

Temporal changes to the geoid and vertical datum

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Summarize the issue

- *If* “the geoid” is to be the zero-height surface used in a future vertical datum, so that all orthometric heights refer to “the geoid” ...
- And if, “the geoid” changes...
- Then heights change too.
- Therefore, NGS must know the changes to “the geoid” to properly serve up the new vertical datum to their customers.

“The geoid”

- In quotes because:
 - It has no official IAG definition
 - Commonly used definitions cause some disagreements when considering temporal changes

The closest thing to an IAG definition



From the Report of the Ad-hoc Group on an International Height Reference System (IHRIS) (Ihde, et al, 2015):

“...the most accepted definition of the geoid is understood to be

the equipotential surface that coincides (in the sense of the least squares) with the worldwide mean ocean surface”*

* Sounds an awful lot like the NGS definition in place since 1986....

Why does this matter?

- Because:
 - Masses move
 - And thus the *shape* of every W =constant surface is changing
 - Sea Level is changing
 - And thus the particular W =constant surface which fits Sea Level changes as Sea Level itself changes.

Some assumptions

- Mass leaves the Earth very slowly
-90,000 metric tons / year (?) of stratospheric ions and free electrons, etc
- Mass joins the Earth very slowly
+ 40,000 metric tons / year (?) of “space dust”

Net change: -50,000 metric tons / year

-0.000000000000000083 % / year = negligible

- $g \sim GM/R^2 = 9.8 \text{ m/s}^2$
- If M loses 50,000 metric tons, g changes by:
-0.0000000000000000837 m/s^2 (= 0.000000084 μGal)

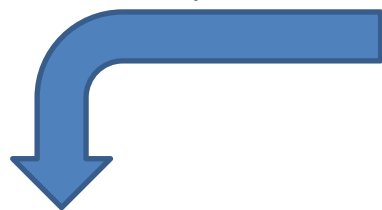
Which we will call “negligible” for this lecture

Further Assumptions

- Assume mass ***quantity*** in the Earth system is effectively constant
- Mass ***distributions*** in the Earth system are time dependent and some are large enough to be measurable
 - Secular
 - *Shape* change to every “ $W=\text{constant}$ ” surface
 - *Size* change to global mean sea level (aka “air/sea boundary”)
 - Periodic
 - Episodic

Secular: Shape vs Size

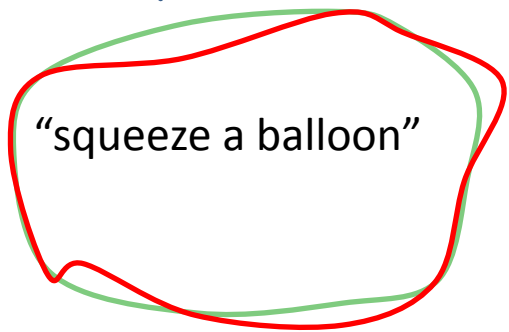
Mass moves around
(ice melts, rebound occurs)



This green surface has 2 properties at t_0 :

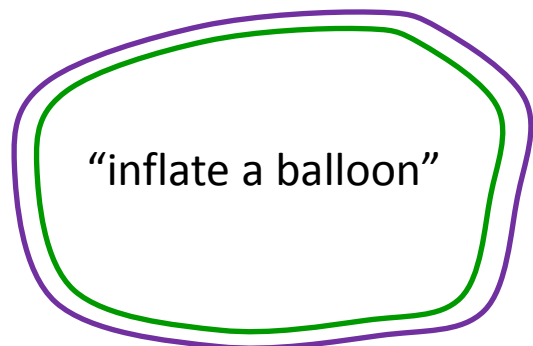
- 1) $W=W_0$
- 2) Fits mean sea level

“Sea level rise” (the average air/sea boundary swells outward from the center of the Earth)



This red surface maintains 1 property at t_1 :

- 1) $W=W_0$
- ~~2) Fits mean sea level (not guaranteed!)~~

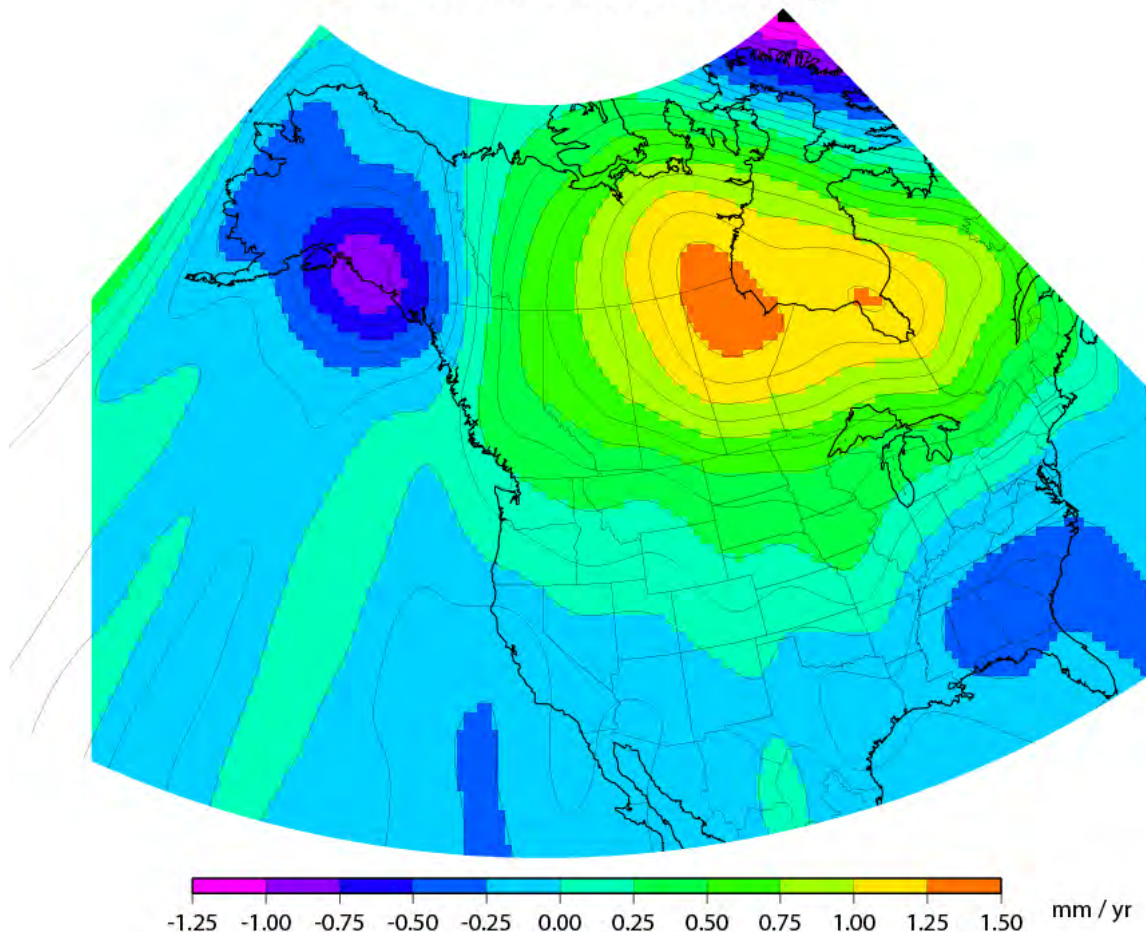


This purple surface maintains 1 property at t_1 :

- ~~1) $W=W_0$ (not guaranteed!)~~
- 2) Fits mean sea level (not guaranteed!)

Secular Change (*shape*) – Glacial

Secular Trend in Geoid. 300 km smoothing.



Uses monthly GRACE fields from the Center for Space Research at U Texas. Complete through degree and order = 60. Fit to April, 2002 – June, 2009.

Secular Change (*shape*) – Glacial



Chapter 81

Secular Geoid Rate from GRACE for Vertical Datum Modernization

W. van der Wal, E. Rangelova, M.G. Sideris, and P. Wu

Abstract GRACE-derived geoid rates are studied for North-America, where the adjustment of the Earth to ancient ice sheets causes a secular geoid increase up to 1.3 mm/year. These significant geoid changes are of particular interest for establishing a new geoid-based vertical datum in Canada and other high accuracy applications. To quantify the uncertainty of the derived rate of change of the geoid, several methods for GRACE error approximation are studied using: (i) calibrated standard deviations, (ii) a full covariance matrix, and (iii) residuals of least-squares fit of a trend and periodic variations to the time series of spectral coefficients. It is found that the residuals give the largest error estimates, probably because correlated errors are captured better. Furthermore, through maximizing the signal-to-noise ratio, it is found that the Swenson and Wahr (2006) filter of correlated GRACE errors should be applied to coefficients above degree 22 and order 4. Measurement errors are largely longitude independent, with magnitude around 0.06 mm/year. The largest geoid rate uncertainty is estimated in the area of present-day ice melt in Alaska and south of the Great Lakes and south-west of Hudson Bay (over 0.3 mm/year) due to uncertainty in continental water storage. For the creation of a geoid rate model based on GRACE data it is important that efforts are focused on reducing uncertainty in these areas, rather than improving post-processing.

81.1 Introduction

The static geoid has reached an accuracy level where time-dependent effects on the geoid become significant. For applications where high geoid accuracy is needed, such as precise georeferencing, oceanography, hazard assessment and monitoring, a static geoid model that is provided by the national survey agency can be accompanied by a model of the secular changes of the geoid (or “dynamical vertical datum”). The GRACE satellite mission provides monthly gravity field solutions from which a secular geoid rate can be estimated. A dynamical vertical datum can be constructed by combining terrestrial and satellite data (Rangelova, 2007). However, in this paper we investigate measurement and systematic errors in the geoid rate from GRACE data alone. It should be mentioned that recently the Swedish national survey agency has incorporated a hybrid of terrestrial data and geophysical model for Glacial Isostatic Adjustment (GIA) to homogenize leveling observations (Ågren and Svensson, 2007) for readjustment of the leveling network.

The study area is North America, where the dominant source of long-term geoid change is GIA. This process is an ongoing response to melting of ice sheets that covered a large part of North America roughly 20,000 years ago. The rebound of the crust is accompanied by mass inflow in the Earth’s mantle, which causes an increase in gravity and a geoid rise at a fixed location. Present-day glacier melting in Alaska and Greenland contributes significantly to the obtained

Secular Change (*shape*) – Glacial

616

W. van der Wal et al.

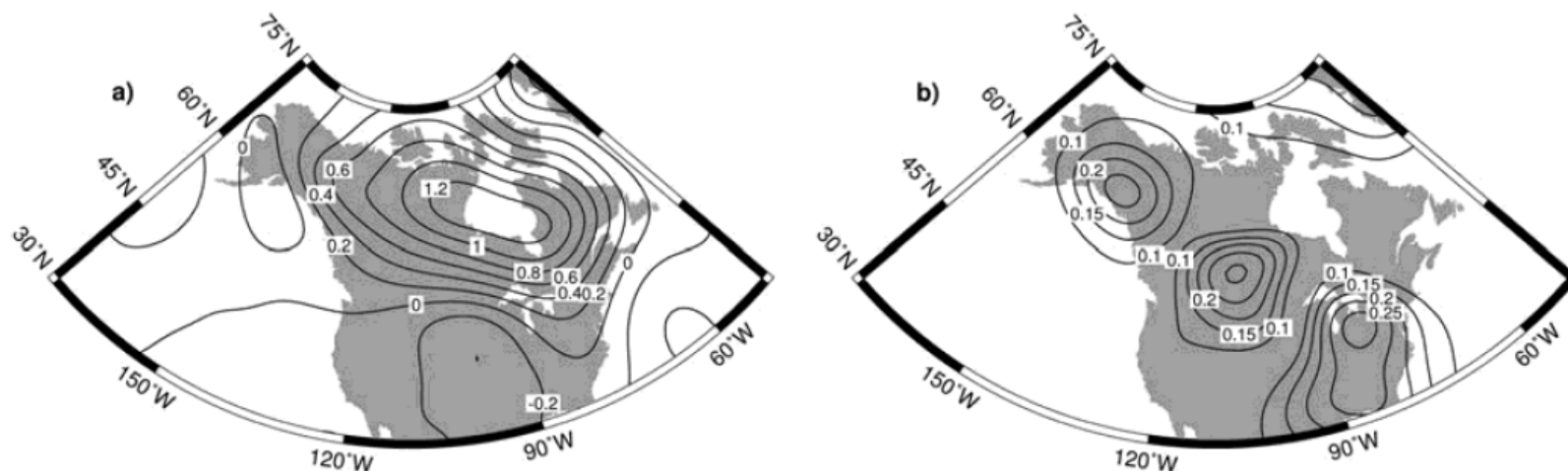


Fig. 81.4 (a): Geoid rate computed from GRACE with WGHM and Alaska and Greenland glaciers subtracted, after the destriping filter (applied to coefficients with degree greater than 22 and order greater than 4) and Gaussian smoothing with a 400 km

halfwidth. The maximum is 1.33 mm/year. (b): uncertainty of the geoid rate computed by Eq. 1 with random errors computed with method 3. The maximum is 0.33 mm/year

Episodic Geoid Change (*shape*)

Geophysical Journal International

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Surface potential and gravity changes due to internal dislocations in a spherical earth—II. Application to a finite fault

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+ Author Affiliations

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Summary

We present a numerical formulation for computing elastic deformations caused by a dislocation on a finite plane in a spherically symmetric earth. It is based on our previous work (Sun and Okubo, 1993). The formulation and gravity changes due to distributed dislocations. In this theory, we make a case study of the 1964 Alaska earthquake. The computed near field gravity changes observed by modern gravimeters. In the far field they are still significantly large. $|\delta g| > 10 \mu\text{gal}$ within the epicentral distance $\theta < 6^\circ$; $|\delta g| > 1 \mu\text{gal}$ within $\theta < 16^\circ$; $|\delta g| > 0.1 \mu\text{gal}$ within $\theta < 40^\circ$; and $|\delta g| > 0.01 \mu\text{gal}$ globally. We also calculate the geoid height changes caused by the 1964 Alaska earthquake and by the same earthquake with revised parameters and an assumed barrier. We find that the earthquake should have caused geoid height changes as large as 1.5 cm.

« Previous | Next Article »
Table of Contents

This Article
Geophys. J. Int. (1998) 132 (1): 79-88.
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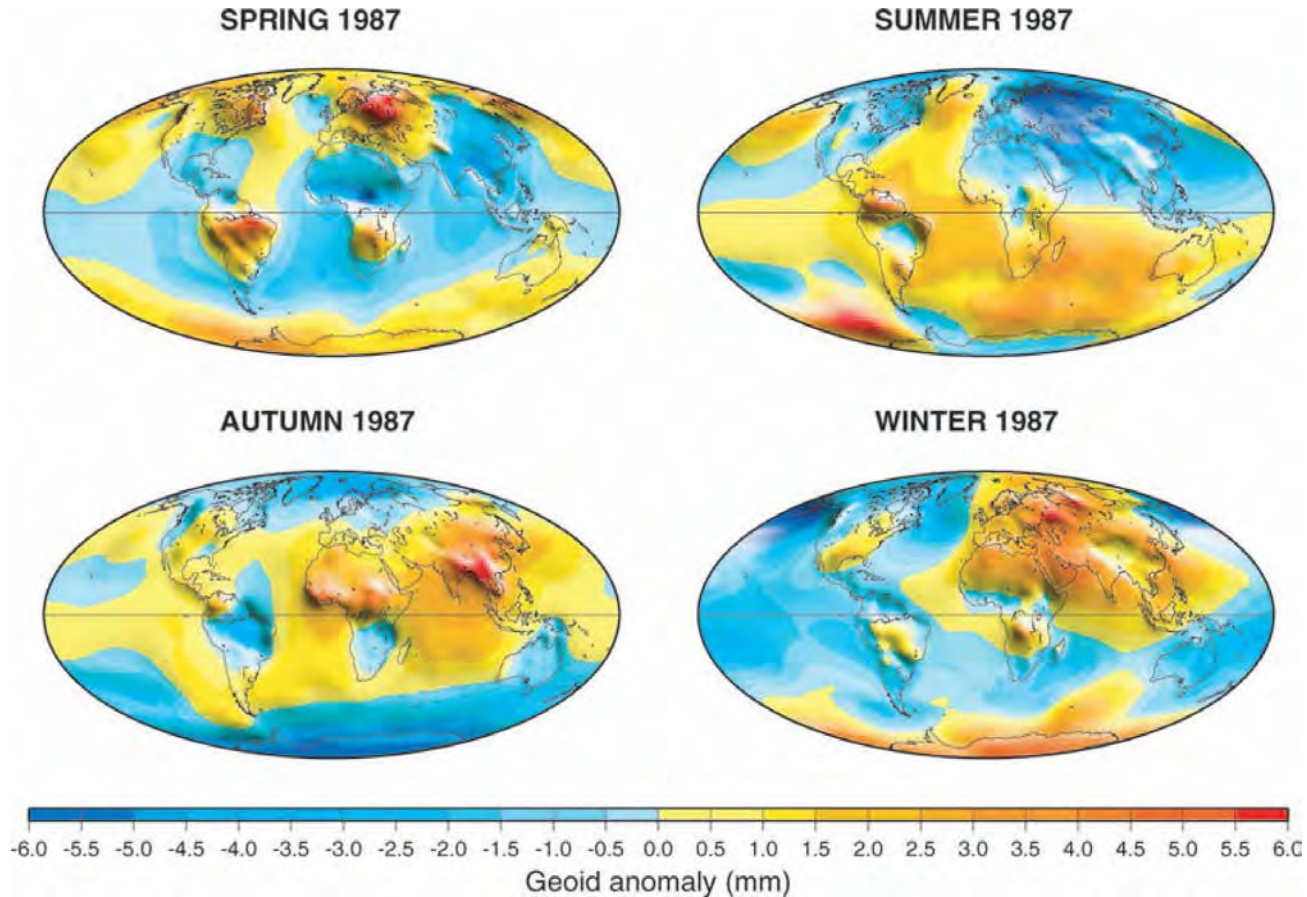
The Royal Astronomical Society

e.g. "earthquakes"

Surface potential and gravity changes due to internal dislocations in a spherical earth—II. Application to a finite fault

$\theta < 16^\circ$; $|\delta g| > 0.1 \mu\text{gal}$ within $\theta < 40^\circ$; and $|\delta g| > 0.01 \mu\text{gal}$ globally. We also calculate the geoid height changes caused by the 1964 Alaska earthquake and by the same earthquake with revised parameters and an assumed barrier. We find that the earthquake should have caused geoid height changes as large as 1.5 cm.

Seasonal/Periodic Change (*shape*)

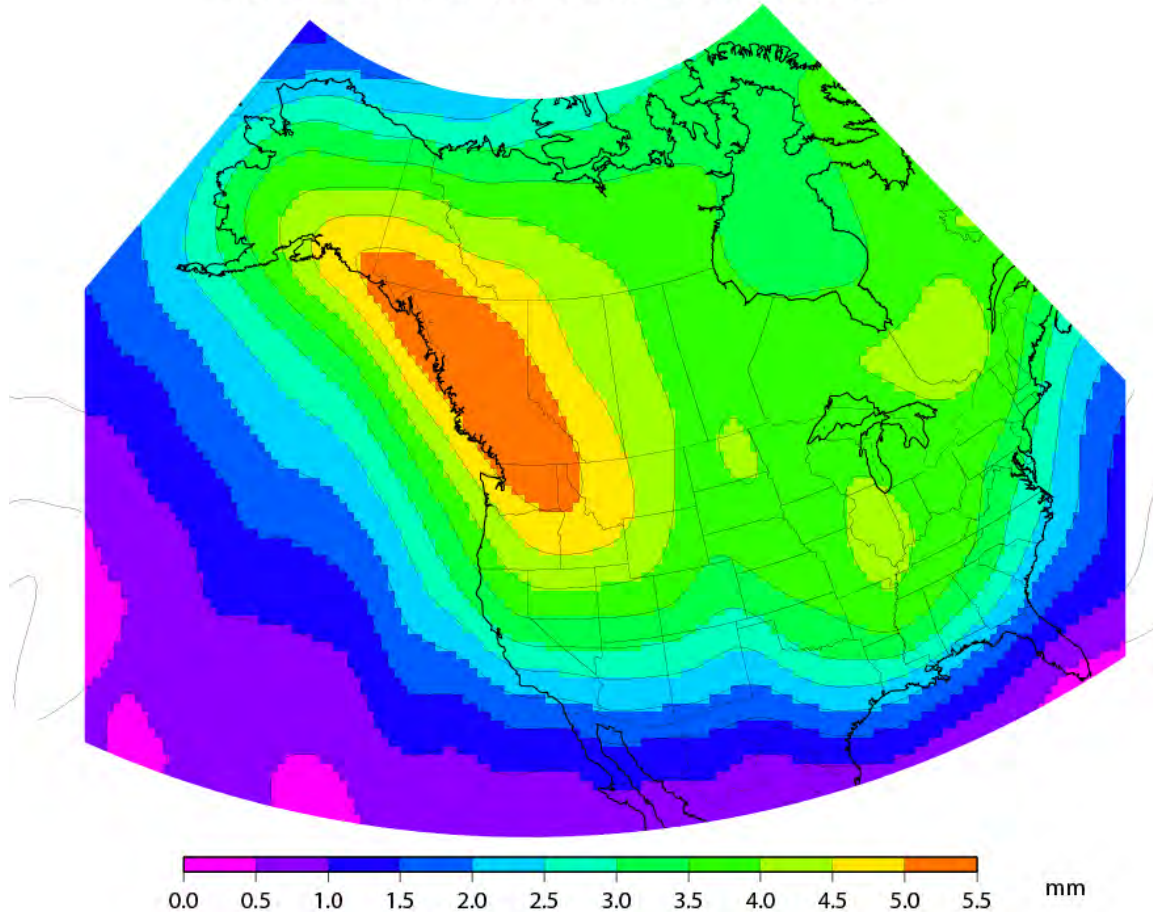


Ramillien G et al. Geophys. J. Int. 2004;158:813-826

Geophysical Journal International

Seasonal/Periodic Change (*shape*)

Amplitude of Annual Cycle in Geoid. 300 km smoothing.



Uses monthly GRACE fields from the Center for Space Research at U Texas. Complete through degree and order = 60. Fit to April, 2002 – June, 2009.

Time for a thought experiment....

Let's introduce some rocky planet...

And now let's fill in its ocean basins with water...

And put some icecaps on the land...

Ignoring all other masses in the universe, and for now assuming this rock isn't spinning, this mix of rock, water and ice generates a three-dimensional field of gravitational potential. Such a field consists of an infinite number of non-intersecting surfaces, one inside another, where each surface is defined as the locus of points where gravitational potential is some constant value. Each such surface is called an "equipotential surface".

Let's show a handful of these surfaces...

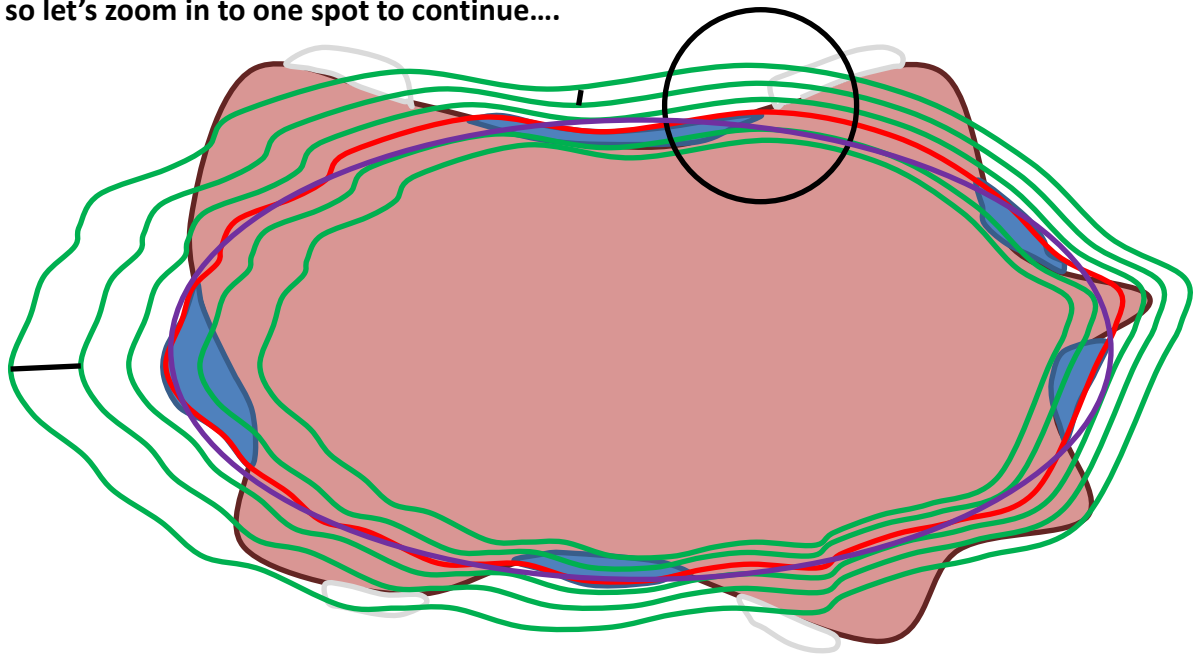
Note that the separation between these surfaces is not fixed, but instead depends on where you are...

And because it is special, let's show that one equipotential surface which best fits to all of the ocean surfaces...

We'll call that red surface "the geoid"

And for the sake of completeness, let's introduce the ellipsoid which best fits the geoid...

This diagram is too complicated for our purposes, so let's zoom in to one spot to continue....



Let's re-introduce this section of our rocky, watery, icy planet...

And note that the geoid does not actually *coincide* with mean sea level, but fits it best only *globally*...

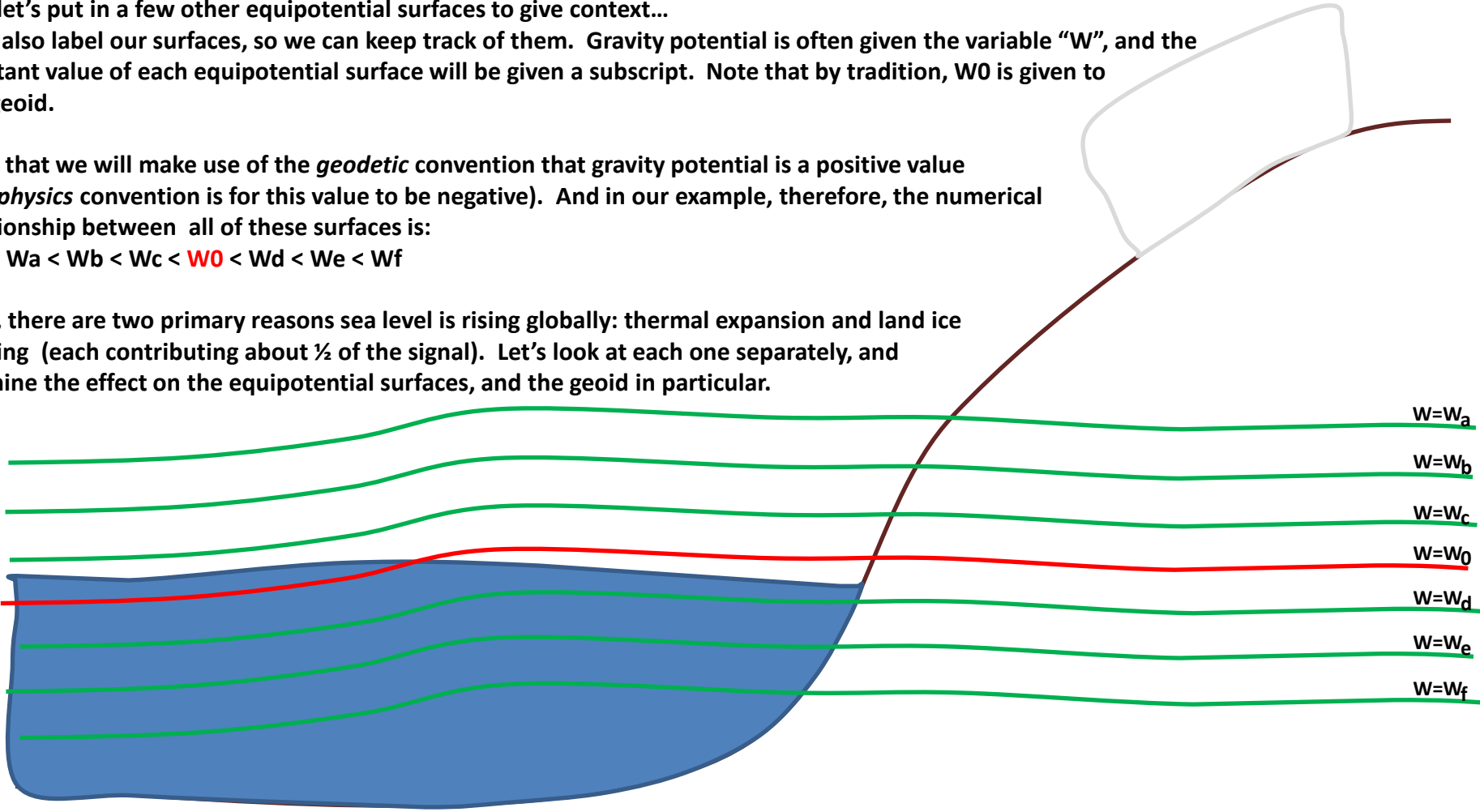
And let's put in a few other equipotential surfaces to give context...

Let's also label our surfaces, so we can keep track of them. Gravity potential is often given the variable "W", and the constant value of each equipotential surface will be given a subscript. Note that by tradition, W0 is given to the geoid.

Note that we will make use of the *geodetic* convention that gravity potential is a positive value (the *physics* convention is for this value to be negative). And in our example, therefore, the numerical relationship between all of these surfaces is:

$$W_a < W_b < W_c < W_0 < W_d < W_e < W_f$$

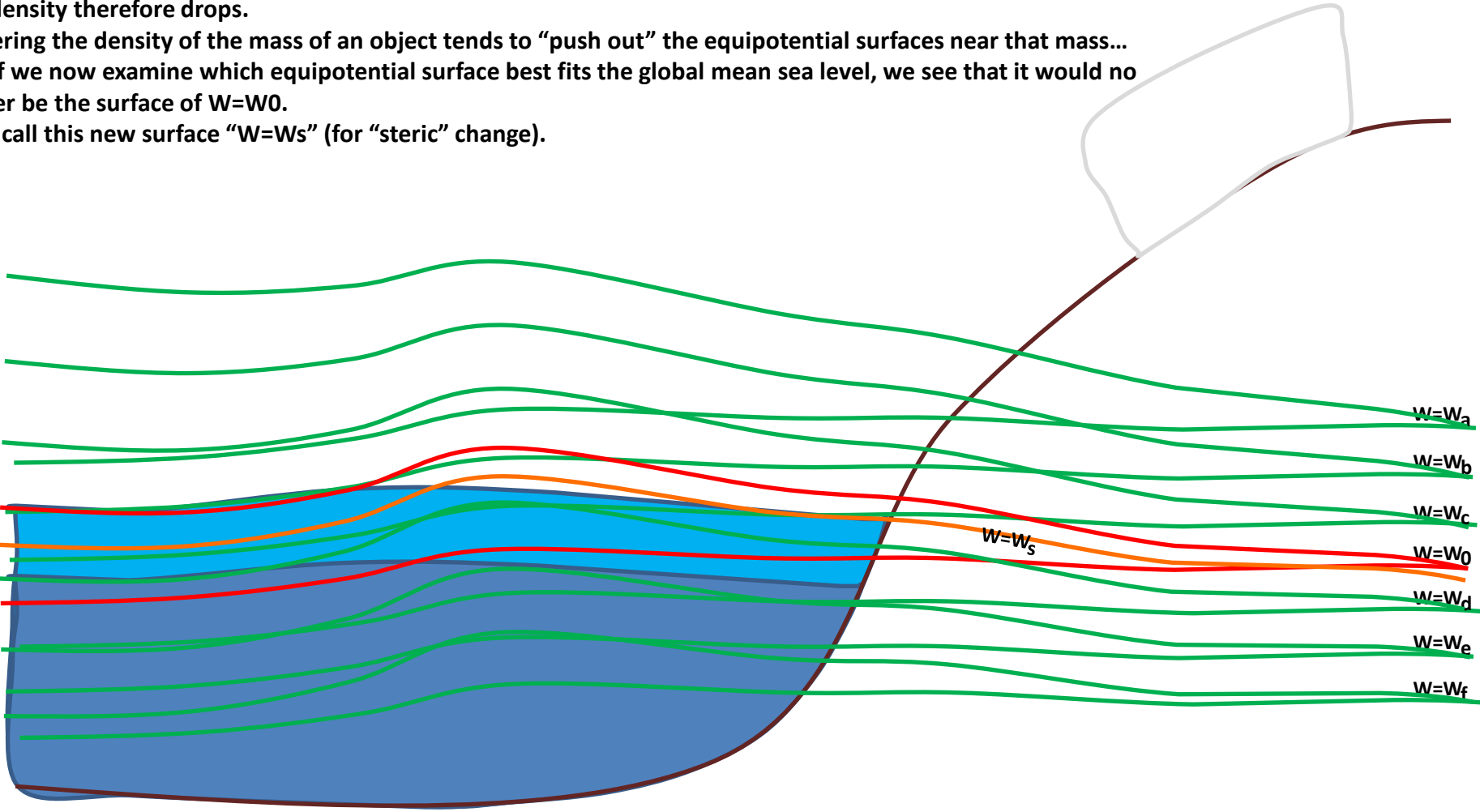
Now, there are two primary reasons sea level is rising globally: thermal expansion and land ice melting (each contributing about 1/2 of the signal). Let's look at each one separately, and examine the effect on the equipotential surfaces, and the geoid in particular.



In thermal expansion, the amount of mass in the oceans does not increase. Rather, the ocean swells with absorbed heat, changing its volume. With increased volume, but no increase in mass, the density therefore drops.

Lowering the density of the mass of an object tends to “push out” the equipotential surfaces near that mass... But if we now examine which equipotential surface best fits the global mean sea level, we see that it would no longer be the surface of $W=W_0$.

Let’s call this new surface “ $W=W_s$ ” (for “steric” change).



Let's examine what happens when ice melts...

Ice (density 0.93) melts...

becoming water (density 1.00)...

and is lost to the sea (density 1.03),

raising sea level...

The density differences are mostly a red herring with respect to the gravitational potential.

* Ocean Density decreased by 0.00000052%

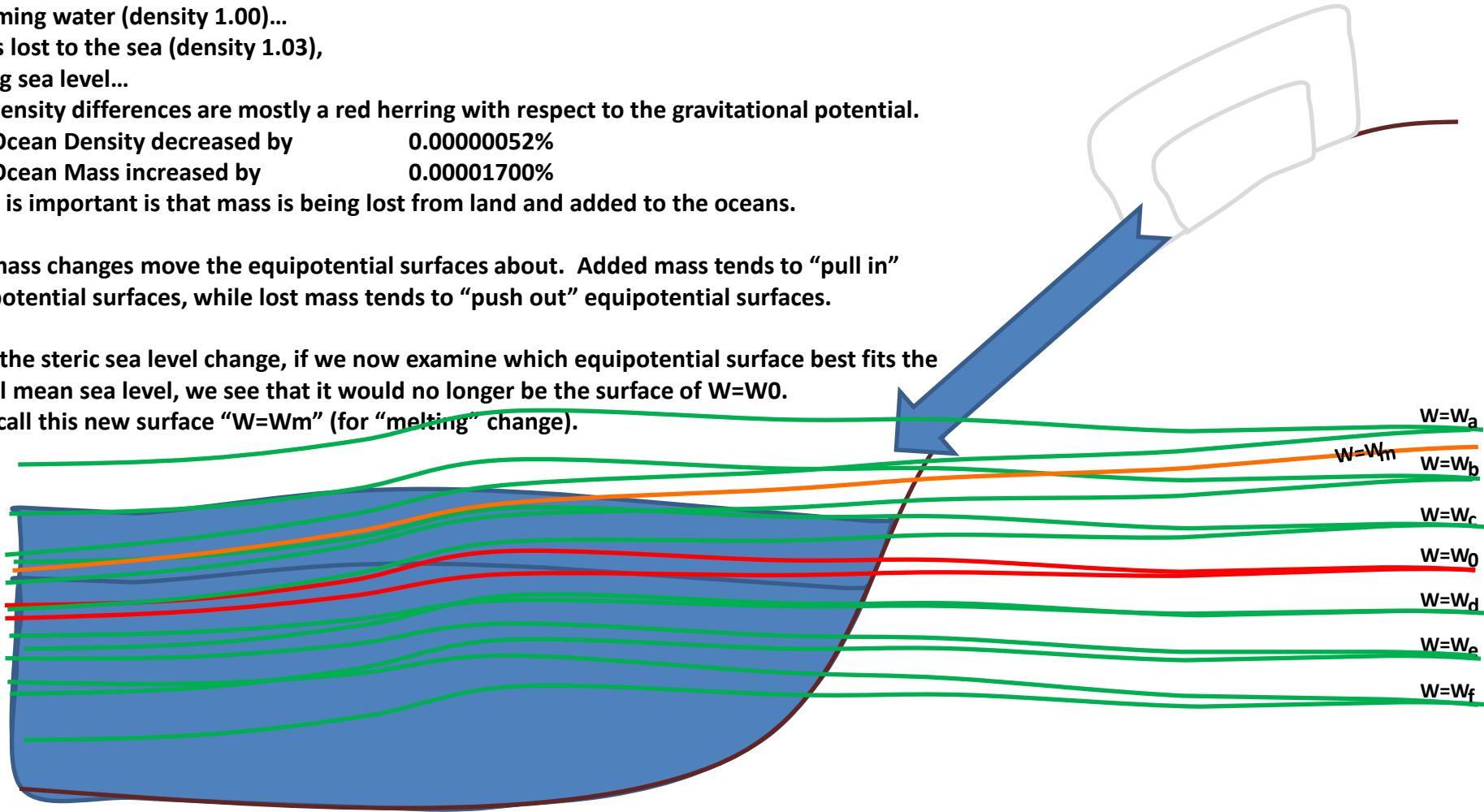
* Ocean Mass increased by 0.00001700%

What is important is that mass is being lost from land and added to the oceans.

The mass changes move the equipotential surfaces about. Added mass tends to "pull in" equipotential surfaces, while lost mass tends to "push out" equipotential surfaces.

As in the steric sea level change, if we now examine which equipotential surface best fits the global mean sea level, we see that it would no longer be the surface of $W=W_0$.

Let's call this new surface " $W=W_m$ " (for "melting" change).



Let' sum up what we've seen so far...

- 1) The **physical location** of global mean sea level, relative to some unchanging datum like an ECEF ellipsoid **is rising** a few mm / year
- 2) Half of the cause of that surface change is from steric (thermal expansion) effects
- 3) The other half is through the addition of new mass from melting land ice
- 4) However, while both effects have the **same sign** regarding **location of sea level**, they have an **opposite sign** regarding **location of equipotential surfaces** near the ocean's surface!

Q: Which effect dominates?

A: It's irrelevant, unless it can be proven that these two effects are 100% in balance with respect to the gravitational potential.

It is therefore enough to state the following:

As Sea Level changes from a combination of steric and land ice melting effects, the **numerical value** of gravitational potential on that one equipotential surface which best fits to global mean sea level **changes**.

A logical question must therefore be asked...

If the geoid is that unique equipotential surface which best fits global mean sea level...

And if the equipotential value on the geoid is W_0 at time "t0"...

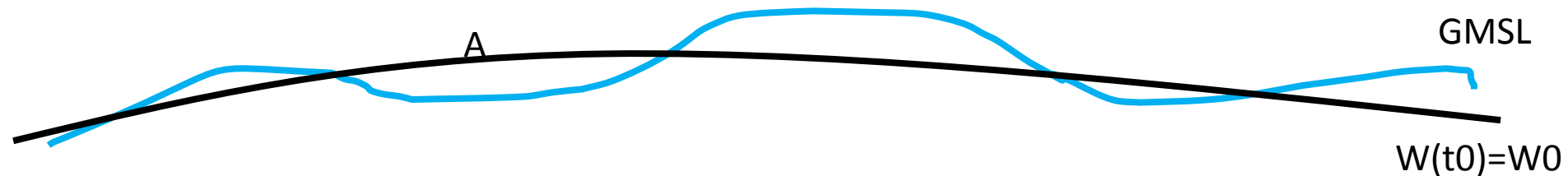
And global mean sea level changes from time "t0" to "t1" due to steric and melting issues....

And a new (not W_0) equipotential surface "best fits" global mean sea level at time "t1" ...

Then doesn't the geoid change as sea level changes?

Let's look at the very simplest cartoon example to understand why this is relevant to the use of "the geoid" as a zero height surface for our new vertical datum...

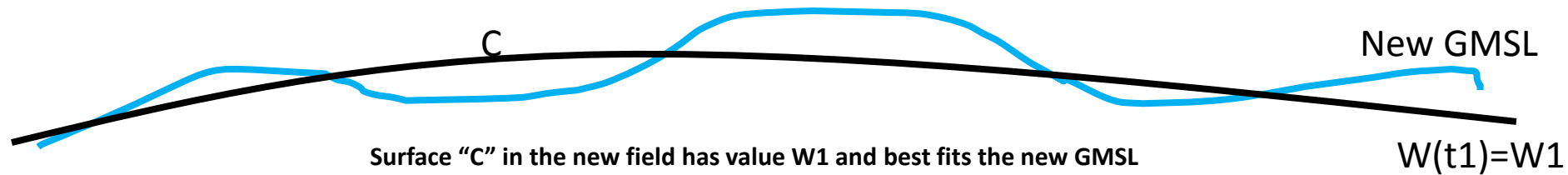
$$T = t_0$$



Because masses will change over time, the potential field “W” must have a time tag “(t₀)”. Every time epoch will have its own potential field....

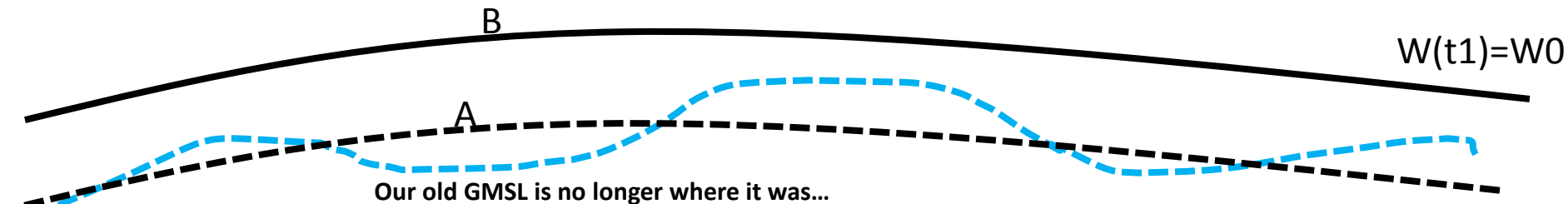
$$T = t_1 = t_0 + \Delta t$$

Masses have changed. The entire gravitational field is different now!



Surface "C" in the new field has value W_1 and best fits the new GMSL

Surface "B" in the new field has value W_0 , but is not in the location where the W_0 surface was in the old field!



Our old GMSL is no longer where it was...

One can remember where surface "A" was, but now that there is a new gravitational field, there is no guarantee that this old surface fulfills $W(t_1)=\text{constant}!!!!$

Questions: What will be the $H=0$ surface at t_1 ?
Will it be A ($W=\text{non-constant}$), B ($W=W_0$) or C ($W=W_1$)?
Which surface is "the geoid"?

This answer must be defensible within the context of answering the question:
"what is the definition of "the geoid?"

Conclusions

- Any meaningful discussion about “temporal changes to the geoid” must begin with *defining* “the geoid”
- Two aspects must be considered:
 - Shape of the geoid (“squeeze a balloon”)
 - $W=W_0$; W_0 remains the same through time, just that $W=W_0$ changes shape
 - Size of the geoid (“inflate a balloon”), in 2 possible, but mutually exclusive aspects
 - W_0 remains the same numerically, but $W=W_0$ enlarges over time, without connection to GMSL
 - Total mass of the Earth system changes over time
 - W_0 changes numerically so that $W=W_0$ fits GMSL at some specified epoch
 - The “ $W=\text{constant}$ ” surface fitting GMSL at one epoch is different than the “ $W=\text{constant}$ ” surface fitting GMSL at a later epoch
- And consider what is trackable and worth tracking
 - Secular
 - Periodic
 - Episodic

NGS's approach



- Continue to use our geoid definition from the 1990's
- Monitor the following:
 - **Secular shape** changes (via GRACE and terrestrial observations)
 - **Permanent episodic shape** changes (using GRACE and/or new airborne surveys), such as earthquakes
 - **Secular size** changes of GMSL (via Altimetry and terrestrial observations)
 - Provide all three of these to the user community independently
- Ignore the following:
 - Periodic changes
 - No “summer geoid” versus “winter geoid”
 - Temporary episodic shape changes
 - No special geoid just for the “7 year Southeast CONUS drought”

Questions

Secular Change

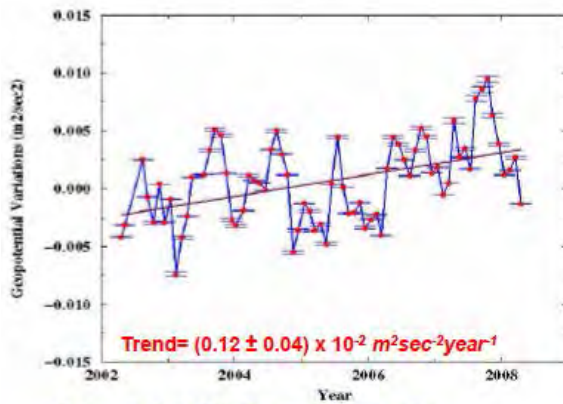


Figure 8. ΔW_0 from ΔC_{20} only

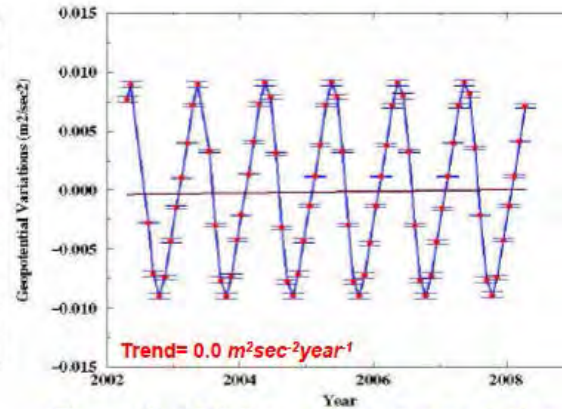


Figure 9. ΔW_0 from geocentre variations.

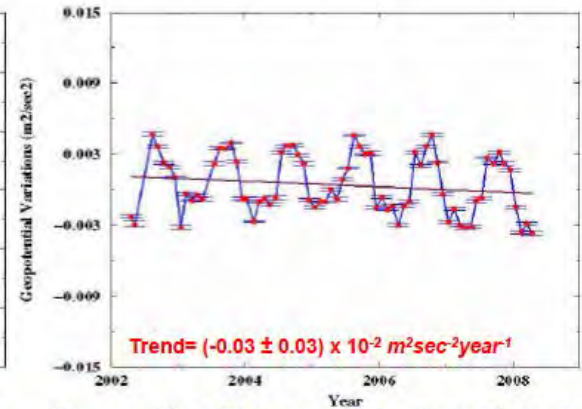


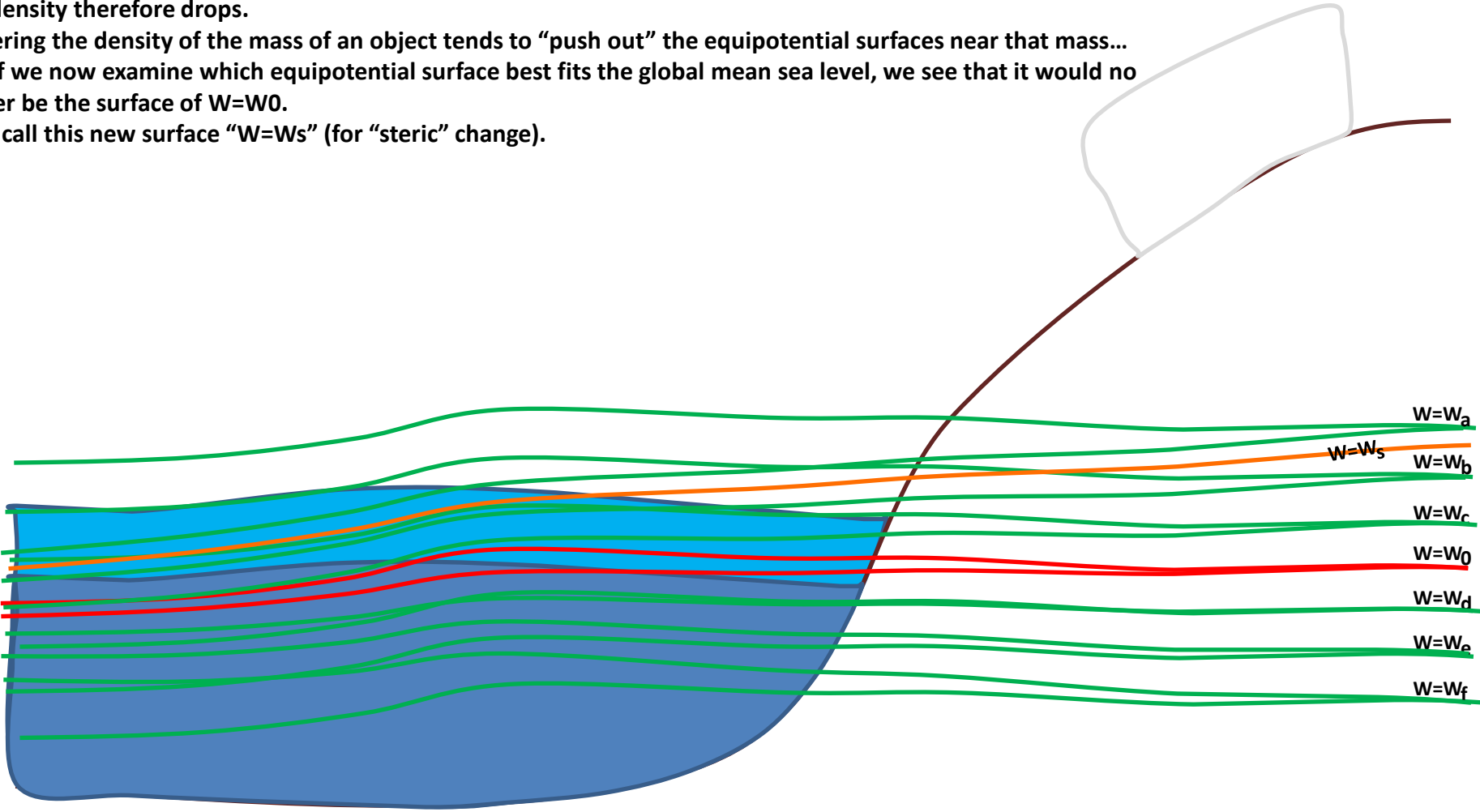
Figure 10. ΔW_0 from Stokes' coefficients variations except C_{20} and Geocentre.

- Original (incorrect) slides, follow

In thermal expansion, the amount of mass in the oceans does not increase. Rather, the ocean swells with absorbed heat, changing its volume. With increased volume, but no increase in mass, the density therefore drops.

Lowering the density of the mass of an object tends to “push out” the equipotential surfaces near that mass... But if we now examine which equipotential surface best fits the global mean sea level, we see that it would no longer be the surface of $W=W_0$.

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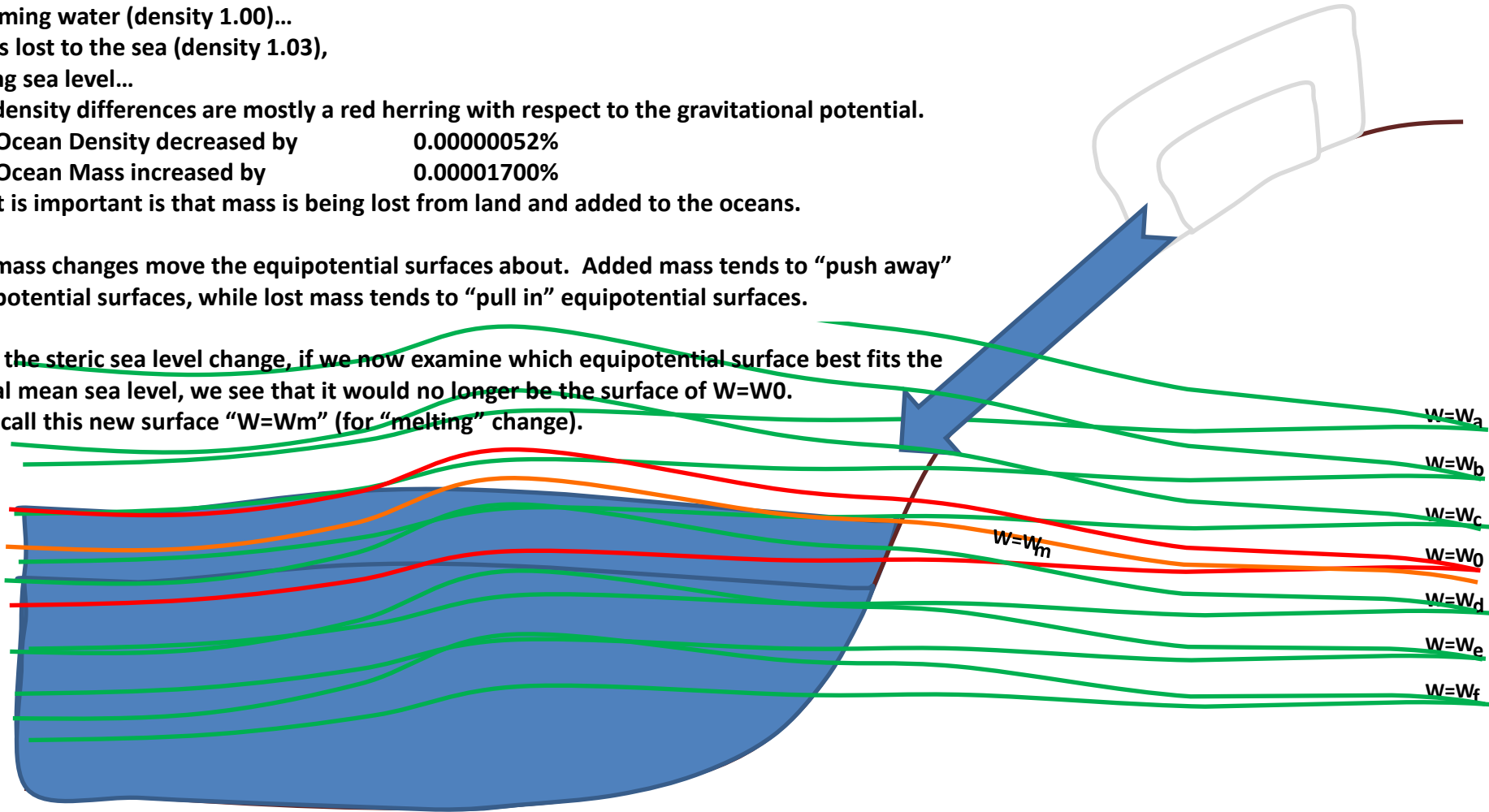
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"The geoid": History

- **1828:** C.F. Gauss first describes the "*mathematical figure of the Earth*"
 - (Heiskanen and Moritz, 1967, p. 49 ; Torge, 1991, p. 2 ; Gauss, 1828)
- **1849:** G.G. Stokes derives the formula for computing the "*surface of the Earth's original fluidity*" from surface gravity measurements. This later became immortalized as "Stokes's integral"
 - (Heiskanen and Moritz, 1967, p. 94; Stokes, 1849)
- **1873:** J.F. Listing coins the term "*geoid*" to describe this mathematical surface
 - (Torge, 1991, p. 2 ; Listing, 1873)
- **1880:** F.R. Helmert presents the first full treatise on "Physical geodesy", including the problem of computing the shape of the geoid.

“The geoid” definitions

- **“The surface of the oceans...after some slight idealization”**
 - *Physical Geodesy* (Heiskanen and Moritz, 1967)
 - Repeated, verbatim, in *Physical Geodesy* (Hofman-Wellenhof and Moritz, 2005)
 - The “slight idealization” is never actually defined
- **“A particular [equipotential surface]...which approximately forms an average surface of the oceans”**
 - *Advanced Physical Geodesy* (Moritz, 1989)
 - The “approximately” is never actually quantified
- **“...an equipotential surface of the Earth’s gravity field chosen to approximate the mean ocean surface...”**
 - *Global Models for the 1cm Geoid* (Rapp, 1997)
 - “chosen” is a dangerous word, letting us approximate the ocean any way we like

“The geoid” definitions

- “...equipotential surface of Earth’s gravity field with value $W_0=62636856.0 \pm 0.5 \text{ [m}^2/\text{s}^2\text{]}^{\prime\prime}$
 - *IERS Conventions* (IERS, 2010)
 - Unfortunately, this translates into +/- 5 cm of geoid location uncertainty, which can easily hide a systematic sea level rise of + 0.17 cm / year for decades.

“The geoid” definitions

- “...coincides with that surface to which the oceans would conform over the entire earth if free to adjust to the combined effect of the earth’s mass attraction and the centrifugal force of the earth’s rotation”
 - *Geodesy for the Layman* (DMA, 1983)
 - Unfortunately, if centrifugal force exists, then the Earth must be rotating .
 - Such a rotation will create **Western Boundary Currents** (such as the Gulf Stream), whose very nature causes a geometric change in the shape of the ocean surface, and thus keep the ocean surface from actually forming an equipotential surface, making this definition deficient.
- “The equipotential surface of the Earth's gravity field which best fits, in the least-squares sense, mean sea level. ”
 - *Geodetic Glossary* (NGS, 1986)
 - Add in the word “global” before “mean sea level” and this has been the official definition in use at NGS since the mid 1990s