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COAST AND GEODETIC SURVEY C&GS 36

Geodetic and Grid Angles - State Coordinate Systems

Rockville, Md.
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FOREWORD

THE OBJECT of this publication is to provide a ready means of determining whether or not observed angles should be corrected to grid angles and a simple method for this conversion, if required. Because of certain approximations and assumptions made herein, the computer usually will not be justified in determining these corrections to an accuracy better than the nearest one-tenth second.

GENERAL STATEMENT

1. In using a state grid as the basis for the computation of horizontal control surveys, the general practice is to consider the observed angles, whether for triangulation or traverse, as grid angles. Not much harm is done in making this assumption as long as the survey lines are relatively short and the accuracy sought is not of a high order. But when the lines are, say, several miles in length, particularly in areas far removed from the axis of the projection, then consideration must be given to correcting the observed angles to grid angles if a reasonably high accuracy is to be maintained.

TRIANGULATION

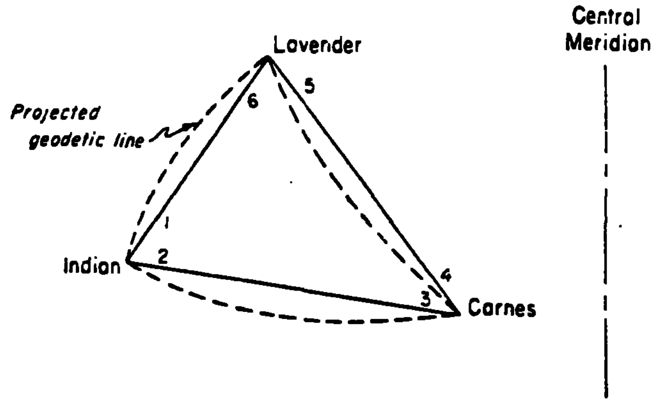
2. It might be well at the outset to define certain terms which are used herein. A geodetic line, a straight line on the earth's surface, and a line of sight are all considered synonymous for our purpose. The axis of the transverse Mercator projection is the central meridian. The axis of the Lambert projection, for our purpose, is the grid line which intersects perpendicularly the central meridian at the point where the y value is equal to y_0 for the zone. The value of y_0 is listed in the state coordinate projection tables as a constant for each zone. For brevity we shall call this the y_0 line. (Actually the axis of the Lambert projection is the parallel through y_0 at the central meridian.)

3. A geodetic line between two points generally projects on a state grid as a curve which is *always* concave toward the axis of the projection. In computing corrections for angles, it is best to compute the corrections for each direction separately and then the correction for the angle as the difference between the corrections of the two directions involved.

4. The correction we are after is the angle formed by: the straight line on the grid between the two points and the tangent at each point to the projected curve of the geodetic line between these same points. This is sometimes referred to as the t minus T or second-term correction. It can be shown from the formulas on page 13 of Special Publication No. 193 that this correction is approximately $(1/2\ \rho_0{}^2 \sin 1'')(\Delta x)(\Delta y)$, in which ρ_0 is the mean radius of curvature of the spheroid for the zone and in which Δx and Δy are next defined. In a transverse Mercator projection Δx is the x' of the center of the line in question, that is, the distance of the center of the line from the central meridian; and Δy is the y-coordinate difference over the line. In the Lambert projection Δx is the x-coordinate difference over the line and Δy is the distance of the center of the line from the y_0 line. The value of $2.36(10^{-10})$ may be used as a constant for the United States for the term, $1/2\ \rho_0{}^2 \sin 1''$. Thus subject to the above definitions, the correction for each direction is $2.36(\Delta x)(\Delta y)\times10^{-10}$.

5. The sign of the correction could be handled algebraically by defining the signs of Δx and Δy in accordance with the position of the point with relation to the axis of the projection and with the azimuth of the line. It is the author's opinion, however, that these signs are best determined with less chance of a blunder by visualizing the curved geodetic line on the grid. For example, consider that a line lies west of the central meridian on a transverse Mercator grid (fig. 1). Remembering that the geodetic line is always concave toward the axis, then the correction is positive at the south end of the line (going from the tangent to the chord) and negative at the north end. Conversely if the line lies east of the central meridian, then the correction is negative at the south end and positive at the north end. Similarly if a line lies north of the y_0 line on a Lambert grid (fig. 2), the correction is positive at the west end, and negative at the east end. The opposite, of course, is true if the line lies south of the y_0 line. If the line crosses the projection axis, then for our purpose, we shall consider it to lie on the side in which the midpoint

Georgia Coordinate System West Zone



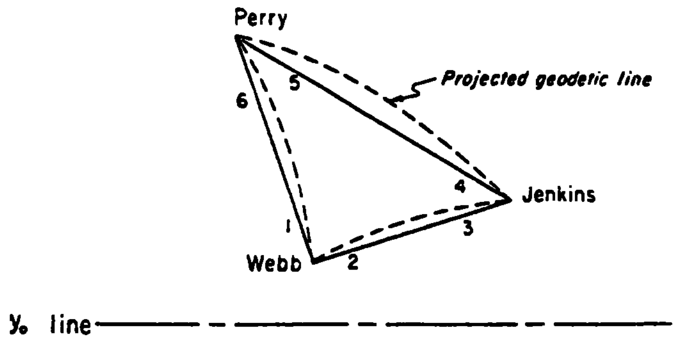
	Geodetic Angles	Corr.	Grid Angles
Lavender	56° 11' 36".21	-0".57	35".64
Carnes	48 59 21.81	+7.39	29.20
Indian	74 49 05.24	-10.06	55.18
	180 00 03.26	-3.24	00.02

All coordinates in units of 100 thousand feet.

	x	y	x
Lavender	1.61	15.74	3.39
Carnes	2.43	14.54	2.57
Indian	1.19	14.68	3.81

	Δx	Δy	Corr. = $2.36(\Delta x)(\Delta y)$	Corr. to Angles	
(1)	3.60	1.06	+9".01	-(1)+(2)	-10".06
(2)	3.19	0.14	-1.05		
(3)	3.19	0.14	+1.05	-(3)+(4)	+7.39
(4)	2.98	1.20	+8.44		
(5)	2.98	1.20	-8.44	-(5)+(6)	-0.57
(6)	3.60	1.06	-9.01		-3.24

FIGURE 1.



	Geodetic Angles	Corr.	Grid Angles
Perry	41° 35' 52″.87	-2″.99	49″.88
Jenkins	45 59 21.30	-1.23	20.07
Webb	92 24 46.37	+3.70	50.07
	180 00 00.54	-0.52	00.02

All coordinates in units of 100 thousand feet.

	x	y
y_c		5.25
Perry	18.65	8.23
Jenkins	19.25	7.88
Webb	18.80	7.76

	Δx	Δy	Corr. = 2.36(Δx)(Δy)	Corr. to Angles	
(1)	0.15	2.74	-0″.97		
(2)	0.45	2.57	+2.73	-(1)+(2)	+3″.70
(3)	0.45	2.57	-2.73		
(4)	0.60	2.80	-3.96	-(3)+(4)	-1.23
(5)	0.60	2.80	+3.96		
(6)	0.15	2.74	+0.97	-(5)+(6)	-2.99
					-0.52

FIGURE 2.

falls. Actually in this case the geodetic line has a reverse curvature, each part being concave toward the axis. Strictly speaking the rules for the signs given above would not completely apply since they would be reversed at one end of the line because of the change in curvature. But in any case for a geodetic line of ordinary length, lying so close to the projection axis, the corrections would be practically negligible.

6. The above indicates that the correction is equal at both ends of the line. This is not quite true since the formulas in Special Publication No. 193, referred to, state that the distances to the central meridian or the y_0 line should be measured from a point not at the center of the line, but one-third the way along the line from the station for which the correction is desired. For most work, however, and for the sake of simplicity, it is sufficient to consider the corrections at each end the same, leaving the refinement to those who wish to obtain the ultimate degree of precision.

7. The data required to compute these corrections involve at least preliminary coordinates of the survey stations, but these need be known only to the nearest one thousand feet for the purpose at hand. These preliminary coordinates may be determined by rough computation or scaled from a map on which the survey scheme, as well as the state grid, has been plotted. The simplest way to handle the factor 10^{-10} is to point off each x and y coordinate 5 decimal places to the left so that they are defined in units of a hundred thousand feet.

8. Figure 1 represents computation on the Georgia Coordinate System, West Zone, transverse Mercator projection and figure 2 on the Tennessee Coordinate System, Lambert projection. These examples should be self-explanatory, but a few comments might be in order. The geodetic angles are adjusted and therefore the three angles of a triangle must add up to 180° plus the spherical excess. After these angles are corrected to grid values, they should add up to 180° exactly, inasmuch as a grid triangle is a plane triangle and the triangle has been adjusted for closure. The examples show that this is so within the limits of the accuracy of the formulas and approximations employed. In both cases the grid angles for each triangle sum up to 0.02, a mere coincidence.

9. The directions are given numbers in the sketch which correspond to those in the computation of each direction. The Δx in figure 1 is the mean x' of the two ends of the line where the x' is the distance to the central meridian in units of a hundred thousand feet. Similarly in figure 2, the Δy is the difference of the mean of the y-coordinates of the two ends of the line and the y_0 of the zone. The corrections to the angles are determined as previously explained by the difference between the corrections of the two directions involved. For example, in figure 2, the correction for the angle at Webb is: minus the correction for direction (1) plus the correction for direction (2), which is +3.70. From the sketch on figure 2 it can easily be seen how the sign of the corrections can be visually determined. For example, at station Webb the correction of the direction to Perry is negative because in swinging from the tangent to the chord, we must go counterclockwise. Similar reasoning indicates that the correction of the direction from Webb to Jenkins must be plus because the swing is clockwise. If observed geodetic angles are used instead of adjusted angles, as in the examples, then the departure of the sum of these angles from 180° exactly, when corrected to the grid, will be the triangle closure. No reference need be made to the spherical excess. Moreover lengths computed through these grid triangles will be grid lengths, provided one starts with a grid length, and no application of scale factors is required.

TRAVERSE

10. The observed angles in a traverse survey are corrected in the same manner as those in triangulation; that is, by determining the corrections for each direction and taking their difference for the angle corrections. In general, the legs of a traverse survey are considerably shorter than triangulation lines and thus the angle corrections are usually much smaller. However, owing to modern electronic distance measuring equipment, this is not always so.

11. For traverse surveys comprised of short lengths, say, well under a mile, the observed angles can be used as grid angles without much harm except possibly in the more precise work. On the other hand, if the lengths of the traverse legs reach, say, 3 miles or more and the survey is a great distance from the projection axis, then consideration must be given to correcting the observed angles to grid angles. For example, assume a traverse is run in an east-west direction on the Lambert projection or a north-south direction on a transverse Mercator projection, has legs of 15,000 feet, and is 450,000 feet from the projection axis, then it can be easily shown that the correction to all the angles is 3.2 and, further, that the effect is cumulative. In order to get the true closure in azimuth on such a traverse, the corrections must either be made or approximated. If the corrections are not used and the traverse adjusted, the systematic effect would be substantially eliminated. However, if the traverse runs in different directions, such an expedient cannot be employed and, for best results, the individual corrections to the angles should be made.

12. Approximations can be safely used in traverse surveys in many cases. A constant

Δy in the Lambert projection or a Δx in the transverse Mercator projection may be applied for a sizable region and the other coordinate difference determined for each line by some rough estimate. It may be convenient to work up a brief table or a graph of corrections using as arguments the Δx and Δy from which individual corrections may be interpolated. In any case such a table or graph could be used to make a quick estimate of the order of magnitude of the corrections to arrive at a decision whether or not these need be applied. After all, in traverse work, and even in most triangulation, usually one or two significant figures will suffice and the computations can be done on a slide rule.

13. As an aid in obtaining a starting grid azimuth for a traverse survey, the following comments may be helpful. If azimuth information available consists of the geodetic azimuth of an azimuth mark, the geodetic azimuth of a distant triangulation station or an intersected point such as a water tank, or a polaris observation, any of these may be converted by an application of the $\Delta\alpha$ or θ angle, only, and the resulting azimuth may be used on the grid with no correction to the observed direction to the azimuth point. If the information is the grid azimuth of an azimuth mark, it may be used *directly* without any correction to the observed direction since the distance to the azimuth mark is usually very short. However, if the information available is the grid azimuth of a distant triangulation station or an intersected point which is derived by an inverse between the plane coordinates, then the observed direction to the azimuth point must be corrected for the so-called "second term" as explained heretofore. The observed direction to the first traverse point needs correction only if it is significant according to previous discussion.