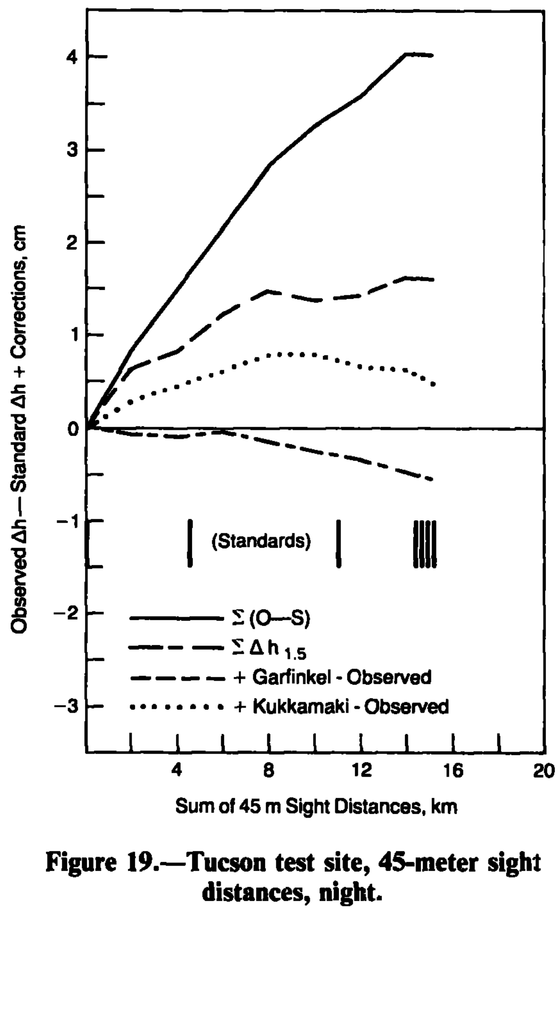

Figure 19.—Tucson test site, 45-meter sight distances, night.

comparing the last columns of tables 4 and 6. The largest signal-to-noise ratio (35.7) is for observations between bench marks 3 and 7 at Tucson where the −209 mm sum of the (O-S) values is reduced to 2 mm after applying Garfinkel's correction, and to −8 mm after applying Kukkamaki's correction.

In table 5, the percent reduction in the sum of the sums of (O-S) values for unbalanced-sight daytime nominal 2-meter elevation differences at Gaithersburg and Tucson is 85 for the Garfinkel correction and 88 for the Kukkamaki correction.

Test results for unbalanced-sight observations between Tucson bench marks 2, 5, and 8, which have the same nominal elevation, are shown at the bottom of table 5, and error estimates are given at the bottom of table 6. The actual elevations differences are: bench marks 2 to 5, 0.038 m; 2 to 8, −0.019 m; and 5 to 8, −0.057 m. The size of the sum of the (O-S) values clearly shows

Table 5.—Unbalanced-sight results

From bench mark No.	*To bench mark No.*	*Nominal backsight distance (m)*	*Nominal foresight distance (m)*	*Backsight minus foresight distance (m)*	*Sum sight distance (m)*	*Number of observations*	*Sum (O-S)[1] (mm)*	*Sum (O-S) plus Garfinkel's correction (mm)*	*Sum (O-S) plus Kukkamaki's correction (mm)*
(Gaithersburg, nominal elevation difference = 2 m)									
3	20	30	50	20	16.6	207	−53.5	9.6	14.4
3	4	30	60	30	20.7	230	−71.3	31.5	43.6
22	4	50	60	10	23.3	212	−44.7	25.3	34.0
Sums					60.6	649	−169.5	66.4	92.0
Percent reduction in sum (O-S)								61	46
(Tucson, nominal elevation difference = 2 m)									
3	4	30	45	15	20.9	278	−84.6	17.6	1.0
3	7	30	60	30	22.0	244	−208.6	1.8	−8.0
6	7	45	60	15	25.7	245	−181.0	12.4	−9.0
Sums					68.6	767	−474.2	31.8	−16.0
Percent reduction in sums								93	97
Sum of sums					129.2	416	−643.7	98.2	76.0
Percent reduction in sum of sums								85	88
(Tucson, nominal elevation difference = 0 m)									
2	5	30	45	15	20.9	278	−39.0	5.9	4.2
2	8	30	60	30	22.0	244	−89.4	19.9	9.5
5	8	45	60	15	25.7	245	−53.8	17.0	8.3
Sums					68.6	767	−182.2	42.8	22.0
Percent reduction in sum (O-S)								77	88

[1](O-S) = observed elevation difference minus standard (adjusted) elevation difference between bench marks.

Table 6.—Unbalanced-sight distances, error estimates for (O-S)[1] sums

Backsight bench mark No.	*Foresight bench mark No.*	*Number of observations*	σ_o *(mm)*	*Number of standards*	σ_s *(mm)*	*Sum* n_i^2	σ *Sum (O-S) (mm)*	*Sum (O-S) (mm)*	*Sum (O-S)/*σ *(signal/noise)*
(Gaithersburg, nominal elevation difference = 2 m)									
3	20	207	0.126	2	0.075	24552	11.89	−53.6	4.5
4	4	230	0.160	2	0.075	31634	13.56	−71.3	5.3
22	4	212	0.188	2	0.075	25844	12.36	−44.7	3.6
(Tucson, nominal elevation difference = 2m)									
3	4	279	0.136	9	0.053	12922	6.44	−84.8	13.2
3	7	244	0.175	9	0.053	9526	5.85	−208.6	35.7
6	7	245	0.195	9	0.053	9544	6.01	−181.1	30.1
(Tucson, nominal elevation difference = 0 m)									
2	5	279	0.136	9	0.053	12922	6.44	−39.1	6.1
2	8	244	0.175	9	0.053	9526	5.85	−89.3	15.3
5	8	245	0.195	9	0.053	9544	6.01	−53.7	8.9

[1](O-S) = observed elevation difference minus standard (adjusted) elevation difference between bench marks.

the existence of refraction errors in leveling observations over a relatively flat surface when sight distances are not balanced. The reductions in the sum of the (O-S) values for Tucson daytime observations, made over nominally zero elevation differences with 15- and 30-meter unbalanced-sight distances, are 77 and 88 percent for the Garfinkel and Kukkamaki single-sight equations, respectively.

SUMMARY AND CONCLUSIONS

Table 7 summarizes the percent reductions in the sum of the sums of the (O-S) values from tables 3 and 5. All the refraction equations and temperatures used in the test gave a net reduction in the sum of the sums of the (O-S) values of at least 70 percent. The Garfinkel, Kukkamaki, and Kukkamaki-Holdahl reductions in the sum of the sums of the (O-S) values for daytime balanced-sight observations at Gaithersburg and Tucson were 99, 88, and 95 percent, respectively. The reduction in unbalanced-sight sum of the sums of the (O-S) values for Gaithersburg and Tucson was 85 and 88 percent for the Garfinkel and Kukkamaki equations when used with observed temperatures. Even the sum of the sums of the (O-S) values for balanced-sight night observations was reduced by 70 percent with the modified Garfinkel equation and by 89 percent with the Kukkamaki equation.

Table 7.—Summary of refraction test results, using 2-meter nominal elevation differences

Percent reduction in (O-S)[1] sums			
Garfinkel	*Kukkamaki*	*Kukkamaki-Holdahl*	*Remarks*
99	88	95	Day, balanced sights
85	88	—	Day, unbalanced sights
70	89	—	Night, balanced sights

[1](O-S) = observed elevation difference minus standard (adjusted) elevaton difference, between bench marks.

The test results greatly exceeded our expectations. The excellent results obtained with Holdahl's predicted temperatures give us great confidence that we can successfully correct old leveling surveys for refraction errors as we prepare for the readjustment of the North American Vertical Datum.

Holdahl's predicted temperature differences apply to the average surface conditions for an area. Sufficient observed air temperatures to compute temperature differences should still be observed on new surveys, especially when surface conditions differ from the average for the area, e.g., as on railroad beds or concrete and asphalt in urban areas. Both the Kukkamaki and Garfinkel equations gave excellent results for the test when used with observed temperatures. The choice of which equation to use may well depend on whether the surveyor wishes to observe two (Kukkamaki) or three (Garfinkel) temperatures at each instrument station.

ACKNOWLEDGMENTS

We are indebted to the foresight of Professor Kukkamaki who developed his refraction theory and equations in the 1930's and has applied them to Finnish leveling surveys since that time, in spite of the lack of application of refraction corrections to leveling by geodesists from other countries.

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APPENDIX A.—GAITHERSBURG TEST DATA

Table A-1.—Gaithersburg, ground elevations between instrument station and bench marks

Bench mark No.	*1*	*2*	*3*	*20*	*21*	*22*	*4*	*5*	*6*
Distance (m)	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*
0.0[1]	136.56	136.56	136.56	136.56	136.56	136.56	136.56	136.56	136.56
5.3	136.79	136.59	136.36	136.73	136.61	136.47	136.70	136.61	136.57
9.9	136.92	136.59	136.13	136.85	136.67	136.38	136.80	136.68	136.49
14.5	137.08	136.62	136.05	136.96	136.84	136.27	136.89	136.69	136.41
19.0	137.25	136.59	135.91	137.07	136.71	136.21	137.00	136.72	136.33
23.6	137.38	136.58	135.76	137.19	136.73	136.11	137.08	136.74	136.29
28.2	137.49	136.57	135.62	137.26	136.75	136.06	137.14	136.76	136.17
29.9	137.55	--	--	--	--	--	--	--	--
30.0	--	136.55	--	--	--	--	--	--	--
30.2	--	--	135.54	--	--	--	--	--	--
32.8	--	--	--	137.37	136.81	136.06	137.25	136.84	136.11
37.3	--	--	--	137.47	136.81	135.98	137.35	136.86	136.16
41.9	--	--	--	137.52	136.74	135.86	137.37	136.80	136.06
46.5	--	--	--	137.60	136.67	135.74	137.42	136.73	135.98
50.0	--	--	--	--	--	135.61	--	--	--
50.1	--	--	--	137.64	136.63	--	--	--	--
51.0	--	--	--	--	--	--	137.46	136.71	135.91
55.6	--	--	--	--	--	--	137.54	136.63	135.75
60.2	--	--	--	--	--	--	--	136.57	135.59
60.4	--	--	--	--	--	--	137.58	--	--
61.4	--	--	--	--	--	--	--	--	135.56
					(Bench mark elevations)[2]				
	137.55	136.54	135.54	137.60	136.64	135.62	137.58	136.59	135.56

[1] Distance origin is at the instrument station.
[2] Distance from the instrument station is the same as for the last elevation above.

Table A-2.—Gaithersburg, elevation standards, 1979.

Bench mark No.	*Aug. 9-10 adjusted elevation (m)*	*Sept. 7-14 adjusted elevation (m)*	*Sept. 24-26 adjusted elevation (m)*	*Combined adjusted elevation (m)*
1	137.55650	137.55700	137.55703	137.55684
2	136.54382	136.54429	136.54426	136.54410
3	135.53434	135.53489	135.53493	135.53469
4	137.58762	137.58870	137.58864	137.58825
5	136.59737	136.59821	136.59824	136.59794
6	135.56135	135.56190	135.56201	135.56172
20	--	137.60298	137.60294	137.60269
21	--	136.64527	136.64532	136.64506
22	--	135.62777	135.62784	135.62758
Class A	137.69994	137.69994	137.69994	137.69994

Table A-3.—Gaithersburg, standard elevation differences (x), for balanced-sight distances

1979 Mo./day	*x* *s*=30[1] 3 to 1 (m)	*x'*−*x* (mm)	*x* *s*=50 22 to 20 (m)	*x'*−*x* (mm)	*x* *s*=60 6 to 4 (m)	*x'*−*x* (mm)
8/9-10	2.02216	−0.04	--	--	2.02627	0.45[2]
9/7-14	2.02211	0.01	1.97521	−0.05	2.02680	−0.08
9/24-26	2.02210	0.02	1.97510	0.06	2.02663	0.09
Mean (x')	2.02212	--	1.97516	--	2.02672	--
Sigma x	--	0.032	--	0.078	--	0.120
Pooled sigma x = 0.075						

[1]Sight distance.
[2]Rejected.

Table A-4.—Gaithersburg, standard elevation differences (x), for unbalanced-sight distances

1979 Mo./day	*x* 3 to 20[1] (m)	*x'*−*x* (mm)	*x* 3 to 4 (m)	*x'*−*x* (mm)	*x* 22 to 4 (m)	*x'*−*x* (mm)
8/9-10	--	--	2.05328	0.48[2]	--	--
9/7-14	2.06809	−0.04	2.05381	−0.05	1.96093	−0.07
9/24-26	2.06801	0.04	2.05371	0.05	1.96080	0.06
Mean (x')	2.06805	--	2.05376	--	1.96086	--
Sigma x	--	0.057	--	0.071	--	0.092
Pooled sigma x = 0.075						

[1]From bench mark 3 to bench mark 20.
[2]Rejected.

APPENDIX B.—TUCSON TEST DATA

Table B-1.—Tucson, ground elevations between instrument station and bench marks

Bench mark No.	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*
Distance (m)	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*	*(m)*
0.0[1]	746.44	746.44	746.44	746.44	746.44	746.44	746.44	746.44	746.44
4.6	746.60	746.45	746.35	746.52	746.45	746.38	746.54	746.45	746.40
9.1	746.83	746.52	746.20	746.73	746.51	746.25	746.76	746.55	746.38
13.7	746.92	746.43	746.05	746.74	746.43	746.20	746.81	746.49	746.33
18.3	746.98	746.44	745.92	746.85	746.42	746.13	746.91	746.53	746.24
22.9	747.16	746.55	745.89	746.97	746.55	746.01	747.08	746.63	746.21
27.4	747.31	746.59	745.71	747.14	746.54	745.98	747.29	746.68	746.17
30.0	--	746.49	745.65	--	--	--	--	--	--
30.6	747.43	--	--	--	--	--	--	--	--
32.0	--	--	--	747.24	746.45	745.87	747.32	746.63	746.20
36.6	--	--	--	747.28	746.58	745.70	747.38	746.65	746.06
41.1	--	--	--	747.42	746.47	745.63	747.53	746.63	746.02
45.1	--	--	--	--	746.47	745.52	--	--	--
45.2	--	--	--	747.49	--	--	--	--	--
45.7	--	--	--	--	--	--	747.58	746.61	745.90
50.3	--	--	--	--	--	--	747.51	746.61	745.78
54.9	--	--	--	--	--	--	747.44	746.56	745.67
59.4	--	--	--	--	--	--	747.46	746.48	745.5[illegible]
59.7	--	--	--	--	--	--	747.46	--	--
59.9	--	--	--	--	--	--	--	--	745.52
60.4	--	--	--	--	--	--	--	746.50	--
				(Bench mark elevations)[2]					
--	747.40	746.45	745.61	747.39	746.49	745.47	747.40	746.43	745.45

[1] Distance origin is at the instrument station.
[2] Distance from the instrument station is the same as for the last elevation above.

Table B-2.—Tucson, elevation standards, 1980

Bench mark No.	*April 8 adjusted elevation (m)*	*April 9 adjusted elevation (m)*	*April 10 adjusted elevation (m)*	*April 11 adjusted elevation (m)*	*April 14 adjusted elevation (m)*	*April 15 adjusted elevation (m)*	*April 17 adjusted elevation (m)*	*April 18 adjusted elevation (m)*	*April 21 adjusted elevation (m)*	*April 23 adjusted elevation (m)*
1	747.39500	747.39498	747.39501	747.39498	747.39497	747.39501	747.39507	747.39508	747.39501	747.39493
2	746.44602	746.44597	746.44595	746.44590	746.44586	746.44589	746.44596	746.44595	746.44592	746.44594
3	745.59944	745.59940	745.59939	745.59941	745.59936	745.59944	745.59948	745.59946	745.59943	745.59945
4	747.38634	747.38637	747.38637	747.38635	747.38639	747.38642	747.38643	747.38642	747.38647	747.38644
5	746.48444	746.48444	746.48444	746.48444	746.48444	746.48444	746.48444	746.48444	746.48444	746.48444
6	745.46463	745.46461	745.46457	745.46454	.745.46461	745.46463	745.46465	745.46466	745.46458	745.46456
7	747.39439	747.39440	747.39439	747.39437	747.39447	747.39444	747.39444	747.39443	747.39453	747.39469
8	746.42697	746.42700	746.42697	746.42698	746.42706	746.42702	746.42703	746.42703	746.42708	746.42702
9	745.44042	745.44041	745.44030	745.44032	745.44046	745.44046	745.44046	745.44048	745.44047	745.44045

Table B-3.—Tucson, standard elevation differences (x), for balanced-sight distances

1980 Mo./Day	x s=30[1] 3 to 1 (m)	x'−x (mm)	x s=45 6 to 4 (m)	x'−x (mm)	x s=60 9 to 7 (m)	x'−x (mm)
4/8	1.79556	0.02	1.92171	0.09	1.95397	0.04
4/9	1.79558	0.00	1.92176	0.04	1.95399	0.02
4/10	1.79562	−0.04	1.92180	0.00	1.95409	−0.08
4/11	1.79557	0.01	1.92181	−0.01	1.95405	−0.04
4/14	1.79561	−0.03	1.92178	0.02	1.95401	0.00
4/15	1.79557	0.01	1.92179	0.01	1.95398	0.03
4/17	1.79559	−0.01	1.92178	0.02	1.95398	0.03
4/18	1.79562	−0.04	1.92176	0.04	1.95395	0.06
4/21	1.79558	0.00	1.92189	−0.09	1.95406	−0.05
4/23	1.79548	0.10	1.92188	−0.08	1.95424	−0.23[2]
Mean (x')	1.79558	--	1.92180	--	1.95401	--
Sigma x	--	0.040	--	0.054	--	0.047
Pooled Sigma x = 0.047						

[1] Sight distance.
[2] Rejected.

Table B-4.—Tucson, standard elevation differences (x), for unbalanced-sight distances

1980 Mo./day	x 3 to 4[1] (m)	x −x (mm)	x 3 to 7 (m)	x'−x (mm)	x 6 to 7 (m)	x'−x (mm)	x 2 to 5 (m)	x'−x (mm)	x 2 to 8 (m)	x'−x (mm)	x 5 to 8 (m)	x −x (mm)
4/8	1.78690	0.07	1.79495	0.06	1.92976	0.06	0.03842	0.08	−0.01905	0.13	−0.05747	0.05
4/9	1.78697	0.00	1.79500	0.01	1.92979	0.03	0.03847	0.03	−0.01897	0.05	−0.05744	0.02
4/10	1.78698	−0.01	1.79500	0.01	1.92982	0.00	0.03849	0.01	−0.01898	0.06	−0.05747	0.05
4/11	1.78694	0.03	1.79496	0.05	1.92983	−0.01	0.03854	−0.04	−0.01892	0.00	−0.05746	0.04
4/14	1.78703	−0.06	1.79511	−0.10	1.92986	−0.04	0.03858	−0.08	−0.01880	−0.12	−0.05738	−0.04
4/15	1.78698	−0.01	1.79500	0.01	1.92981	0.01	0.03855	−0.05	−0.01887	−0.05	−0.05742	0.00
4/17	1.78695	0.02	1.79496	0.05	1.92979	0.03	0.03848	0.02	−0.01893	0.01	−0.05741	−0.01
4/18	1.78696	0.01	1.79497	0.04	1.92977	0.05	0.03849	0.01	−0.01892	0.00	−0.05741	−0.01
4/21	1.78704	0.03	1.79510	−0.09	1.92995	−0.13	0.03852	−0.02	−0.01884	−0.08	−0.05736	−0.06
4/23	1.78699	−0.02	1.79524	−0.23[2]	1.93013	0.31[2]	0.03850	0.00	−0.01892	0.00	−0.05742	0.00
Mean (x')	1.78697	--	1.79501	--	1.92982	--	0.03850	--	−0.01892	--	−0.05742	--
Sigma x	--	0.041	--	0.0600	--	0.058	--	0.046	--	0.072	--	0.037
Pooled sigma x = 0.053												

[1] From bench mark 3 to bench mark 4.
[2] Rejected.

(Continued from inside front cover)

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