NOAA Technical Memorandum NOS NGS 31

A MODEL OF TEMPERATURE STRATIFICATION FOR CORRECTION OF LEVELING REFRACTION

Rockville, Md.
April 1981

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Proceedings of the Second International Symposium on Problems Related to the Redefinition of North American Geodetic Networks. Sponsored by U.S. Department of Commerce; Department of Energy, Mines and Resources (Canada); and Danish Geodetic Institute; Arlington, Va., 1978, 658 pp (GPO #003-017-0426-1).

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A MODEL OF TEMPERATURE STRATIFICATION FOR CORRECTION OF LEVELING REFRACTION

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ABSTRACT. Refraction causes leveled height differences to be too small. It is usually considered to be the largest known systematic error in leveling measurements, amounting to as much as a cm/km. In 1937, long before refraction was widely accepted as a significant error source, T.J. Kukkamaki developed a correction which is proportional to the temperature difference, Δt, between heights of 0.5 and 2.5 m. Δt was to be measured, requiring extra effort and equipment. Few countries adopted the correction, but it is now realized that the correction is necessary, especially in the middle and lower latitudes.

Removing refraction bias from old leveling measurements requires a temperature stratification model. The model described here is based on historical records of solar radiation, sky cover, precipitation, and ground albedo from many locations in the conterminous United States. The average difference between the predicted Δt and observed Δt is -0.12°, +0.14°, and -0.22°C for data sets from Maryland, California, and Arizona, respectively. This modeling method can be adopted by any country with records of solar radiation. The model is an important asset to leveling computations because it provides a means of eliminating extreme refraction bias in the absolute heights and makes profiles of relative vertical movements more reliable. An economic advantage is realized by eliminating the need to observe vertical temperature profiles during most leveling surveys.

INTRODUCTION

A refraction correction for leveling was first developed by T.J. Kukkamaki in 1937 (Kukkamaki 1939). The correction for a single instrument setup is

$$R = -10^{-5} A \left(\frac{L}{50}\right)^2 \Delta h \, \Delta t.^{*} \qquad (1)$$

L is the sight length, Δh is the measured difference of elevation, and A is a function dependent on an assumed temperature function ($t=a+bz^c$). A is sometimes assumed constant, but is rigorously calculated as

$$A = \frac{5.95}{z_2^c - z_1^c} \left[\frac{1}{c+1}\left(Z_1^{c+1} - Z_2^{c+1}\right) - Z_0^c\left(Z_1 - Z_2\right)\right] \qquad (2)$$

where Z_0 is the height of the instrument, and Z_1 and Z_2 are the heights of the line of sight on the fore and back rods, respectively. Δt is a difference in air temperature between two chosen heights, usually z_1=50 cm and z_2=250 cm. Leveling refraction is proportional to the height difference observed at the instrument station and to the square of the sight length, thus accumulating most quickly on long gentle slopes that can be found almost anywhere in the United States.

Like many countries, the United States has changed its surveying specifications with time, corresponding to the early need for rapid development of the national level net, followed by a later requirement for more accuracy in regions of special interest. To obtain increased accuracy and take advantage of improvements in leveling instrumentation, the sight lengths have been gradually reduced from a maximum of 150 m to a present maximum of 50 m. When old surveys are compared to new ones over the same route, anomalous height changes between the levelings can often be attributed to differing amounts of refraction error in each survey caused by the changed sight length. Too often these anomalous height changes are interpreted as real crustal movements.

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*R is in millimeters, L in meters, Δh in half-centimeters, and Δt in degrees Celsius.

From 1878, when geodetic leveling first began in the United States until 1977, Δt was not measured and the refraction correction was not applied to leveling measurements incorporated into the national leveling network. Refraction error was minimized procedurally by balancing foresights and backsights, by not reading below the 0.5-meter point on the level rods, and by limiting the sight length to what was thought to be a reasonable maximum. The refraction correction was considered small because circuit misclosures did not seem to be influenced by it, and most documented experience with the measurement of Δt originated in England and Finland, where it seldom exceeded 1°C.

In December 1977, the author measured temperatures in California at heights of 50, 150, and 250 cm above the ground. As suspected, Δt was much larger in the United States than in Scandinavia or England. It was evident that the refraction correction could not be ignored. Therefore, a method of estimating Δt was sought to enable the recomputation of all historical leveling data with the correction applied. The inconsistencies in the leveling data caused by refraction error should be removed prior to adjustment of level networks and crustal movement investigations.

The magnitude of the vertical temperature gradient near the ground depends primarily on the intensity of solar radiation. Solar radiation at mid-latitudes is highly variable depending on season and time of day. This causes temperature gradients near the ground to fluctuate similarly. Thus the amount of refraction error in leveling surveys will generally depend on "when" the measurements were made. Rainfall, cloud cover, and ground reflectivity also have regional and temporal variations which influence vertical temperature gradients.

SOLAR RADIATION

Solar radiation has been recorded daily at U.S. weather stations for a period averaging about 30 years and is the foundation for modeling temperature variation with height. The National Weather Service (NWS) provided the author with mean daily totals of solar radiation for each month at 192 stations in the conterminous United States.

The radiation received on a horizontal surface is recorded by a pyranometer, which measures both direct and diffuse radiation. Solar radiation was observed at the 26 stations indicated by asterisks in figure 1. From these stations and from data observed from 1952 to 1975, the NWS developed basic regression equations for solar radiation versus solar zenith angle, amount of opaque cloudiness, and precipitation. These equations were then applied to the remaining stations (indicated by a dot or dot surrounded by a square in fig. 1) where long-period meteorological records were available, but solar radiation data were not. A few of the stations in figure 1, indicated by a dot surrounded by a circle, provided data to help model precipitation and temperature, but these did not contribute solar radiation information. Because the solar radiation data are well distributed, it was possible to find a representative mathematical surface to express the variability of solar radiation at locations other than where they were measured. The surface has the following form:

$$S(x,y,D) = F_1(x,y) + F_2(x,y)\cos(2\pi D/365) + F_3(x,y)\sin(2\pi D/365). \qquad (3)$$

S is the mean daily total of solar radiation, x and y are the geographic coordinates of the point, and D is the number of days since December 21. F_1, F_2, and F_3 each used 49 coefficients in a series involving all cross products of x and y up to the sixth power. The 2304 observations, mean daily totals for each month, were weighted according to the number of years used to generate the mean. A satisfactory fit was obtained. The standard deviation of a predicted daily total, having weight 30 (corresponding to 30 years), was 19 cal/cm^2. This is usually about 5 percent or less of the predicted daily total for the warmer months when leveling is performed.

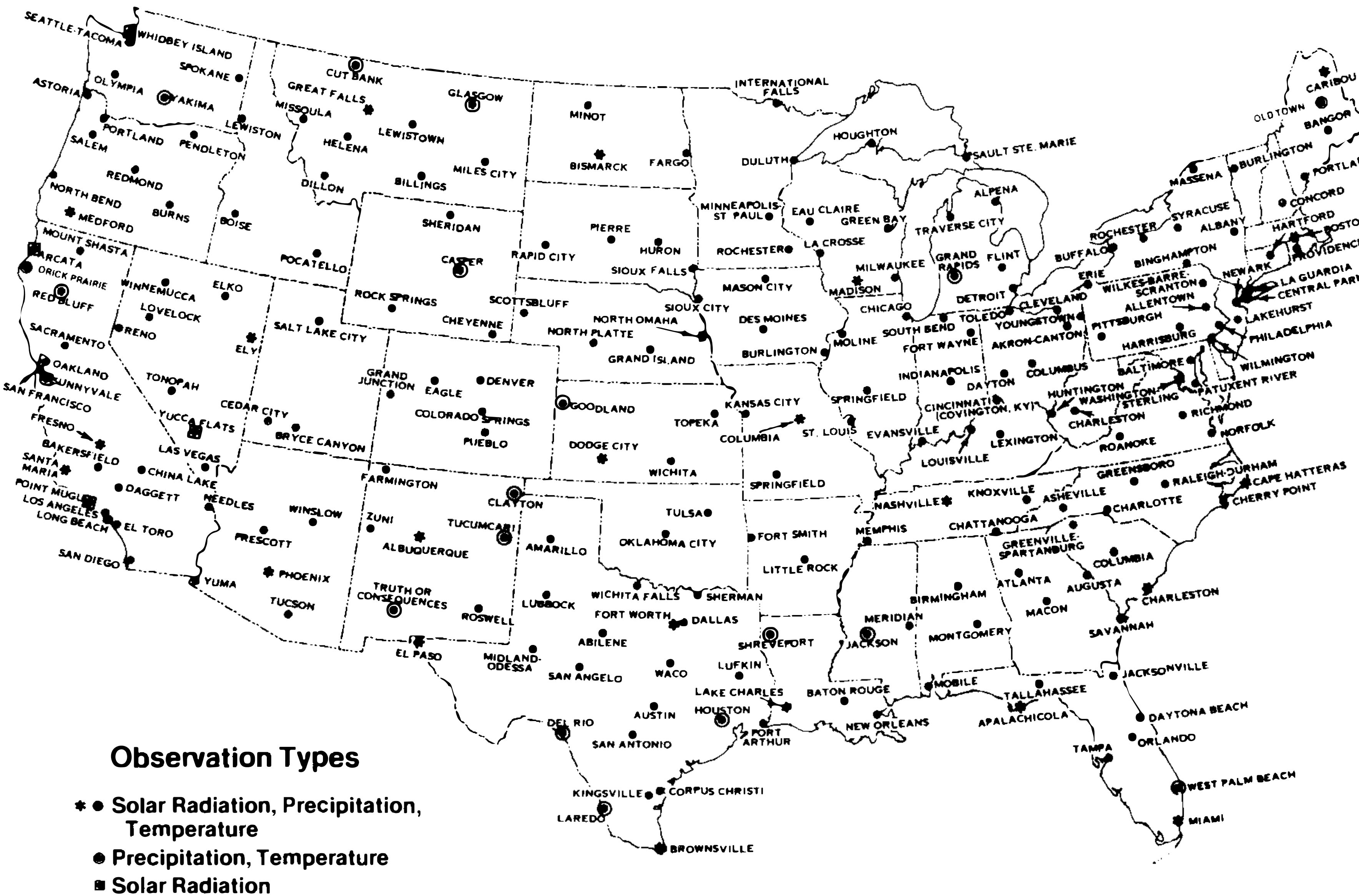

Figure 1.--Weather stations used to model solar radiation, precipitation, and temperature.

Figures 2 and 3 are contour plots of mean daily total solar radiation for the months of January and July, respectively. The July diagram vividly illustrates that solar radiation is not a simple function of latitude. Solar radiation is greater in the Western States because the sky is overcast less frequently, and the filtering effect of the atmosphere is less for the high elevations of the western mountains. The comparison of the two plots also reveals the large seasonal fluctuation of solar radiation, particularly in the higher latitudes. The diagrams are generated from the coefficients obtained by a surface fit of the type described by eq. (3).

The following notation is adopted:

S = mean daily total of solar radiation measured between sunrise and sunset, which is incident on a horizontal surface (cal cm^{-2}).

S' = instantaneous solar radiation on a horizontal surface (cal cm^{-2} min^{-1}).

S'' = instantaneous solar radiation on an inclined surface.

S_{max} = maximum value of instantaneous solar radiation, S', on a given day .

S_n = instantaneous net radiation.

The mean daily total solar radiation measured on a level surface must be converted to instantaneous solar radiation on a surface that is usually inclined.

The plotted diurnal variation of solar radiation between sunrise and sunset can be mathematically approximated by part of a sine curve, peaking at noon, and zero at approximately sunrise and sunset. (See fig. 4 and eq. 4.)

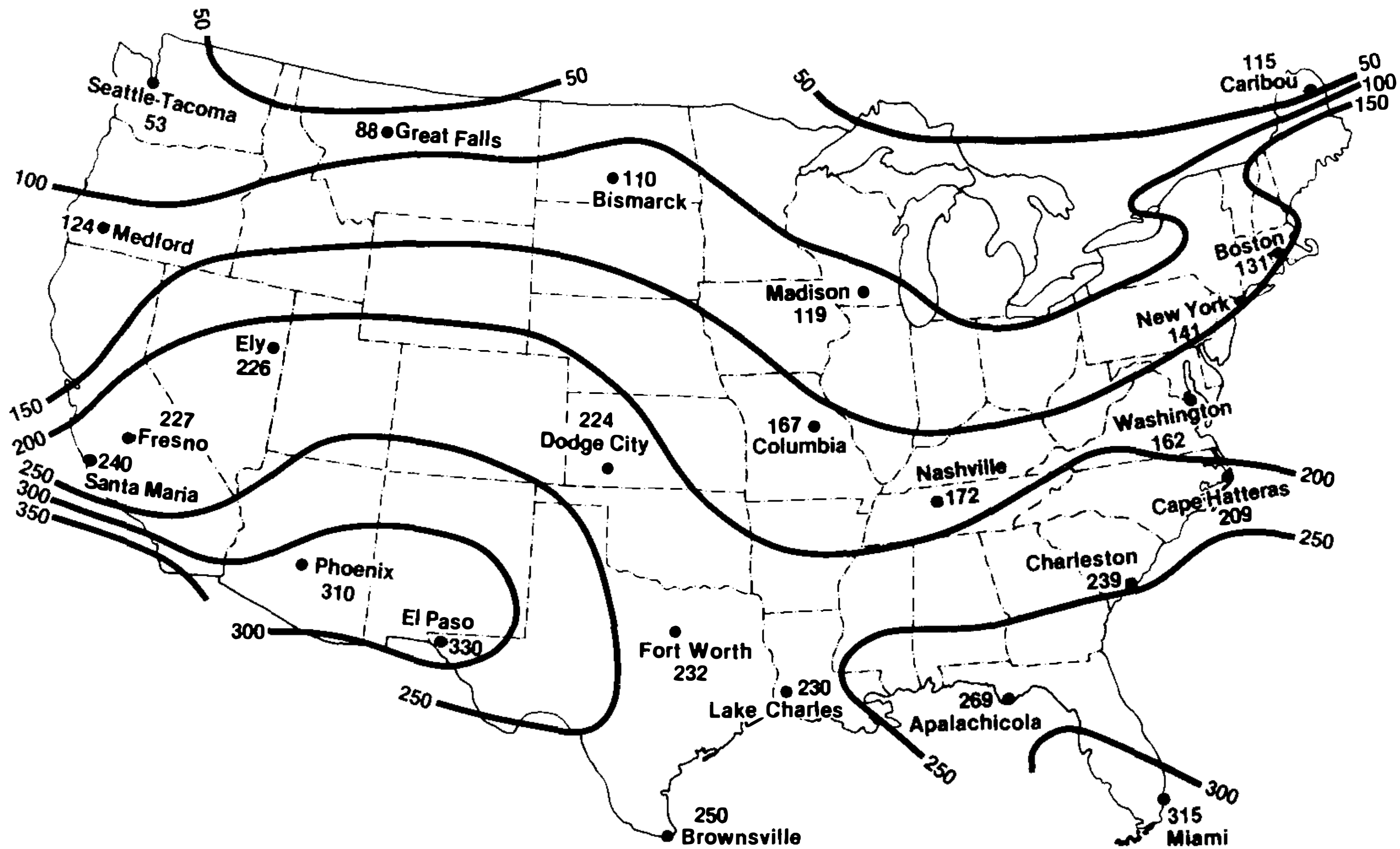

Figure 2.--Contours of mean daily total solar radiation for January. (Units = cal/cm^2)

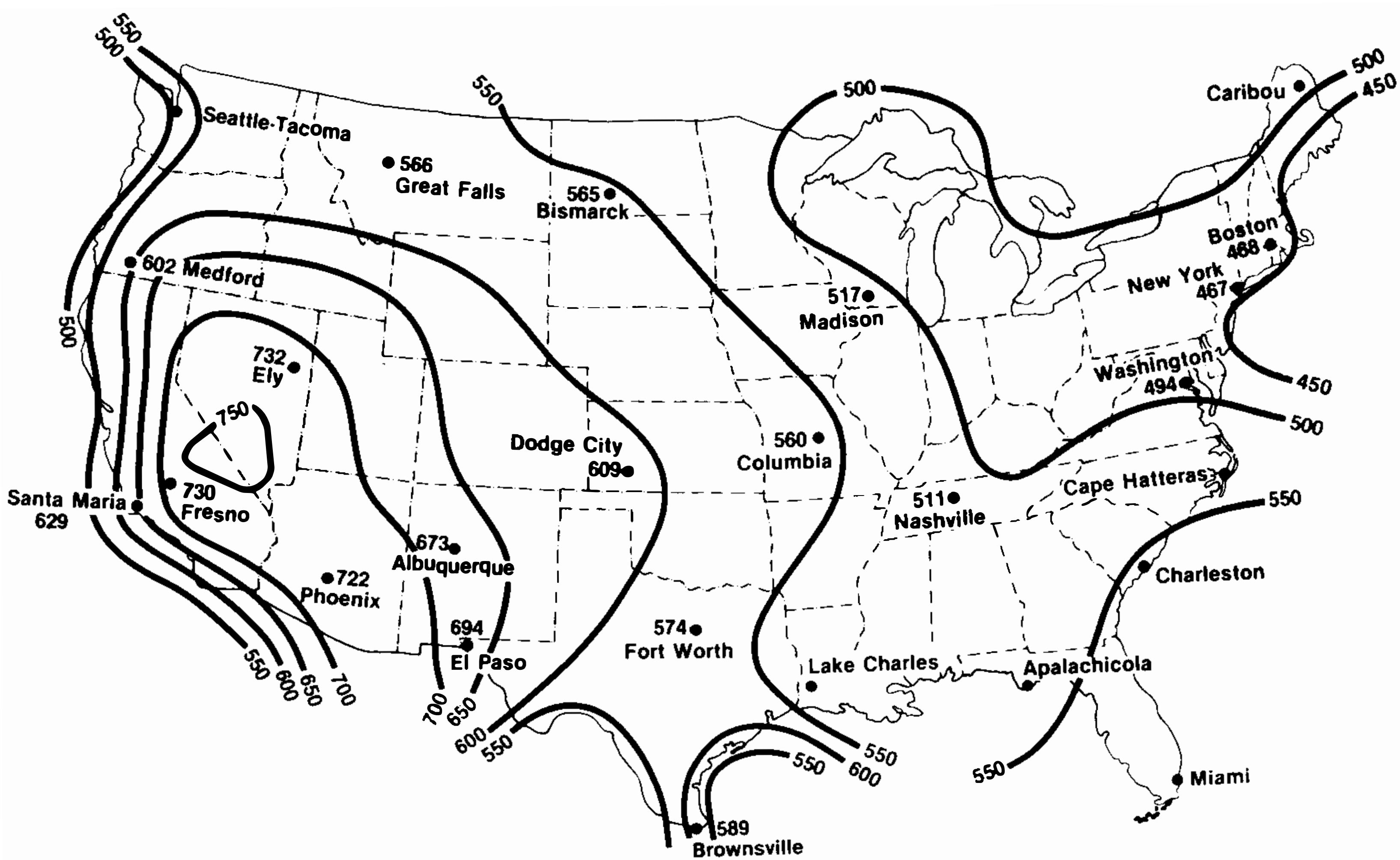

Figure 3.--Contours of mean daily total solar radiation for July. (Units = cal/cm^2)

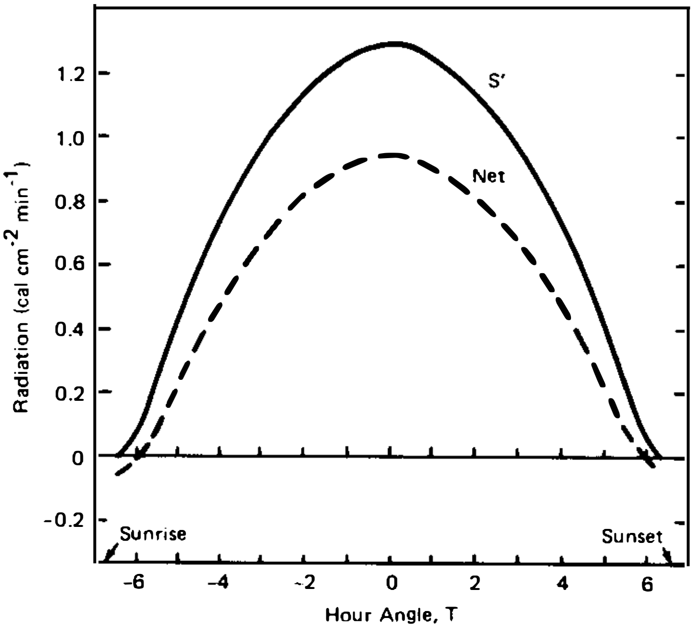

Figure 4.--Diurnal variation of solar radiation.

$$S' = \sin\left[\frac{(T + a - b)\,\pi}{2(a - b)}\right] S_{max}. \tag{4}$$

T is the hour angle of the Sun, a is half the number of hours between sunrise and sunset, and b reflects the difference in time between the onset and finish of radiation receipts with respect to sunrise and sunset. A value of 30 minutes has tentatively been assigned to b.

The value "a" can be calculated according to

$$a = \frac{1}{15}\cos^{-1}(-\tan\delta\,\tan\phi) \tag{5}$$

where δ is the declination of the Sun and ϕ is the latitude of the location where leveling is being done.

We can integrate eq. (4) to obtain the daily total of energy received, and get

$$S = \int_{b-a}^{a-b} S' \, dT = \frac{4(a-b)}{\pi} S_{max}. \tag{6}$$

Rearranging,

$$S_{max} = \frac{\pi S}{4(a-b)}. \tag{7}$$

Substituting eq. (7) into eq. (4), and putting T, a, and b in minutes gives S', the instantaneous solar radiation on a level surface, as a function of mean daily total solar radiation.

$$S' = \frac{\pi S}{4(a-b)} \sin \left[\frac{(T+a-b)\pi}{2(a-b)} \right]. \tag{8}$$

The following equation uses a variation of Lambert's Cosine Law to convert the instantaneous solar radiation, S', on a level surface to solar radiation, S", received on an inclined surface:

$$S'' = \frac{S' \sin B_1}{\sin B_0}. \tag{9}$$

B_0 is the incidence angle between the Sun's rays and a level surface, and B_1 is the incidence angle between the Sun's rays and the ground surface.

$$B_0 = 90° - \gamma \tag{10}$$

where γ is the zenith distance of the Sun, and

$$\begin{aligned} \sin B_1 &= \cos\gamma \cos\alpha + \sin\gamma \sin\alpha \cos\beta \\ &\text{if } \alpha \le 0,\ \beta = A^* - A' \\ &\text{if } \alpha > 0,\ \beta = A^* - A' + \pi \ . \end{aligned} \tag{11}$$

α is the slope of the terrain in the direction of the level line:

$$\alpha = \tan^{-1} \frac{\Delta h}{2L}. \tag{12}$$

A^* and A' are azimuths of the Sun and level line, respectively.

Net radiation S_n is the difference between total upward and downward radiation fluxes and is a measure of the energy available at the ground surface. This parameter is important because it is the fundamental quantity of energy available at the Earth's surface to drive the processes of evaporation, air, and soil heat flux as well as other smaller energy consuming processes, such as photosynthesis.

The following expression (Polavarapu 1970) is used to express net radiation as a function of instantaneous solar radiation

$$S_n = (1-r)\, S'' + L^*. \tag{13}$$

The quantity r is ground reflectivity, and L^* is the long wave radiation balance (incoming minus outgoing).

$$L^* = m\, S'' + q. \tag{14}$$

Combining eqs. (13) and (14), we get

$$S_n = (1-r-m)\, S'' + q. \tag{15}$$

The value q is a regression coefficient which, for the author's model, has been assigned a value of -0.037 cal cm^{-2} min^{-1} (Polavarapu 1970).

The reflectivity, r, varies by geological regions, season, and time of day. Reflectivity is greater for light-colored soils and increases somewhat as the Sun's elevation decreases. Kung et al. (1964) developed U.S. maps of reflectivity (albedo) for winter, summer, and transitional seasons. The author took data values from each map using a grid of 114 points covering the United

States and fitted a time-varying surface of the type described by eq. (3). The albedo values predicted by the derived surface coefficients, denoted r', are representative of midday. The following modification accounts for diurnal variation:

$$r = r' + 15\ (T/a)^2. \tag{16}$$

The long wave radiation balance, L^*, depends mainly on sky cover, C_*, the portion of the sky (usually given in tenths) that is covered by clouds. The coefficient m is estimated by

$$m = -0.5 + 0.4\ C_*. \tag{17}$$

The sky cover has been modeled by the author with data from 141 stations using monthly means from the Climatic Atlas of the United States (Environmental Data Service 1968). The form of the fitted surface is given by eq. (3). The 141 stations with sky cover data are a subset of those shown in figure 1.

VERTICAL TEMPERATURE PROFILE

Net radiation combines with heat flux into the ground, G, and evaporation flux, λE, to yield upward sensible heat flux, H:

$$H = S_n - G - \lambda E \quad \text{cal cm}^{-2}\ \text{min}^{-1}. \tag{18}$$

E is the evaporation rate and λ the latent heat of vaporization of water. Evaporation flux depends on the amount of water in the soil, and can be 20 to 100 percent of the incoming net radiation. λE is highly correlated with net radiation (Geiger 1975: 234-235, Rosenberg 1974: 197-280.) The largest normal monthly precipitation at any place in the conterminous United States is 26 cm. If we equate soil moisture with precipitation and express λE as a fraction of net radiation, (S_n), we get the empirical formula:

$$\lambda E = S_n\ (0.3 + 0.027\ \psi) \tag{19}$$

where ψ is the monthly average precipitation in centimeters. This is a simple and practical expression devised here to account for variability of λE. Precipitation has been modeled by the author using the same surface-fitting technique that was used for solar radiation. Monthly averages of precipitation from the period 1941-70 were obtained at the weather stations shown in figure 1. Figure 5 shows monthly means of precipitation for selected weather stations in the United States. It gives a clear indication that λE should generally be higher in the Eastern States.

It is important to consider the high seasonal variation of precipitation in the Western States. A profile of relative vertical movement in Oregon might give a very misleading result if evaporation flux is assumed to be invariant with season. For example, Oregon's precipitation may range from 0 to 15 cm, depending on the month the survey was taken. When the ground is moist, refraction is reduced because much of the Sun's radiation is used to evaporate water. When the Sun bakes the ground dry, more of its radiation is returned as upward sensible heat flux. Thus, on the west coast of the United States we can expect more extreme seasonal variation in refraction because in winter both the greater precipitation and lower declination of the Sun serve to minimize Δt, and the opposite conditions increase Δt in the summer.

Heat flux into the ground is estimated by using the following equation (Deacon 1969)

$$G = \sqrt{2}\,\xi\,A_0\,K_0\,\sin(\omega T + \pi/4 + J)$$

where (20)

$$\xi = \sqrt{\pi/Pd}\ .$$

P is the period of the daily surface temperature cycle (24 hrs), A_0 is its amplitude (°K), d is thermal diffusivity of the soil (25 $cm^2\ min^{-1}$), K_0 is the thermal conductivity of the soil (cal $cm^{-1}\ min^{-1}\ {}^{\circ}K^{-1}$), T is the hour angle of the Sun, $\omega = 2\pi/24$, and J is a phase lag (0.3π).

The amplitude of the daily temperature cycle depends on the amount of solar radiation and sky cover. The clear skies and high solar radiation in the Southwestern United States establish a very high amplitude:

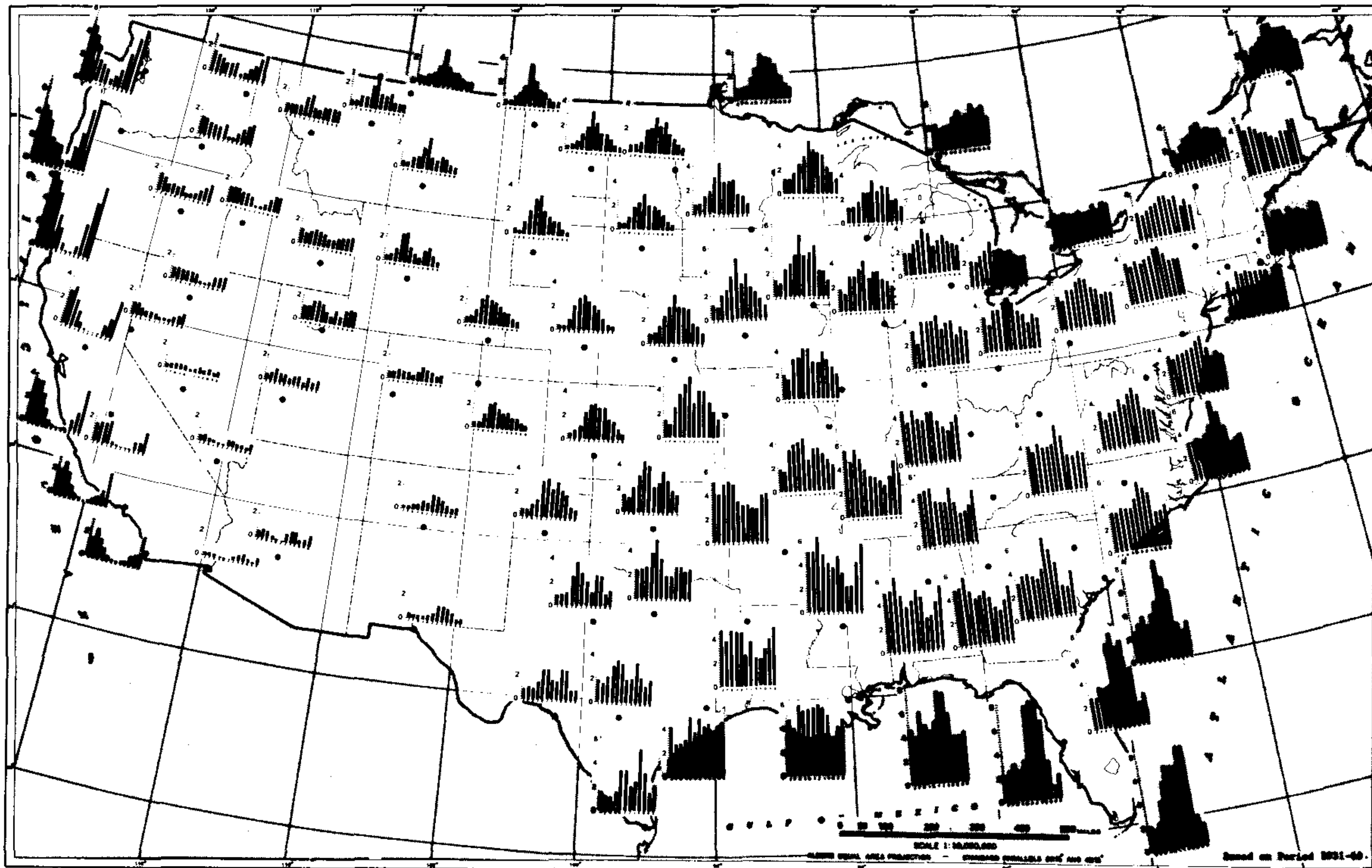

Figure 5.--Normal monthly total precipitation (inches) for selected stations in the United States.

$$A_0 = 15\ (S_{max}/1.5)(1-C_*). \tag{21}$$

Thermal conductivity of the soil depends on its moisture content, and therefore precipitation, ψ. See values given by Geiger (1975: 29) for wet and dry sand or clay.

$$K_0 = 0.033 + 0.327\ (\psi/26). \tag{22}$$

Equations (21) and (22) are empirical formulas devised here for input to eq. (20).

At this point we wish to use upward sensible heat flux, H, to calculate a vertical temperature difference, Δt. The formula to be used depends on the stability of the atmosphere. Before daylight, the air temperature is warmer than the ground, and the atmosphere is said to be "stable." After the Sun has been up long enough to make the ground hotter than the air, the atmosphere becomes "unstable." During the first hour or more of daylight, the lowest part of the atmosphere is in a "neutral" or transitional stage.

For the neutral condition, Δt is calculated using eq. (23). This formula was recommended by Brunner (1978), and its origins and details are discussed by Dyer (1974), Priestley (1959), Webb (1965), and Angus-Leppan (1980). Here H is in units of watts/m^2.

$$\Delta t = -\frac{H}{\rho\ C_p\ k\ u_*}\ \ln(z_2/z_1). \tag{23}$$

k is the von Karman constant, k=0.4; C_p is the specific heat of the air at constant pressure; ρ is the density of the air, $C_p\rho$=1200; and u_* is the friction velocity (m/s).

The friction velocity is related to wind velocity, u, measured at a height z_w above the surface, and the roughness factor z_r of the surface. z_r is approximately one-tenth of the height of the vegetation or roughness on the surface, ranging between 1 mm for sand to 10 mm over grazed grassland. Assumed model values for z_w and z_r are 2.0 m and 0.01 m, respectively.

$$u_* = \frac{k\ u}{\ell n(z_w/z_r)}. \tag{24}$$

An average wind speed of 17 km/hr is normally assumed, but this value is changed to 25 km/hr if climatic records indicate strong winds.

For the unstable atmospheric condition characteristic of midday, the formula to be used is independent of wind and mildly dependent on absolute temperature. The temperature t_1 at height z_1 is given by

$$t_1 = t_0 + 3\left[\frac{H^2\ t_0}{(C_p\rho)^2 g}\right]^{1/3}\left(z_1^{-1/3} - z_0^{-1/3}\right) - 0.0098(z_1 - z_0). \tag{25}$$

To obtain the temperature difference between two heights, eq. (25) is applied twice,

$$\Delta t = t_2 - t_1 = 3\left[\frac{H^2\ t_0}{(C_p\rho)^2 g}\right]^{1/3}\left(z_2^{-1/3} - z_1^{-1/3}\right) - 0.0098(z_2 - z_1). \tag{26}$$

In the preceding two equations t_0 is the air temperature at z_0 in °K. The value can be obtained from the old leveling records where it was needed to correct for expansion or contraction of the graduated invar strips of the level rods. The height, z_0, would ordinarily be 1.5 m. g is the acceleration of gravity (m/s^2).

Equations (18) and (25) were presented previously in more detail by Webb (1969), and later by Angus-Leppan (1970, 1971). The equations were suggested as being applicable for reducing electronic distance measurements for heights from less than a meter above the ground up to tens of meters. Consequently, they should also be suitable for estimating Δt for input to the refraction correction for leveling.

Neglecting the last term for the moment, we can rewrite eq. (25) as the difference of two temperature functions:

$$t_1 - t_0 = 3\left[\frac{H^2\ t_0}{(C_p\rho)^2 g}\right]^{1/3}\left(z_1^{-1/3} - z_0^{-1/3}\right). \tag{27}$$

Letting

$$b = 3\left[\frac{H^2\ t_0}{(C_p\rho)^2 g}\right]^{1/3},\ \text{and } c = -1/3, \tag{28}$$

then we see that the temperature functions are compatible with the form suggested by Kukkamaki:

$$t_1 = a + bz_1^c \text{ and } t_0 = a + bz_0^c. \tag{29}$$

For leveling computations the adiabatic lapse rate (0.0098 °K/m) has very little influence on eq. (25) or eq. (26) because the separation between z_0, z_1, and z_2 is never more than 2.5 m. It can be neglected because Δt will not be altered by more than 0.025°C.

Given that $c = -1/3$, and z_1, z_0, and z_2 are equal to 50, 150, and 250 cm, respectively, the value A of Kukkamaki's formula is calculated to be 80.7 and is constant. All variation of Δt with time and season must come from a change in b. This is mainly a function of upward sensible heat flux, which in turn is primarily dependent on solar radiation. In fact, solar radiation is dependent on time, place, and season. This differs somewhat from the interpretation given by Kukkamaki which calls for the exponent c, and consequently A, to change with time, date, and latitude.

It may be impossible or impractical to apply the refraction correction to Δh at every setup of the instrument. Instead, the above formulas, beginning with (3), are used to generate only one Δt value for input to Kukkamaki's formula. The correction is applied to the observed difference of elevation for the section of leveling, $\Delta h'$, which is usually the sum of the height differences from several setups. Average sight length $\bar{L}$ is substituted for the individual setup values. The total correction is then

$$R = -10^{-5} A \left(\frac{\bar{L}}{50}\right)^2 \Delta t \; \Delta h'. \tag{30}$$

For c = -1/3, and changing the units of $\Delta h'$ to meters, (30) simplifies to

$$R = C \; \bar{L}^2 \; \Delta t \; \Delta h' \tag{31}$$

where R is in millimeters, C equals -0.00006456, Δt is in degrees Celsius, and $\bar{L}$ is in meters.

INFLUENCE OF CLOUDS AND WIND

The model for predicting Δt is based on averages of solar radiation and the other observed parameters described previously. On overcast days, a strong deviation from the average can be expected, and the predicted vertical temperature difference (Δt) must be diminished.

The predicted Δt is refined by considering the following sun and wind codes which have traditionally been recorded by the National Geodetic Survey.

Wind code. A one-character numerical code is used to denote the approximate wind conditions prevailing during the survey. The three specific wind codes are as follows:

0 - Wind speed less than 10 km per hour (calm).

1 - Wind speed from 10 to 25 km per hour (moderate).

2 - Wind speed greater than 25 km per hour (strong).

Sun code. A one-character numerical code is used to denote the approximate conditions of insolation prevailing while leveling. The three specific sun codes are as follows:

0 - Less than 25 percent of setups under sunny conditions (overcast).

1 - From 25 percent to 75 percent of setups under sunny conditions (partly cloudy).

2 - More than 75 percent of setups under sunny conditions (clear).

The wind code influences Δt thru eqs. (23) and (24),

$$u_* = 0.356 \text{ if wind code is 0 or 1.}$$

$$u_* = 0.524 \text{ if wind code is 2.}$$

During the part of the day when the atmosphere is unstable, wind does not significantly alter Δt. Note that u_* does not appear in eq. (25).

Clouds can cause Δt to either decrease or increase. When clouds obscure the Sun, the average Δt must be reduced. Under total cloudiness only 35 percent of the radiation is received (Rosenberg 1974: 18). Therefore, the NGS model reduces Δt by 60 percent for a sun code of 0 (overcast). Sky cover, the general cloud condition, may have the opposite effect if the Sun can shine through. The Earth gives off long wave radiation which is lost to outer space unless reflected back to Earth by clouds. The ratio S_n/S'' may be higher for a partly cloudy day than for a clear bright day. This is accounted for in eqs. (15) and (17), and no distinction is made between sun codes 1 or 2. Sky cover also plays a minor role in eq. (21).

EVALUATION AND TESTING

The model for predicting Δt was tested by comparing observed temperature differences with the predicted values. The observed data were sufficiently numerous and were obtained under a variety of typical weather conditions, thus making the comparisons meaningful. The temperatures were measured in Gaithersburg, Md., Tucson, Ariz., Maui, Hawaii, and various sites in California.

The California temperatures were observed in December 1977, over a period of three weeks. Illumination readings were made when the temperatures were recorded, and small fans aspirated the temperature probes. The accuracy of the observed Δt (the difference between the higher and lower observed temperatures) was approximately 0.2°C. The mean difference (predicted minus observed) was +0.14°C. The rms was ±0.65°C.

The temperatures in Gaithersburg were acquired at one site during 19 days in August-September, 1979. The Tucson data were obtained during 10 days of stationary observation in April, 1980. The details of the Gaithersburg and Tucson measurements have been described by Whalen (1980). Table 1 shows the results of the comparisons with predicted values. Also included are data from Maui, which were obtained while leveling was underway. The range of Δt values during observation hours was generally between 0.0° and -4.0°C for Maui, 0.0° to -1.5°C in Maryland, 0.5° to -3.0°C in California, and 0.0° to -2.5°C in Tucson.

Table 1.--Comparison of predicted Δt minus observed Δt

Location	Md.	Calif.	Hawaii	Ariz.	Model type
*Mean difference	-0.12°C	+0.14°C	+0.22°C	-0.22°C	Solar radiation by Holdahl
rms	±0.32	±0.65	±0.90	±0.47	
Mean observed Δt	-0.56	-0.75	---	-1.03	---
No. of observations	838	714	3760	844	---

*Δt is usually negative throughout the daylight hours. A positive mean difference means the predicted Δt was generally lower in magnitude than the observed values.

During the Maui survey, temperatures were measured at five different heights. The predicted Δt values were compared to the corresponding observed vertical temperature differences between heights of 50 to 250 cm. The result was a mean difference of +0.22°C and an rms of ±0.90°C. The high rms results partly from significant moment to moment cloud variation. It is also due to very significant variations in roadside foliage, which intermittently shaded the leveling operation or altered winds. However, it is encouraging that the mean difference is low, and it should be expected that the rms would be approximately proportional to the Δt range, which is large on Maui.

Maui did not have its own weather station. Four weather stations elsewhere on the Hawaiian Islands provided solar radiation information, from which a crude prediction algorithm was constructed. The four stations were regarded as one, and only "time" was considered as an independent variable. Albedo, precipitation, and sky cover were not considered. Since Maui was not within the circuit delineated by the four weather stations, the prediction algorithm should not be considered to be as good as the mainland version.

The rms value is an indication of the average instantaneous agreement, whereas the mean difference is an indication of the tendency towards predominance of overprediction or underprediction. A zero mean difference is desired.

If it were not for the influence of the sun code, the plot of predicted Δt values for a day would be a nearly smooth curve with only a small discontinuity, where H becomes less than 69.7 w/m^2. When looking at a diurnal plot of Δt, such as figure 6, it is obvious that Δt has large fluctuations in short time intervals (3-5 minutes). Nevertheless, there is a definite pattern of the type being modeled. The rms difference between the observed and predicted Δt values shown in figure 6 is 0.53°C, which is typical of the data sets used for evaluating the model. It is clear that using the modeled Δt values will remove most of the refraction error, and that the somewhat high rms should not be discouraging. The mean difference between the two plotted

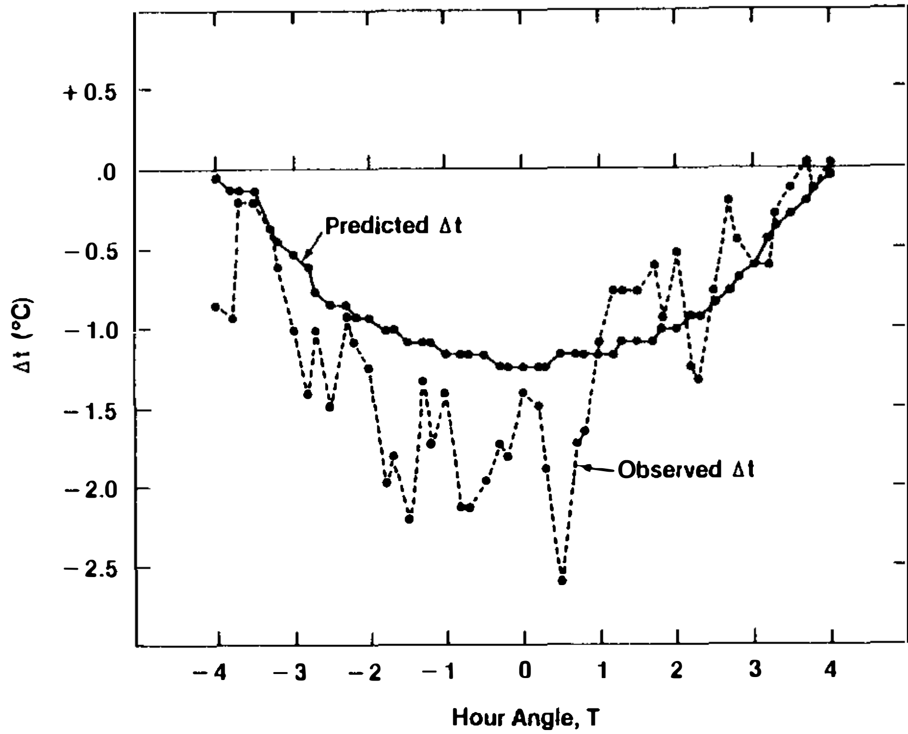

Figure 6.--Comparison of observed and predicted Δt values (z_1 = 50 cm, z_2 = 250 cm), Gorman, Calif., December 11, 1977.

lines is only 0.27°C. A shift in the line of predicted values to eliminate the 0.27°C mean difference would not alter the rms very much. For this reason predicted values are nearly as good as observed values. The primary benefit in observing Δt is to ensure against the possibility that modeled values might be badly biased for a particular region. The mean differences in table 1 are less than 0.25°C and are an indication of bias. Knowing that observed Δt values are not free of error, it would be difficult to justify the expense of measuring Δt in the hope of eliminating bias altogether.

REFRACTION TEST

Testing at the National Bureau of Standards (NBS) has verified the usefulness of the refraction correction for leveling. The equipment at the test site was arranged so that refraction error would adversely influence observed height differences. This was accomplished by placing leveling rods at specified distances from the level instruments: 30, 50, and 60 m. As shown in figure 7, a set of three rods was placed at each of these distances on sloping terrain such that the line of sight intercepted the rods at heights of approximately 0.5, 1.5, and 2.5 m. Temperatures were measured at these same heights, near the instrument and at the 60-meter distance. At each of the distances it was possible to observe a height difference of about 2 m between the high and low rods. Because several distances were used, it was possible to assess whether greater refraction would occur as sighting distance increased. The "true" or standard height differences were determined by leveling between the level rods, using very short sight lengths.

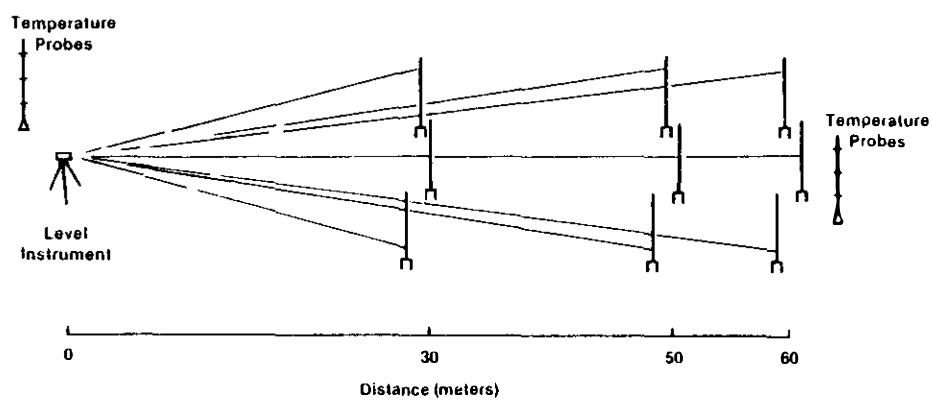

Figure 7.--Configuration of instrumentation at the refraction test site, National Bureau of Standards, August-September, 1979.

An almost identical test was performed at Tucson, Arizona in April, 1980. The testing at the National Bureau of Standards had been performed over short mown grass, but at Tucson the ground contained a mixture of light-colored gravel and sand with very little vegetation. The sighting distances were 30, 45, and 60 m.

Preliminary results indicate that the Kukkamaki refraction correction removes above 75 percent of the refraction error, and the modeled values of Δt work about as well as observed values. Preliminary results have been reported by Whalen (1980). The test data are now being refined and final results will be published in 1981.

SUMMARY

Adequate Δt values can be predicted using a model for temperature stratification near the ground. This now provides geodesists with the ability to correct leveling data so that when network adjustments are made, the resulting heights will be as free of refraction bias as possible. To ignore the correction would be to accept an error of several decimeters, of known sign, for the heights of high terrain features. Of perhaps greater importance is the need for geophysicists to use properly reduced leveling data when estimating vertical crustal motions.

The application of the correction is painless because the computer performs the computations quickly. The model is accurate enough to make the extra expense of measuring Δt in ordinary first-order surveys unnecessary. For future surveys requiring especially high accuracy and reliability, Δt should be measured in the field.

Most modern countries have recorded histories of solar radiation, rainfall, and temperature for at least several regions. If possible, the meteorological data from several countries should be combined for surface fits of solar radiation, sky cover, and precipitation. The author has also modeled monthly average temperature values in the United States, which permits prediction of temperatures not found in the old leveling records. Calculation of the average sight length, $\bar{L}$, in eq. (31) requires that the number of setups be known or estimated. An algorithm for estimating the number of setups has been devised. This algorithm is appropriate only for the United States because of its dependence on dates associated with specific changes in instrumentation and procedures, but it is not complicated.

It is important that all of the leveling data in a country be corrected for refraction, not just the new data. Refraction corrections are large, amounting to several mm/km at times; therefore, mixing corrected and uncorrected data in network adjustments or crustal movement computations is not advised.

Attempting to correct for refraction during the network adjustment by adding terms to the observation equations is also not advisable. Circuit misclosures usually contain only a small fraction of the refraction signal. The refraction error remaining in a circuit misclosure is often mixed with accumulations of random errors, other systematic errors, and contributions caused by crustal movements. Refraction errors as well as other known systematic errors are best eliminated prior to adjustment, so that rates of vertical crustal motion can be extracted from the adjustment with minimal confusion.

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