# A. Theoretical Fundamentals of Airborne Gravimetry, Parts I and II (Monday, 23 May 2016)

## I. Introduction - Airborne Gravity Data Acquisition

## II. Elemental Review of Physical Geodesy

## III. Basic Theory of Moving-Base Scalar Gravimetry

## IV. Overview of Airborne Gravimetry Systems

# B. Theoretical Fundamentals of Gravity Gradiometry and Inertial Gravimetry (Thursday, 26 May 2016)

## V. Theoretical Fundamentals of Inertial Gravimetry

## VI. Theoretical Fundamentals of Airborne Gradiometry

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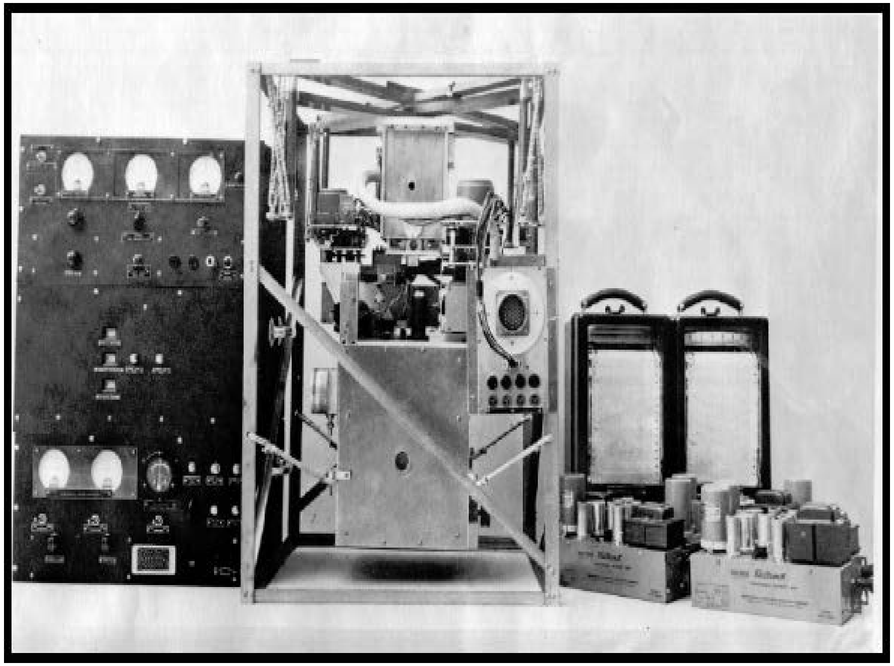
# I. Introduction - Airborne Gravity Data Acquisition

- A very brief history of airborne gravimetry
- Why airborne gravimetry?

# A Brief History of Airborne Gravimetry

- Natural evolution of successes in 1st half of 20th century with ocean-bottom, submarine, and shipboard gravimeters operating in dynamic environments
  - airborne systems promised rapid, if not highly accurate, regional gravity maps for exploration reconnaissance and military geodetic applications
- Special challenges
  - critical errors are functions of speed and speed-squared
  - difficulty in accurate altitude & vertical acceleration determination
  - trade accuracy for acquisition speed
- 1958: First fixed-wing airborne gravimetry test (Thompson and LaCoste 1960)
  - 5-10 minute average, 10 mgal accuracy
  - high altitude, 6-9 km
- Further tests by exploration concerns
  - LaCoste & Romberg, Austin TX
  - Gravity Meter Exploration Co., Houston, TX
  - 10 mGal accuracy, 3 minute averages (Nettleton et al. 1960)

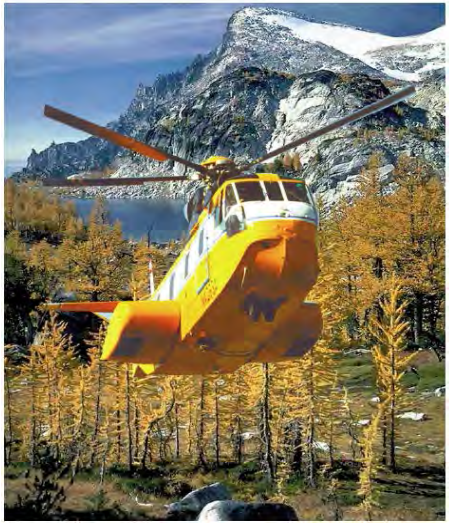
# First Airborne Gravity Test – Air Force Geophysics Lab 1958




Instruction Manual LaCoste Romberg Model "S" Air-Sea Dynamic Gravity Meter, 2002; with permission

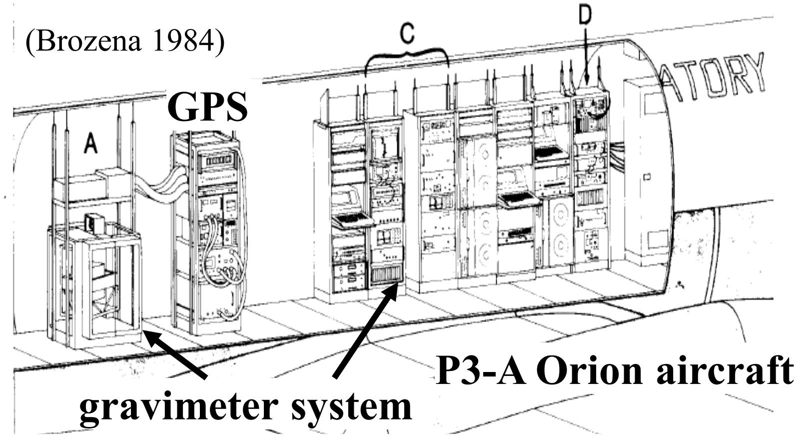
- **The first LaCoste-Romberg Model "S" Air-Sea Gravimeter**
- **KC-135 jet tanker**
  - **Doppler navigation system – elevation above mean sea level determined from the tracking range data**
  - **flights over an Askania camera tracking range at Edwards Air Force Base**

# First Successful Helicopter Airborne Gravimetry Test 1965



- **Carson Services, Inc. (Carson Helicopter)**
  - gimbal-suspended LaCoste and Romberg Sea gravimeter
  - 5 mGal accuracy, hovering at 15 m altitude (Gumert 1998)
  - Navy sponsored
- **Further tests and development by exploration companies**
  - principally, Carson Services throughout the 1960s and 1970s

# Rapid Development with Advent of GPS (1980s and 1990s)




- **Naval Research Laboratory – John Brozena**
- **National Survey and Cadastre of Denmark (DKM) – Rene Forsberg**
- **Academia (in collaboration with industry and government)**
  - University of Calgary (K.P. Schwarz)
  - University FAF Munich (G. Hein)
  - Swiss Federal Institute of Technology (E.E. Klingele)
  - Lamont-Doherty Earth (Geological) Observatory (R. Bell)
  
  …
- **Industry …**

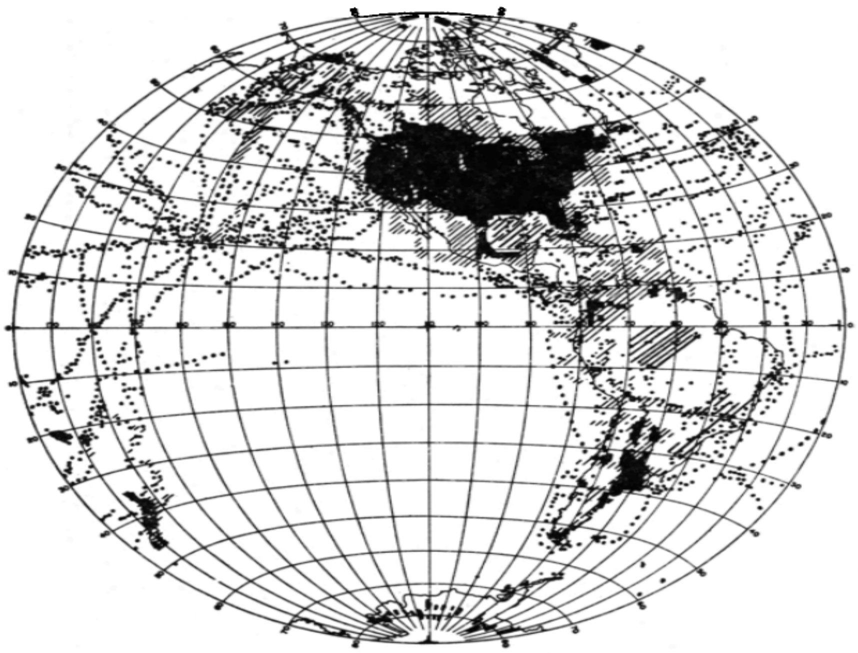
Olesen (2003)

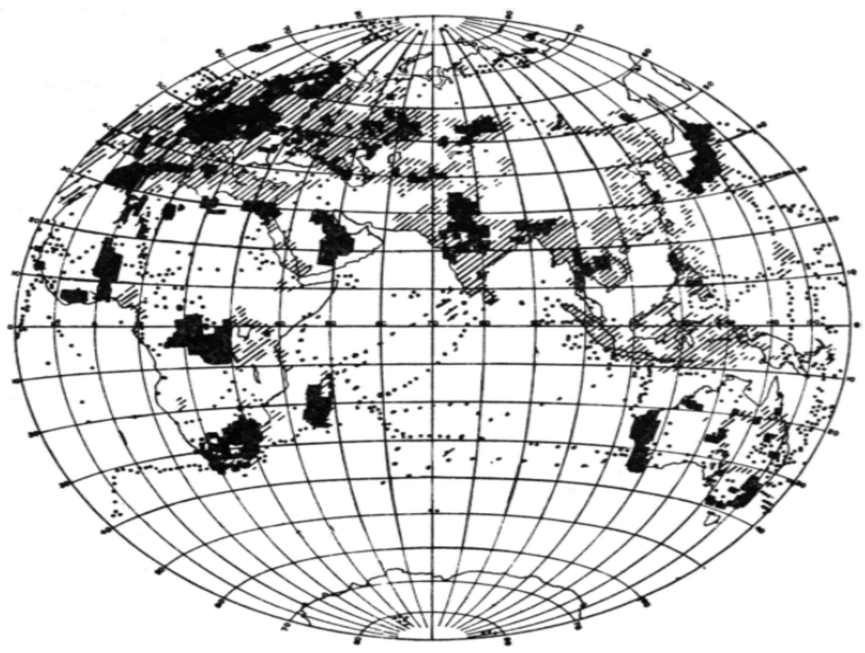
**Twin-Otter Aircraft**

# Dedicated International Symposia & Workshops

# The Need for Global Gravity Data

- **Gravity data until the early 1960s were obtained primarily by point measurements on land and along some ship tracks.**
  - **map of data archive of 1963 (Kaula 1963)**





# 1990s – More Data, Still Many Gaps

- **Greater uniformity, but only at relatively low resolution**
  - **map of terrestrial 1°×1° anomaly archive of 1990 (Rapp and Pavlis 1990)**

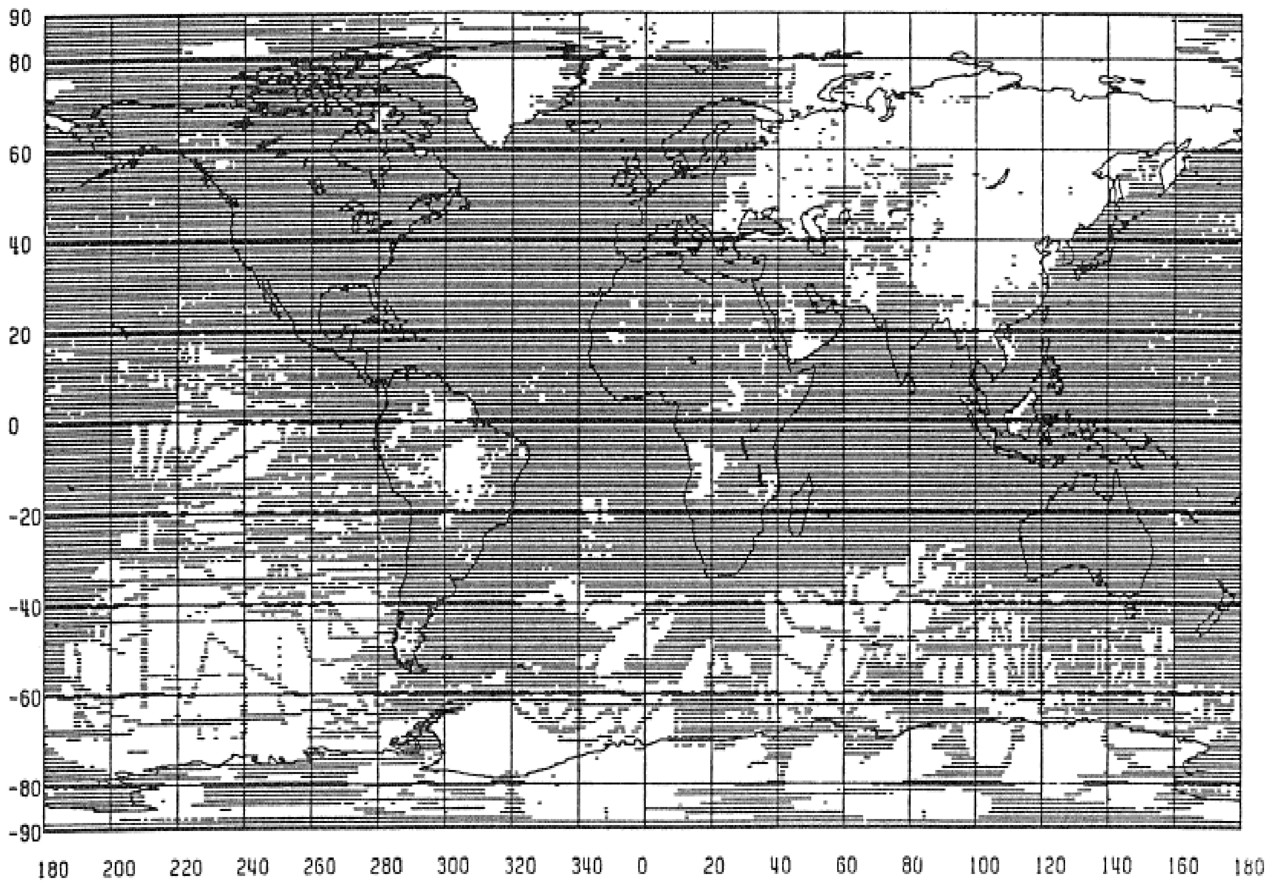


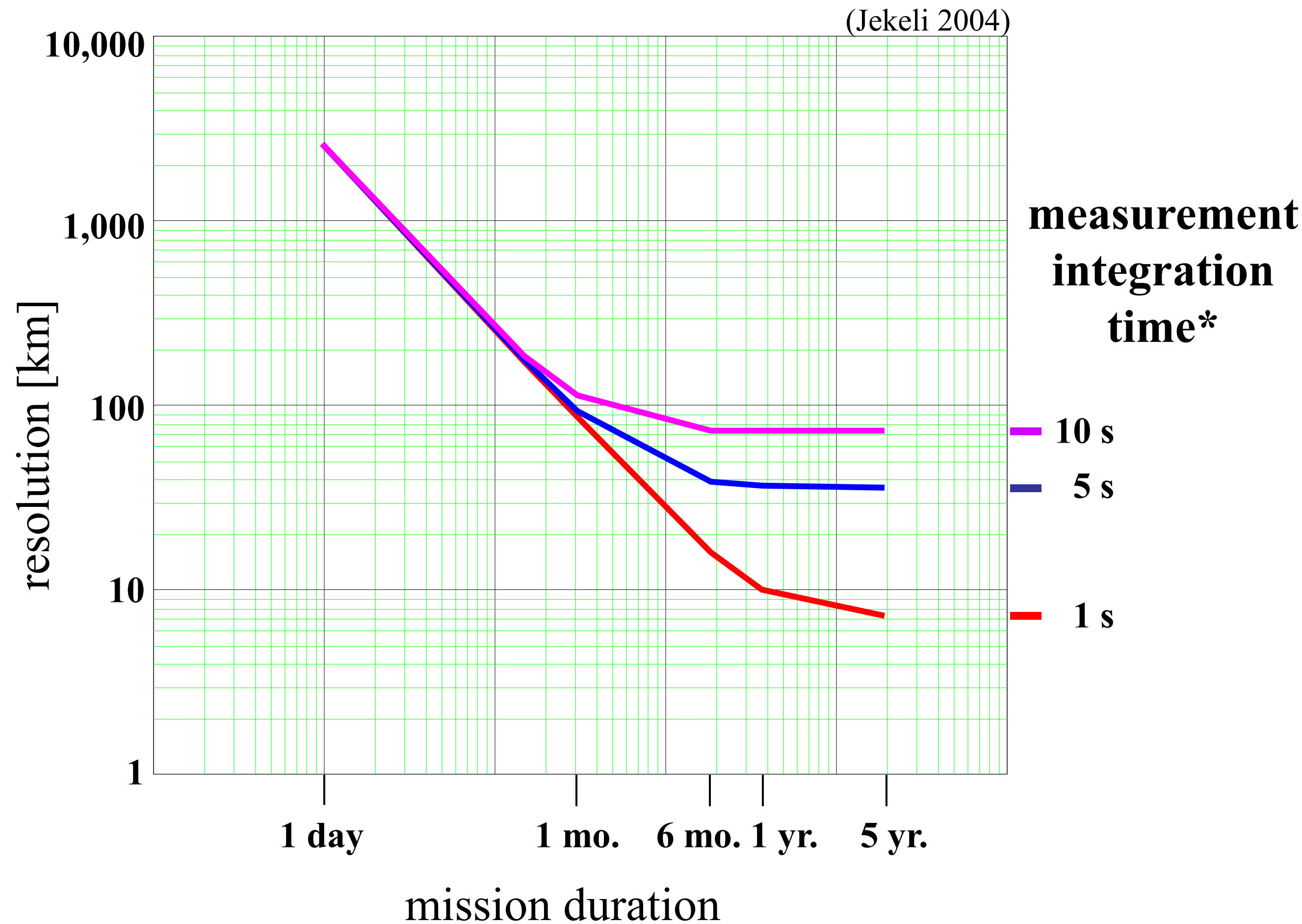

Fig. 3. Location of 45,126 1° × 1° free-air anomalies in the OSU July 1989 file excluding geophysically predicted values.

# Why Airborne Gravimetry?

- **Satellite-derived gravitational models are limited in spatial resolution because of high inherent satellite speed**

- **Only airborne gravimetry yields higher resolution efficiently**

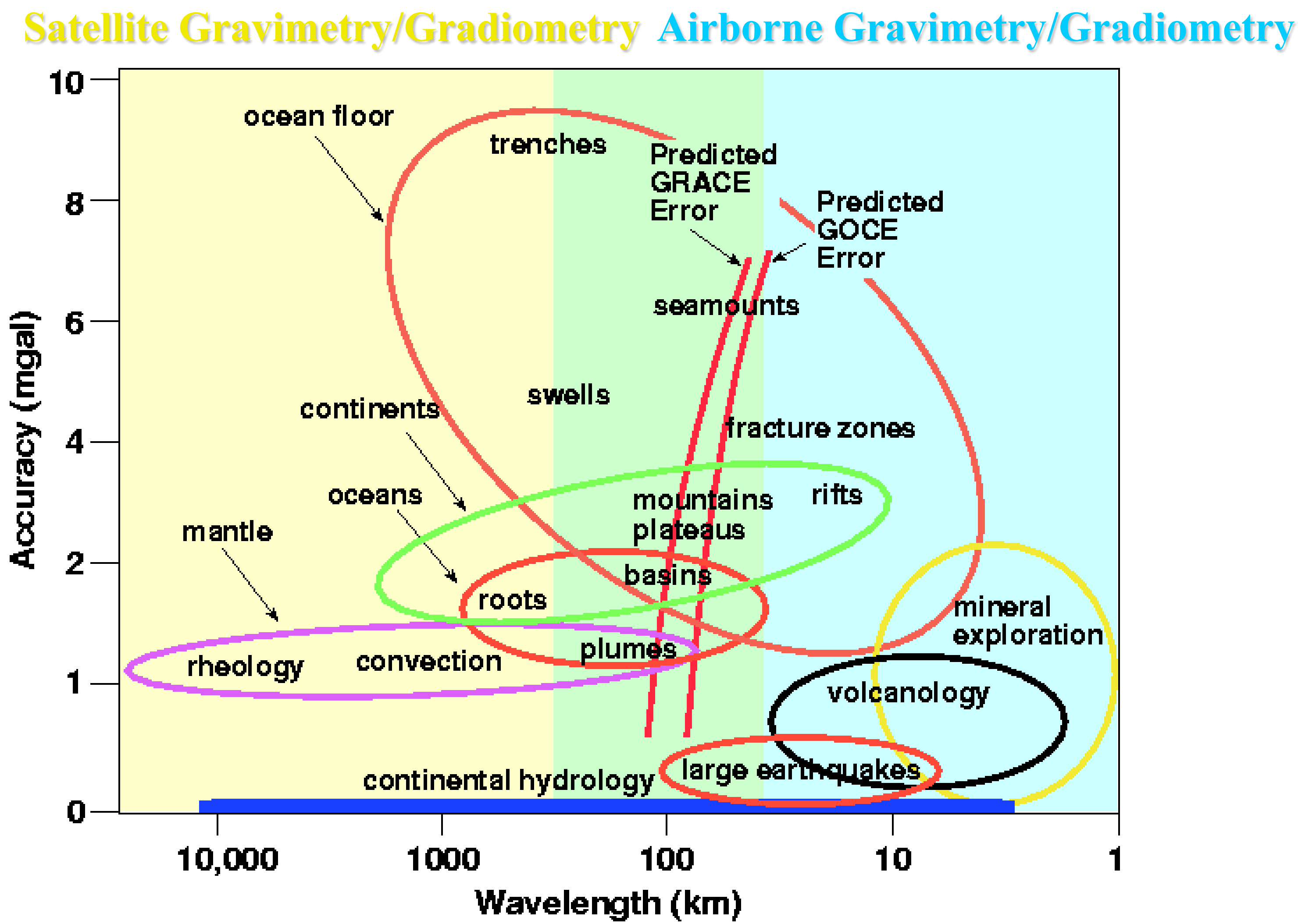
## Satellite Resolution vs Mission Duration and Integration Time




*

| | |
|---|---|
| **CHAMP: 10 s** | **GOCE: 5 - 10 s** |
| **GRACE: 5 s** | **Laser-interferometry GRACE follow-on: 1 - 10 s** |

# Gravity Resolution vs Accuracy Requirements in Geophysics



# Geodetic Motivation

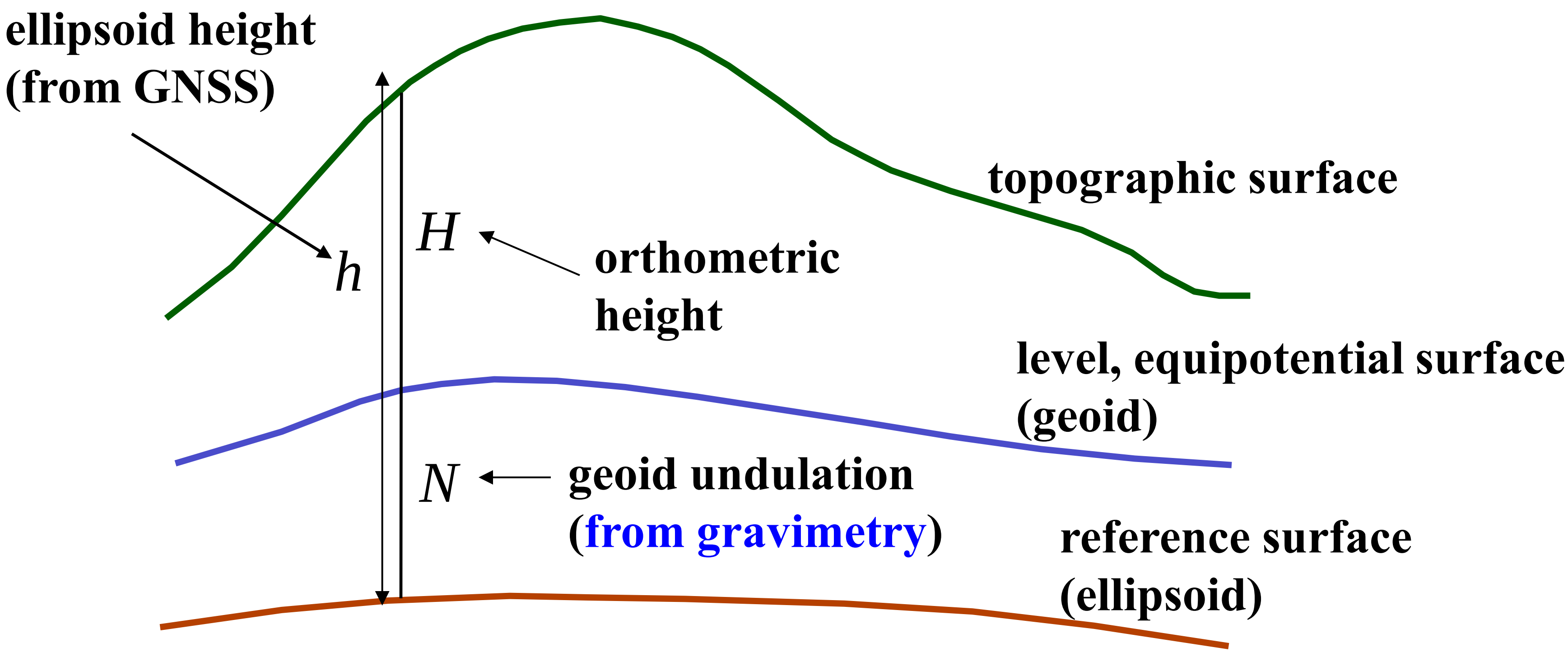

http://images.nationalgeographic.com/wpf/media-live/photos/000/166/cache/article-sea-level-rise_16648_600x450.jpg


http://www.fourwinds10.net/resources/uploads/images/missouri%20river%20flooding(1).jpg


http://thelastgringo.com/2012/11/northeast-united-states-future-climate-change-to-the-year-2099/homes-flooded-nj-hurricane-sandy-oct-2012_620_422_s_c1-2/

# GNSS and Geopotential

- **Traditional height reference surface: equipotential surface (geoid)**
  - **needed for determining and monitoring the flow of water, from flood control to sea level rise**
  - **replace arduous spirit leveling with GNSS:** $H = h - N$

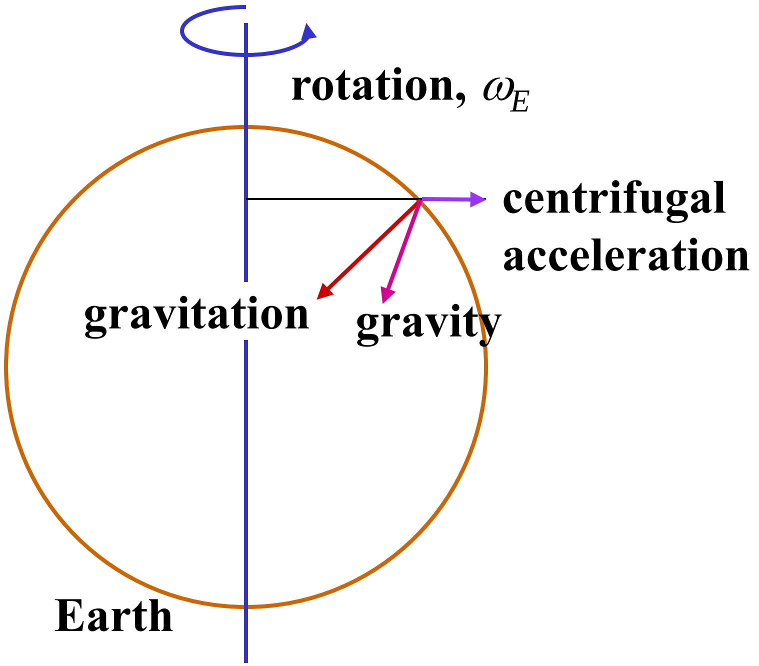


# II. Elemental Review of Physical Geodesy

- Gravitational potential, gravity
- Normal gravity
- Disturbing potential, gravity anomaly, deflection of the vertical
- Geoid determination aspects

# Basic Definitions

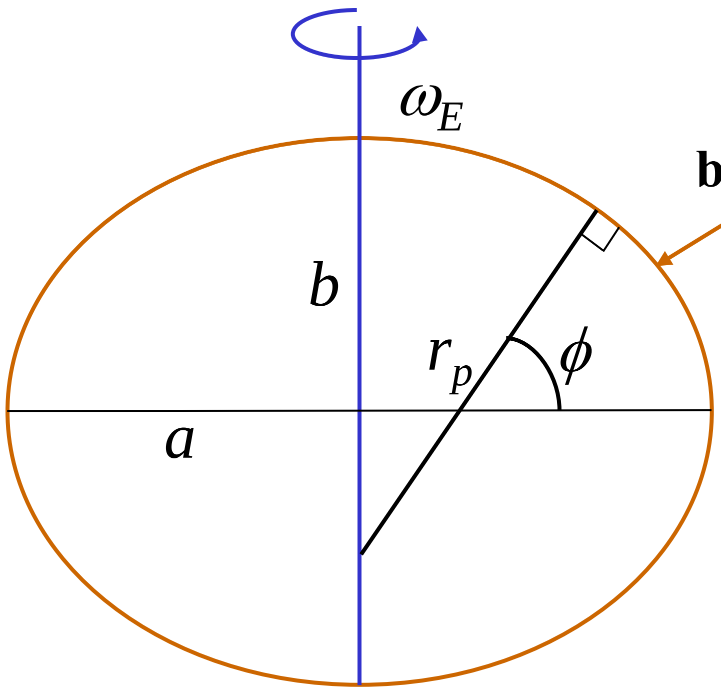
- **Gravitational potential, $V$**
  - due to mass attraction
  - gravitational acceleration: $\boldsymbol{g} = \nabla V$
- **Centrifugal "potential", $\phi$**
  - due to Earth's rotation
  - centrifugal acceleration: $\boldsymbol{a}_{\text{cent}} = \nabla \phi$
- **Gravity potential, $W = V + \phi$**
  - gravity acceleration: $\bar{\boldsymbol{g}} = \boldsymbol{g} + \boldsymbol{a}_{\text{cent}}$




Physical geodesy makes the distinction between gravitation and gravity, especially in terrestrial gravimetry

# Normal Gravitational Potential

- **Mathematically simple potential and boundary**
  - approximates Earth's potential and geoid to about 5 ppm
  - approximates Earth's gravity to about 50 ppm
  - rotates with the Earth




**centrifugal potential on ellipsoid**

$$V\big|_{\text{boundary}} = U_0 - \underbrace{\frac{1}{2}\omega_e^2 r_p^2 \cos^2\phi}$$

- **boundary function is chosen so that gravity potential on boundary is a constant,** $U_0$

# Normal Gravity Potential

- **Expressed as spherical harmonic series in spherical coordinates**

$$U(r,\theta) = V(r,\theta) + \phi(r,\theta)$$

$$= \frac{GM}{a} \sum_{n=0}^{\infty} \left(\frac{a}{r}\right)^{2n+1} C_{2n}^{N} P_{2n}(\cos\theta) + \frac{1}{2}\omega_e^2 r^2 \sin^2\theta$$

  - **closed expression exists in ellipsoidal coordinates**
  - $C_{2n}^{N}$ **depends on only 4 parameters:** $\omega_e$**,** $C_2^N$**,** $a$**,** $GM$
    - **(e.g., WGS84 parameters)**

$$\omega_e = 7.292115 \times 10^{-5} \text{ rad/s}$$

$$C_2^N = -0.484166774985 \times 10^{-3}$$

$$a = 6378137.\text{ m}$$

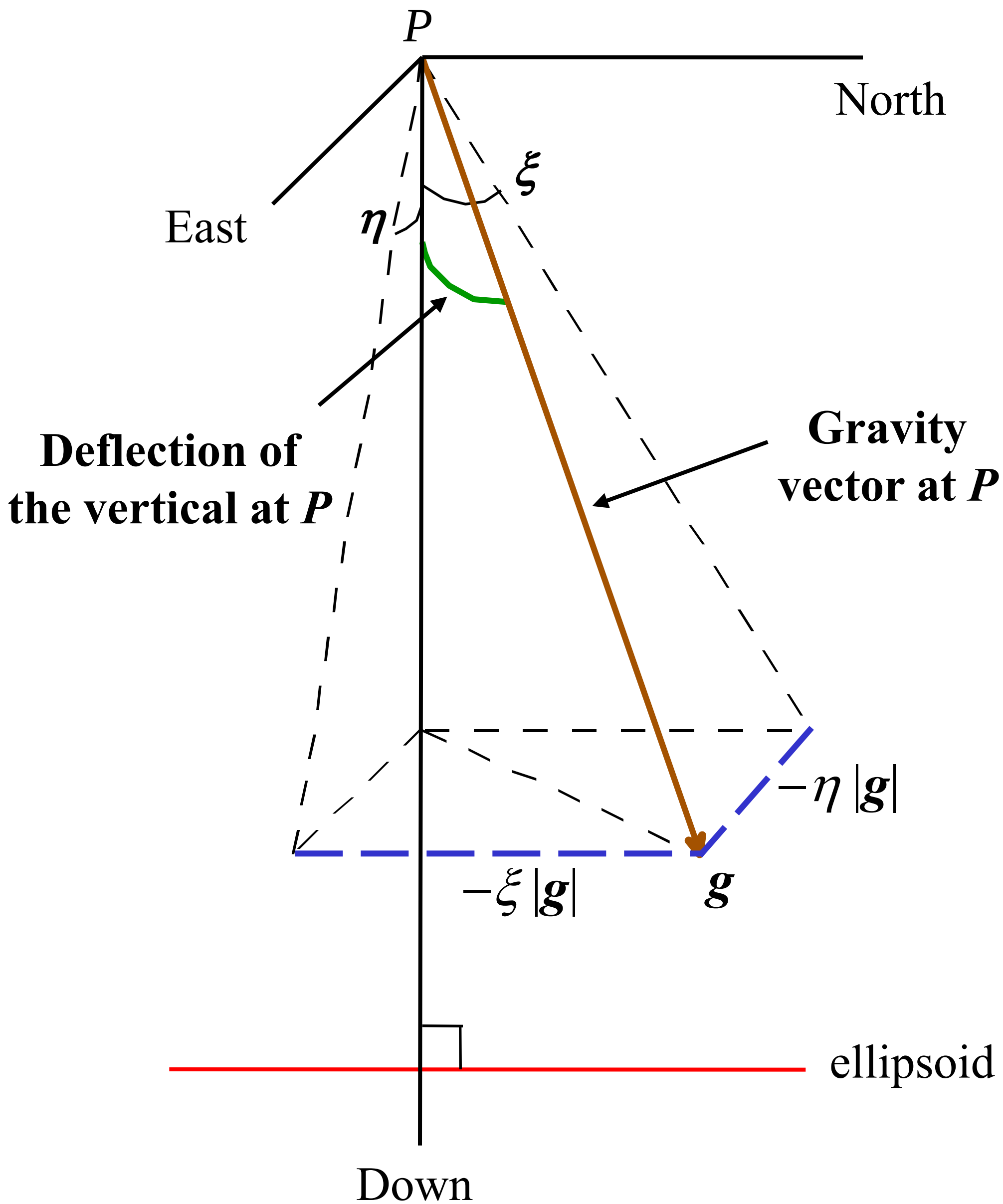
$$GM = 3.986004418 \times 10^{14} \text{ m}^3/\text{s}^2$$

- **Normal gravity vector:** $\gamma = \nabla U$

# Gravity Disturbance and Anomaly

- **Disturbing potential:** $T = W - U$ **($W$ = total gravity potential)**
- **Gravity disturbance vector:** $\delta \boldsymbol{g} = \nabla W - \nabla U = \boldsymbol{g} - \boldsymbol{\gamma}$
  - **gravity disturbance:** $\delta g = |\boldsymbol{g}| - |\boldsymbol{\gamma}|$
  - **in $n$-frame (North-East-Down):** $\delta \boldsymbol{g}^n = \begin{pmatrix} g_N - \gamma_N \\ g_E \\ g_D - \gamma_D \end{pmatrix} \approx \begin{pmatrix} g_N \\ g_E \\ g_D - \gamma_D \end{pmatrix}$
    - due to symmetry, $\gamma_E = 0$
    - near Earth's surface, $\gamma_N \approx 0$
- **Gravity anomaly vector:** $\Delta \boldsymbol{g}_P = \nabla W_P - \nabla U_Q = \boldsymbol{g}_P - \boldsymbol{\gamma}_Q$
  - $P$ **and** $Q$ **are points on the ellipsoid normal such that** $W_P = U_Q$
  - **gravity anomaly:** $\Delta g_P = |\boldsymbol{g}_P| - |\boldsymbol{\gamma}_Q|$

# Deflection of the Vertical




$\xi =$ **north deflection**

$\eta =$ **east deflection**

$$\nabla T = \delta \boldsymbol{g}^n \approx \begin{pmatrix} -\xi g \\ -\eta g \\ \delta g \end{pmatrix} \approx \begin{pmatrix} g_N \\ g_E \\ g_D - \gamma_D \end{pmatrix}$$

– **linear approximation**

– **signs agree with convention of astronomic deflection of the vertical**

# Geoid Determination

- **Bruns's Formula:** $N_{P_0} = \frac{1}{\gamma_{Q_0}} T_{P_0} + N_0$ **where $N_0$ is a height datum offset**

- **Boundary-value Problem:** $\nabla^2 T = 0$ **above geoid (by assumption)**

$$\Delta g_{P_0'} = -\left.\frac{\partial T}{\partial h}\right|_{P_0'} + \frac{1}{\gamma_{Q_0}} \left.\frac{\partial \gamma}{\partial h}\right|_{Q_0} T_{P_0'}$$ **boundary condition**

$\Delta g_{P'}, \Delta g_{P_a'}$

$+$

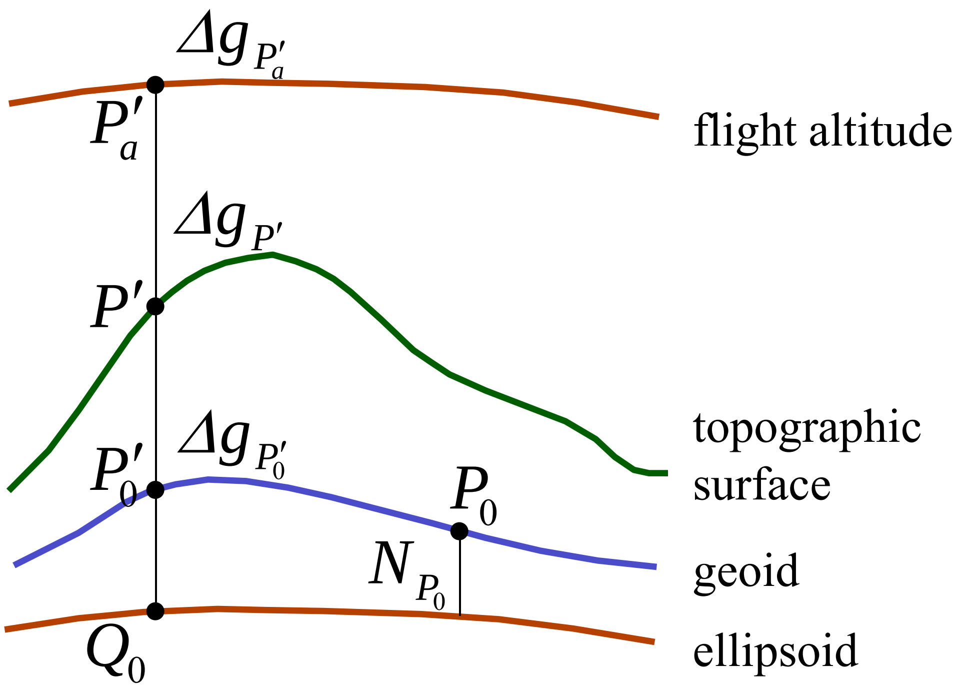
**gravity reductions**

$\|$

$\Delta g_{P_0'}$

- **Stokes's formula**

$$N_{P_0} = N_0 + \frac{R}{4\pi\gamma_{Q_0}} \iint_{\Omega} \Delta g_{P_0'} S\left(\psi_{P_0, P_0'}\right) d\Omega$$



# Details

- Gravity reductions to satisfy the boundary-value conditions
  - re-distribution of topographic mass; consequent indirect effect
  - downward continuation (various methods)
- Ellipsoidal corrections
  - account for spherical approximation of geoid, boundary condition
- Include existing spherical harmonic model (satellite-derived)
  - remove-compute-restore techniques
- Back to Motivation
  - use airborne gravimetry to improve spatial resolution of data (boundary values) – few km to 200 km wavelengths

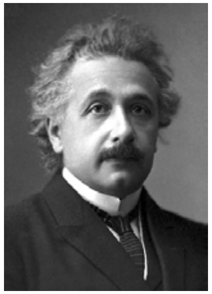
# III. Basic Theory of Moving-Base Scalar Gravimetry

- **Fundamental laws of physics and the gravimetry equation**
- **Coordinate frames**
- **Mechanizations and methods of scalar gravimetry**
- **Rudimentary error analyses**

# Fundamental Physical Laws

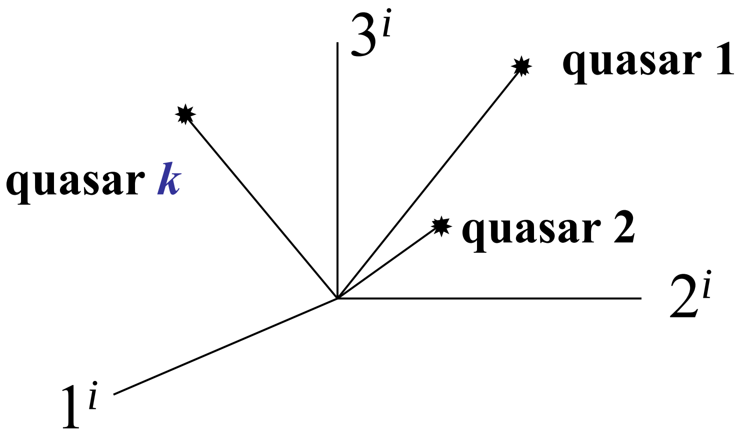
- Moving-Base Gravimetry and Gradiometry are based on 3 fundamental laws in physics
  - Newton's Second Law of Motion
  - Newton's Law of Gravitation
  - Einstein's Equivalence Principle
- Laws are expressed in an inertial frame
- General Relativistic effects are not yet needed
  - however, the interpretation of space in the theory of general relativity is used to distinguish between applied and gravitational forces

Issac Newton
1643 - 1727



Albert Einstein
1879 – 1955

# Inertial Frame




**notation convention:**
**- axis identified by number**
**- superscript identifies frame**

- **The realization of a system of coordinates that does not rotate (and is in free-fall, e.g., Earth-centered)**

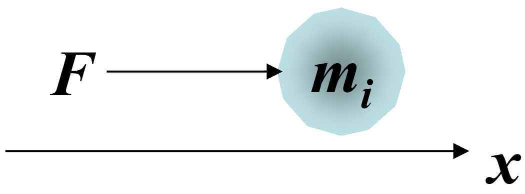
- **Modern definition: fixed to quasars – which exhibit no relative motion on celestial sphere**

- **International Celestial Reference Frame (ICRF) based on coordinates of 295 stable quasars**

# Newton's Second Law of Motion

- **Time-rate of change of linear momentum equals applied force, $F$**

$$\frac{d}{dt}\left(m_i \dot{\boldsymbol{x}}\right) = \boldsymbol{F}$$




- – $m_i$ **is the inertial mass of the test body**

$$\left(m_i = \text{constant} \;\rightarrow\; m_i \ddot{\boldsymbol{x}} = \boldsymbol{F}\right)$$

- **In the presence of a gravitational field, this law must be modified:**

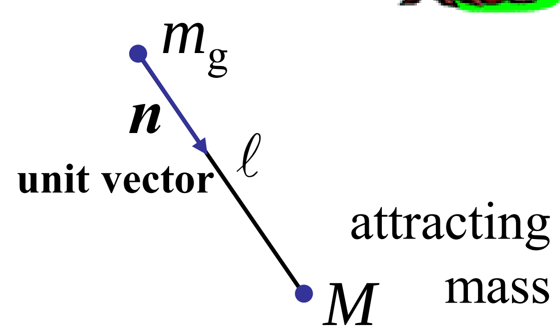
$$m_i \ddot{\boldsymbol{x}} = \boldsymbol{F} + \boldsymbol{F}_g$$

- – $\boldsymbol{F}_g$ **is a force associated with the gravitational acceleration due to a field (or space curvature) generated by all masses in the universe, relative to the freely-falling frame (Earth's mass and tidal effects due to moon, sun, etc.)**
- – **action forces, $\boldsymbol{F}$, and gravitational forces, $\boldsymbol{F}_g$, are fundamentally different**

# Newton's Law of Gravitation

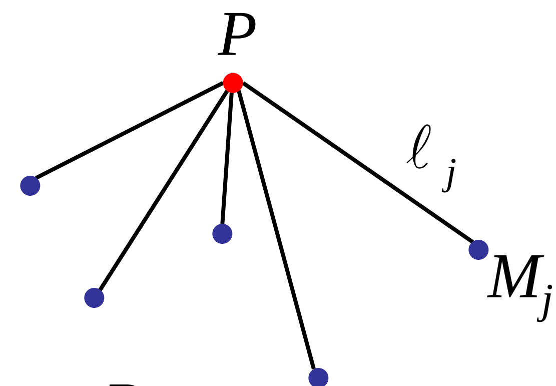
- **Gravitational force vector**

$$\boldsymbol{F}_g = G\frac{Mm_g}{\ell^2}\boldsymbol{n} = m_g\boldsymbol{g}$$

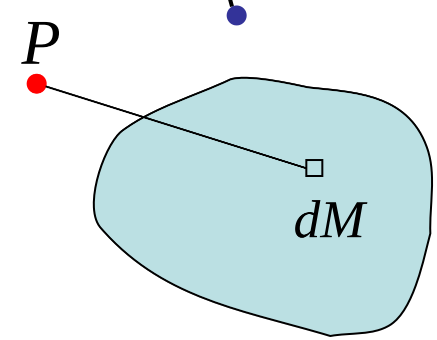



  - **$G$ = Newton's gravitational constant**
  - **$\boldsymbol{g}$ = gravitational acceleration due to $M$**
  - **$m_g$ is the gravitational mass of the test body**
  - **it's easier to work with field potential, $V$**

$$\boldsymbol{g} = \nabla V \qquad V = GM/\ell$$




- **Many mass points**
  - **law of superposition:** $$V_P = G\sum_j \frac{M_j}{\ell_j}$$
  - **mass continuum:** $$V_P = G\int_M \frac{dM}{\ell}$$

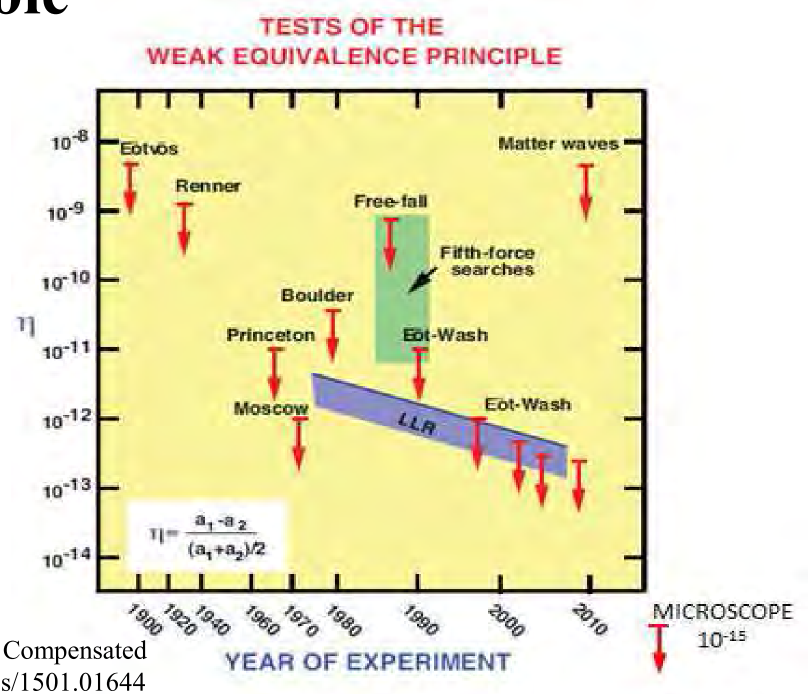


# Equivalence Principle (1)

- **A. Einstein (1907): No experiment performed in a closed system can distinguish between an accelerated reference frame or a reference frame at rest in a uniform gravitational field.**
  - **consequence: inertial mass equals gravitational mass**

$$m_i = m_g = m$$

- **Experimental evidence has not been able to dispute this assumption**
  - **violation of the principle may lead to new theories that unify gravitational and other forces**
  - **proposed French Space Agency mission, MICROSCOPE*, aims to push the sensitivity by many orders of magnitude**




* Micro-Satellite à traînée Compensée pour l'Observation du Principe d'Equivalence (Drag Compensated Micro-satellite to Observe the Equivalence Principle); Berge et al. (2015) http://arxiv.org/abs/1501.01644

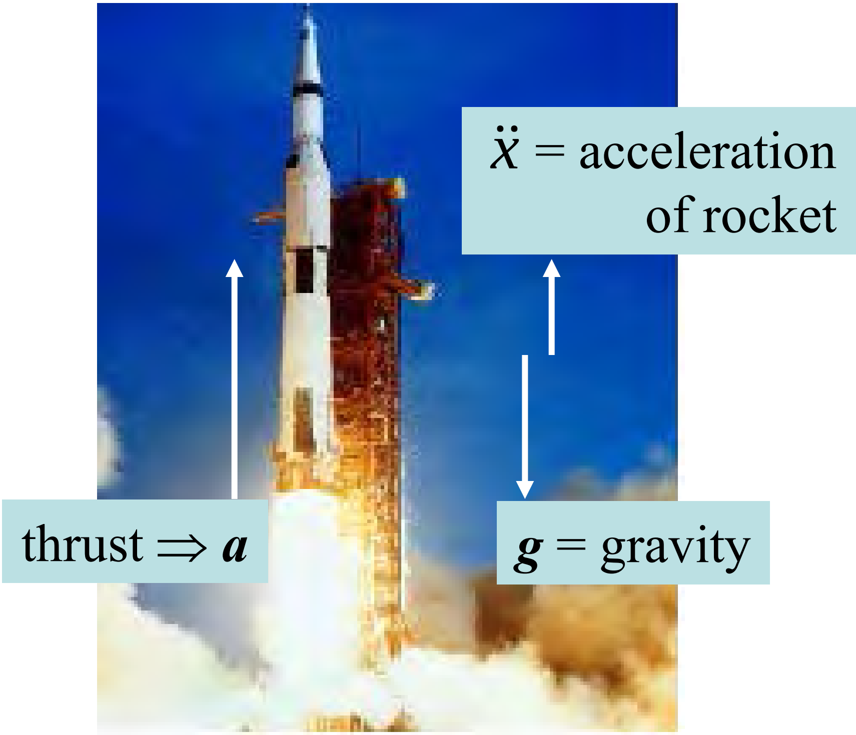
# Equivalence Principle (2)

- **Equation of motion in the inertial frame**

$$\ddot{x}^i = \frac{F^i}{m} + g^i$$

  - $\ddot{x} = \dfrac{d^2x}{dt^2}$, **vector of total kinematic acceleration**
  - $\dfrac{F^i}{m} = a^i$, **specific force, or the acceleration resulting from an action force; e.g., thrust of a rocket**

$$\boxed{\ddot{x}^i = a^i + g^i}$$

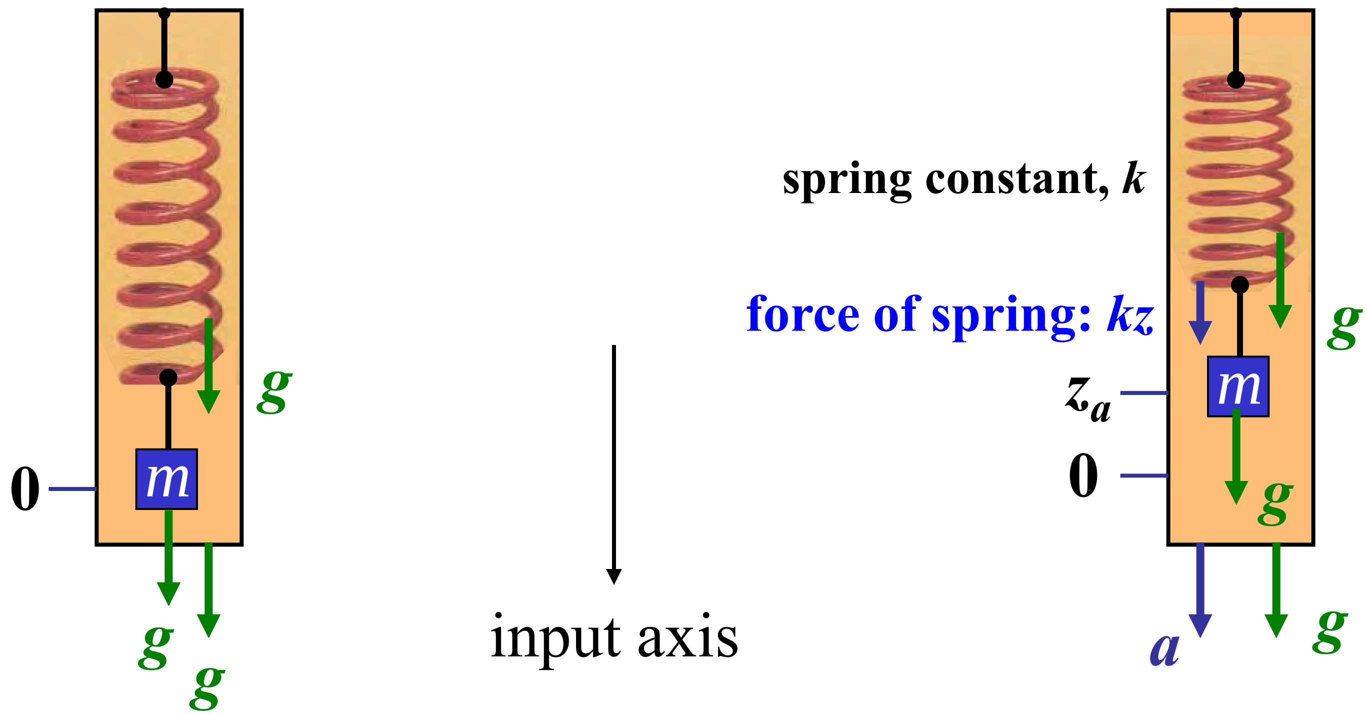



$$|a| \le |g| \quad \Rightarrow \quad \text{no lift-off !}$$

# What Does an Accelerometer Sense?

**gravitational field, *g***
***no* applied acceleration**

**gravitational field, *g***
**applied acceleration, *a***




**accelerometer indicates: 0**

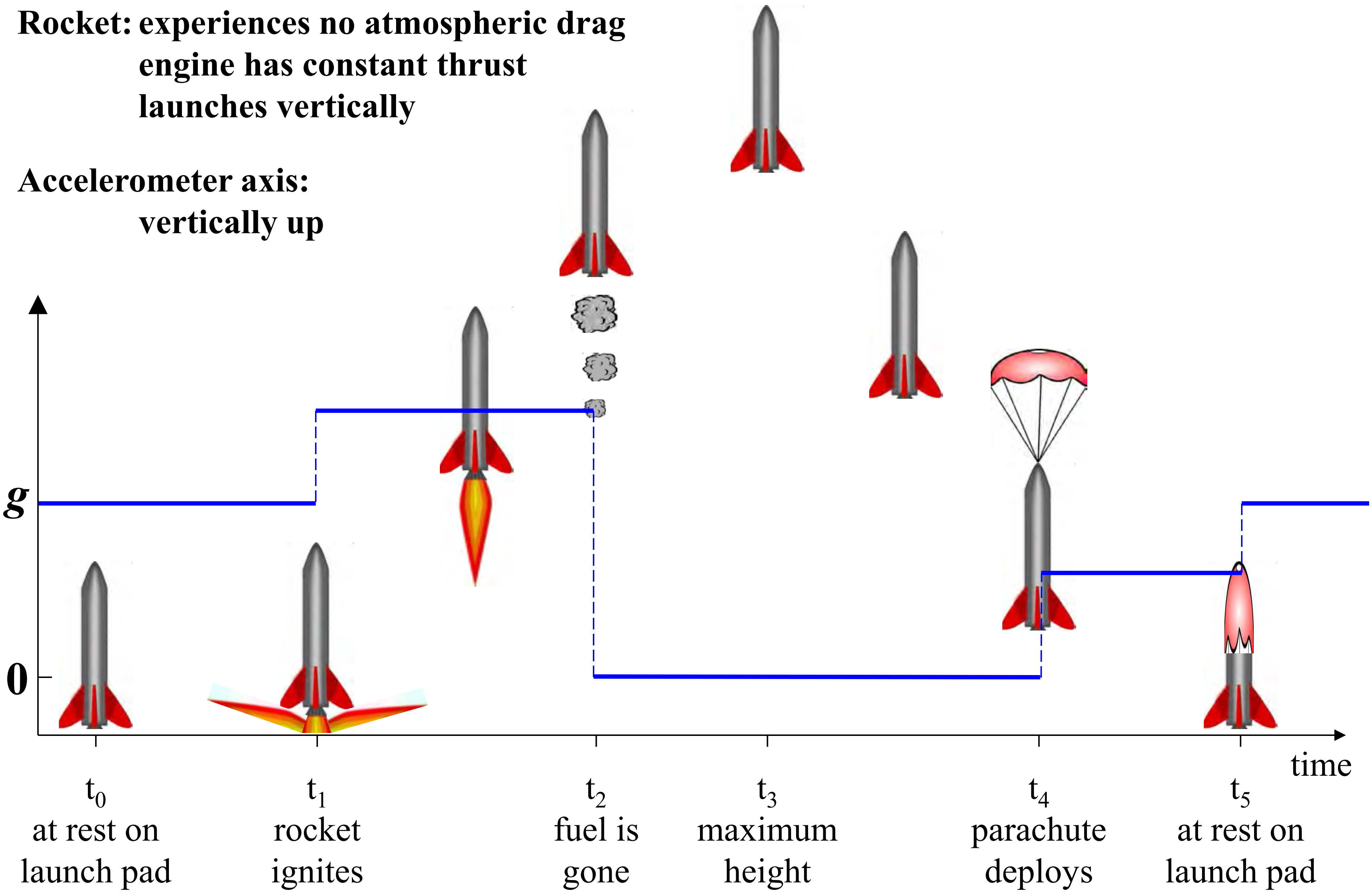
**accelerometer indicates: $z_a \sim a$**

- **Accelerometer does *not* sense gravitation, only acceleration due to action force (including *reaction* forces!)**

# What Does Accelerometer (or Gravimeter) on Rocket Sense?

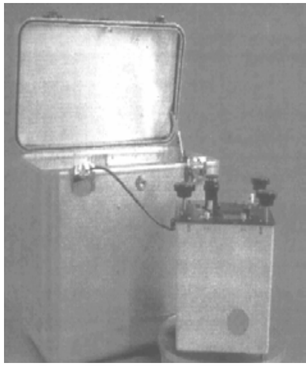
**Rocket:** **experiences no atmospheric drag**
**engine has constant thrust**
**launches vertically**

**Accelerometer axis:**
**vertically up**

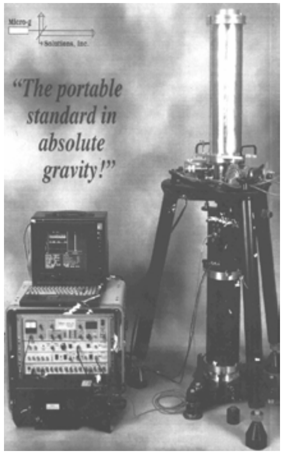


# Static Gravimetry – Special Case

- Assume non-rotating Earth (for simplicity) $g = \ddot{x} - a$
- Relative (spring) gravimeter: $\ddot{x} = 0 \quad \Rightarrow \quad a = -g$
  - it is an accelerometer that senses specific force, $a$
  - with sensitive axis along plumb line, $a$ is the reaction force of Earth's surface that keeps the gravimeter from falling



- Absolute (ballistic) gravimeter: $a = 0 \quad \Rightarrow \quad \ddot{x} = g$
  - it tracks a test mass in vacuum (zero spring force)
  - indirectly, it senses the reaction force that keeps the reference from falling

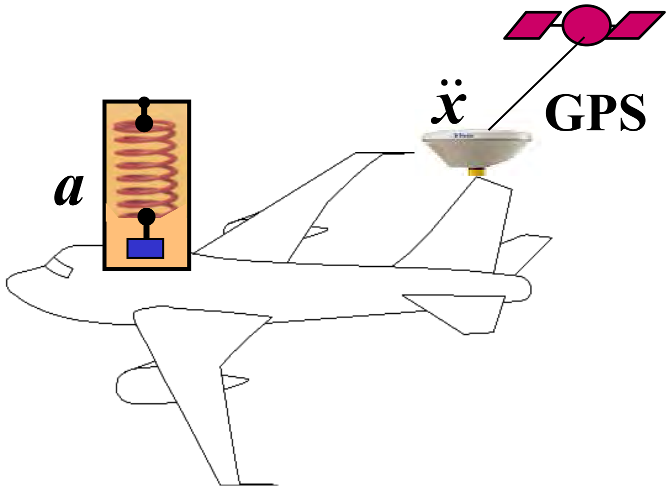



- All operational moving-base gravimeters are relative sensors

# Basic Equation for Moving-Base Gravimetry

- **In the inertial frame:** $$g^i = \ddot{x}^i - a^i$$

- **Because:**
  - **specific forces are measured in a non-inertial frame attached to a rotating body (vehicle)**
  - **specific forces and kinematic accelerations refer to different measurement points of the instrument-carrying vehicle**
  - **generally, gravitation is desired in a local, Earth-fixed frame**




- **Need to introduce:**
  - **coordinate frames**
  - **rotations and lever-arm effects**

- **Get more complicated expressions for gravimetry equation**

# Two Possible Approaches to Determine *g* (1)

- **For concepts, consider inertial frame for simplicity:** $\ddot{\boldsymbol{x}}^i = \boldsymbol{a}^i + \boldsymbol{g}^i$
- **Position (Tracking) Method to determine the unknown:** $\boldsymbol{g}$
  - **Integrate equations of motion**

$$\boldsymbol{x}^i(t) = \boldsymbol{x}^i(t_0) + \dot{\boldsymbol{x}}^i(t_0)(t - t_0) + \int_{t_0}^{t} (t - t')\left(\boldsymbol{a}^i(t') + \boldsymbol{g}^i(t')\right) dt'$$

  - **Positions, $x$: from tracking system, like GPS or other GNSS**
  - **Specific forces, $a$: from accelerometer**
  - **method is used for geopotential determination with satellite tracking, and was used also with ground-based inertial positioning systems**
  - **Advantage: do not need to differentiate $x$ to get $\ddot{x}$**
  - **Disadvantage: $g$ must be modeled in some way to perform the integration (e.g., spherical harmonics in satellite tracking, with statistical constraint)**
  - **Not used for scalar airborne gravimetry due to vertical instability of integral**
    - **but can be (is) used for horizontal components of gravity!**

# Two Possible Approaches to Determine *g* (2)

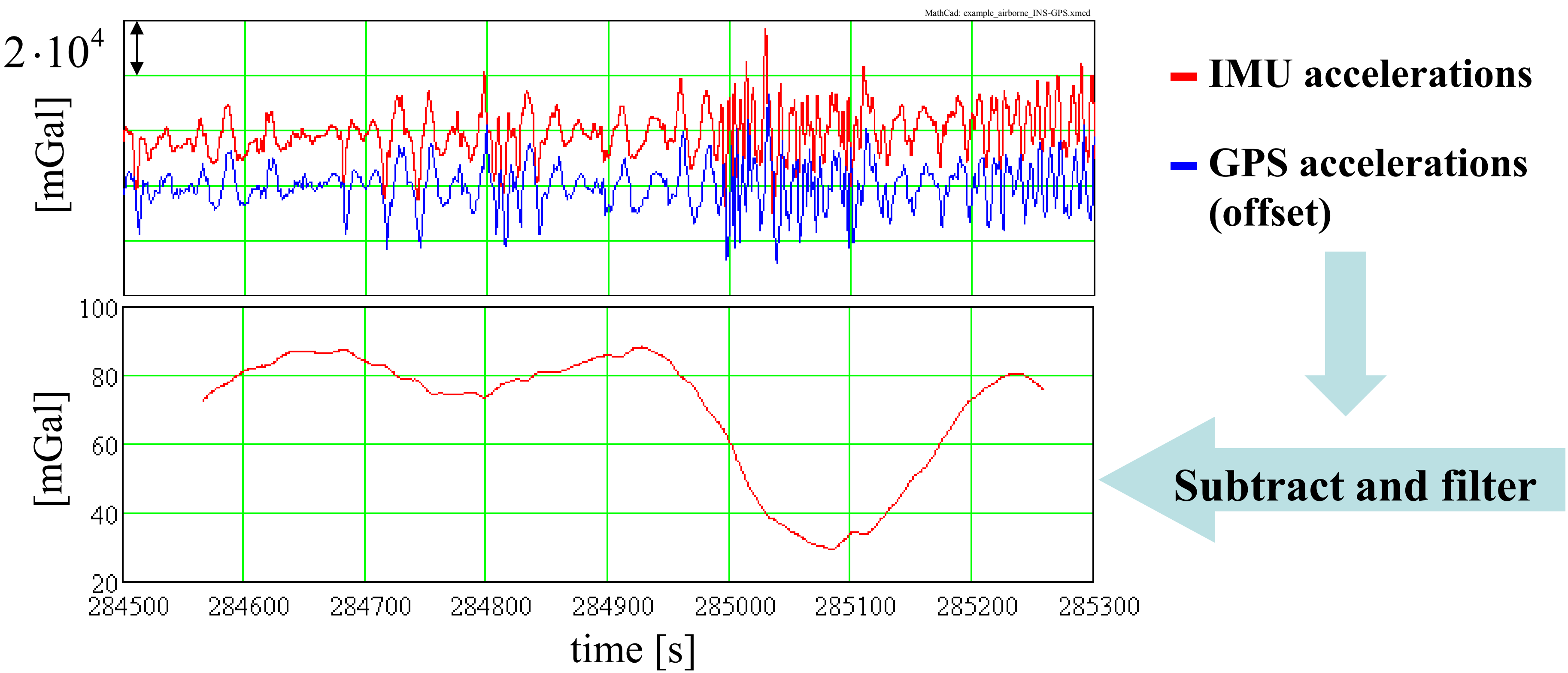
- **Accelerometry Method to determine the unknown: *g***

$$g^i = \ddot{x}^i - a^i$$

  - **Specific force, $a$ : from accelerometer**
  - **Kinematic acceleration, $\ddot{x}$ : by differentiating position from tracking system, like GPS (GNSS)**
  - **Advantage: $g$ does not need to be modeled**
  - **Disadvantage: positions are processed with two numerical differentiations**
    - advanced numerical techniques → may be less serious than gravity modeling problem
- **Either position method or accelerometry method requires two independent sensor systems**
  - **Tracking system**
  - **Accelerometer (gravimeter)**
  - **Gravimetry accuracy depends equally on the precision of both systems**

# The Challenge of Airborne Gravimetry

- Both systems measure large signals e.g., (> ± 10000 mGal)
- Desired gravity disturbance is orders of magnitude smaller
  - signal-to-noise ratio may be very small, depending on system accuracies
- e.g., INS/GPS system – data from University of Calgary, 1996



# Coordinate Frames

- **Other coordinate frames**
  - **rotating with respect to inertial frame,**
  - **may have different origin point,**
  - **have different form of Newton's law of motion,**
  - **all defined by three mutually orthogonal, usually right-handed axes (Cartesian coordinates).**

- **Specific frames to be considered:**
  - **navigation frame: frame in which navigation equations are formulated; usually identified with local North-East-Down (NED) directions (*n*-frame).**
  - **Earth-centered-Earth-fixed frame: frame with origin at Earth's center of mass and axes defined by conventional pole and Greenwich meridian (Cartesian or geodetic coordinates) (*e*-frame).**

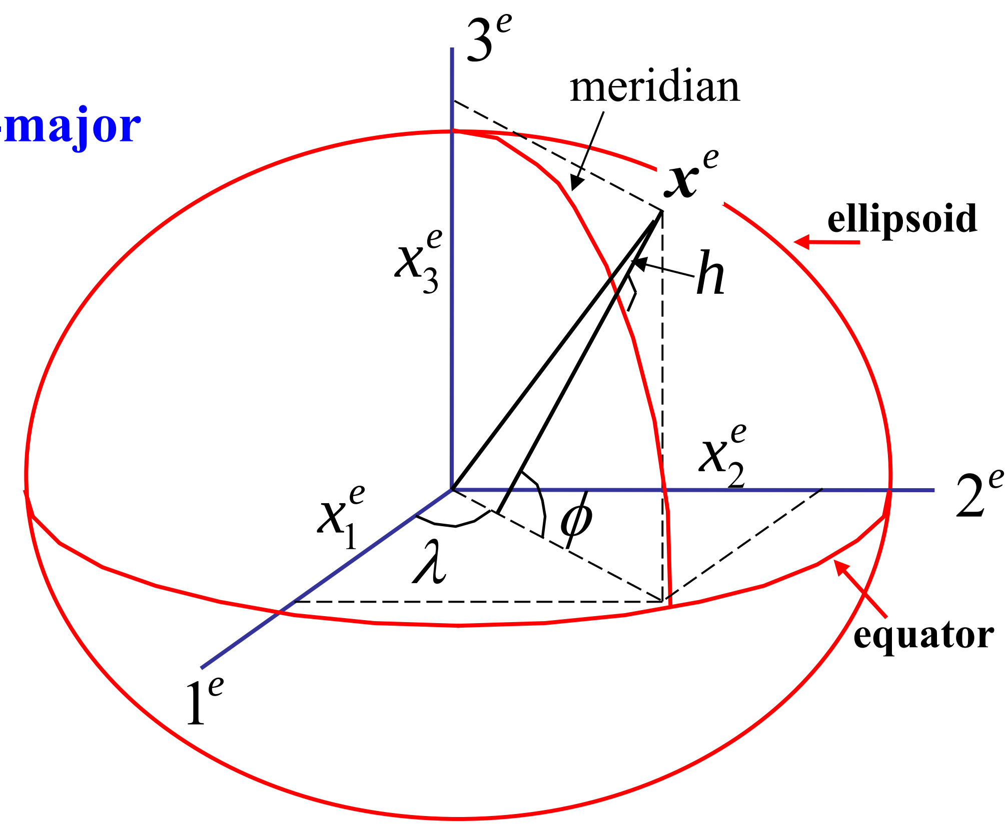
# Earth-Centered-Earth-Fixed Coordinates

- **Cartesian coordinate vector in *e*-frame:** $\boldsymbol{x}^e = \begin{pmatrix} x_1^e & x_2^e & x_3^e \end{pmatrix}^{\mathrm{T}}$

- **Geodetic coordinates, latitude, longitude, height:** $\phi, \lambda, h$

  – **refer to a particular ellipsoid (assume geocentric) with semi-major axis, *a*, and first eccentricity, *e***

  – **are orthogonal curvilinear *e*-frame coordinates**



# Transforming Between Cartesian & Geodetic Coordinates

$$x_1^e = (N+h)\cos\phi\cos\lambda$$

$$x_2^e = (N+h)\cos\phi\sin\lambda$$

$$x_3^e = \left(N\left(1-e^2\right)+h\right)\sin\phi$$

$$\phi = \tan^{-1}\left(\frac{x_3^e}{\sqrt{\left(x_1^e\right)^2+\left(x_2^e\right)^2}}\left(1+\frac{e^2 N\sin\phi}{x_3^e}\right)\right)$$

$$\lambda = \tan^{-1}\left(x_2^e / x_1^e\right)$$

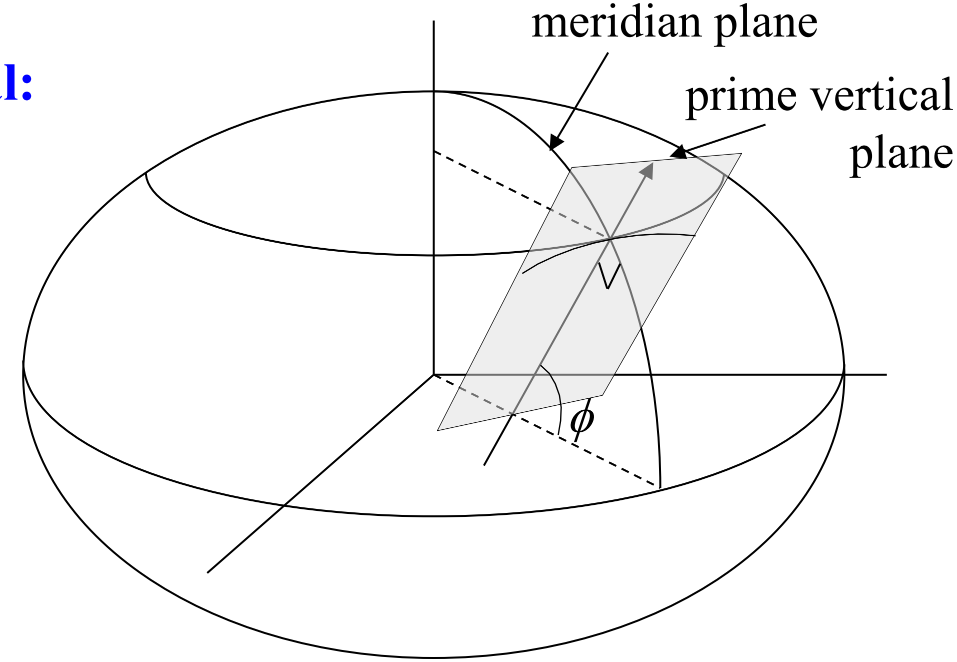
$$h = \sqrt{\left(x_1^e\right)^2+\left(x_2^e\right)^2}\cos\phi + x_3^e\sin\phi - a^2/N$$

– **radius of curvature in prime vertical:**

$$N = \frac{a}{\sqrt{1-e^2\sin^2\phi}}$$

– **radius of curvature in meridian:**

$$M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\phi\right)^{3/2}}$$




• **Cartesian and geodetic coordinates may be used interchangeably**

# Rotations and Angular Rates Between Frames

- **Assume common origin for frames**
- $\mathbf{C}_t^s$ **= matrix that rotates coordinates from *t*-frame to *s*-frame**
  - vector, $x$: $\quad \boldsymbol{x}^s = \mathbf{C}_t^s \boldsymbol{x}^t$
  - matrix, $A$: $\quad \mathbf{A}^s = \mathbf{C}_t^s \mathbf{A}^t \mathbf{C}_s^t$
  - $\mathbf{C}_t^s$ is **orthogonal**: $\mathbf{C}_s^t \equiv \left(\mathbf{C}_t^s\right)^{-1} = \left(\mathbf{C}_t^s\right)^{\mathrm{T}}$
- $\boldsymbol{\omega}_{st}^t$ **= angular rate vector of *t*-frame relative to *s*-frame; components in *t*-frame**
- **Let** $\boldsymbol{\omega}_{st}^t = \begin{pmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{pmatrix}$ **then** $\left[\boldsymbol{\omega}_{st}^t \times\right] \equiv \mathbf{\Omega}_{st}^t = \begin{pmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{pmatrix}$
  - cross-product is same as multiplication by **skew-symmetric** matrix
- **Time-derivative:** $\dot{\mathbf{C}}_t^s = \mathbf{C}_t^s \left[\boldsymbol{\omega}_{st}^t \times\right] = \mathbf{C}_t^s \mathbf{\Omega}_{st}^t$

# Earth-Fixed vs. Inertial Frames

- **Transformation of coordinates between *i*-frame and *e*-frame is just a rotation about the 3-axis**

$$\boldsymbol{x}^i = \mathbf{C}_e^i \boldsymbol{x}^e \quad \Rightarrow \quad \ddot{\boldsymbol{x}}^i = \ddot{\mathbf{C}}_e^i \boldsymbol{x}^e + 2\dot{\mathbf{C}}_e^i \dot{\boldsymbol{x}}^e + \mathbf{C}_e^i \ddot{\boldsymbol{x}}^e$$

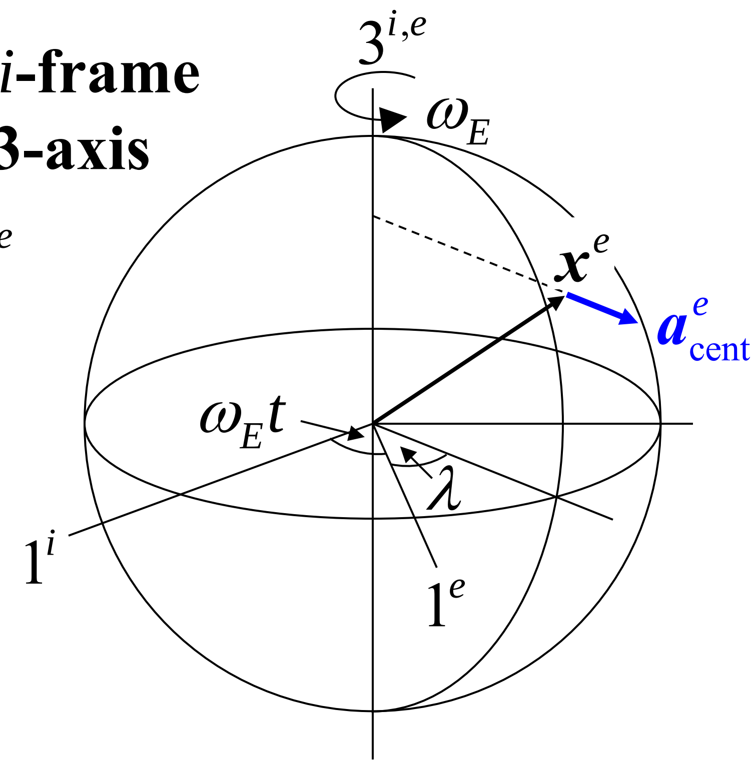
$$\boldsymbol{\omega}_{ie}^e = \begin{pmatrix} 0^* & 0^* & \omega_E \end{pmatrix}^{\mathrm{T}}$$

$\omega_E$ = **Earth's rotation rate**

$0^*$ = **neglect rates of polar motion and precession/nutation**

$$d\boldsymbol{\omega}_{ie}^e / dt = \mathbf{0}$$

$$\ddot{\boldsymbol{x}}^i = \mathbf{C}_e^i \left( \boldsymbol{\Omega}_{ie}^e \boldsymbol{\Omega}_{ie}^e + \dot{\boldsymbol{\Omega}}_{ie}^e \right) \boldsymbol{x}^e + 2\mathbf{C}_e^i \boldsymbol{\Omega}_{ie}^e \dot{\boldsymbol{x}}^e + \mathbf{C}_e^i \ddot{\boldsymbol{x}}^e$$




- **Extract centrifugal acceleration from other kinematic accelerations**

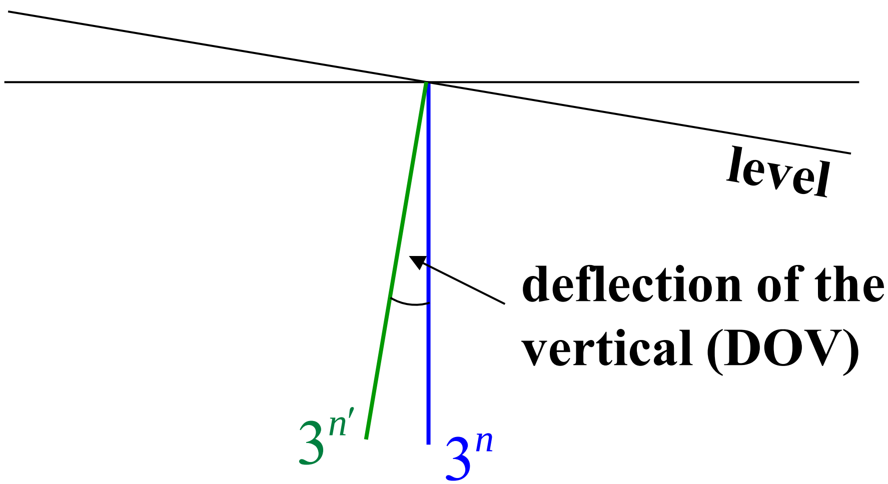
$$\mathbf{C}_i^e \ddot{\boldsymbol{x}}^i = \boldsymbol{\Omega}_{ie}^e \boldsymbol{\Omega}_{ie}^e \boldsymbol{x}^e + 2\boldsymbol{\Omega}_{ie}^e \dot{\boldsymbol{x}}^e + \ddot{\boldsymbol{x}}^e = \boldsymbol{a}^e + \boldsymbol{g}^e$$

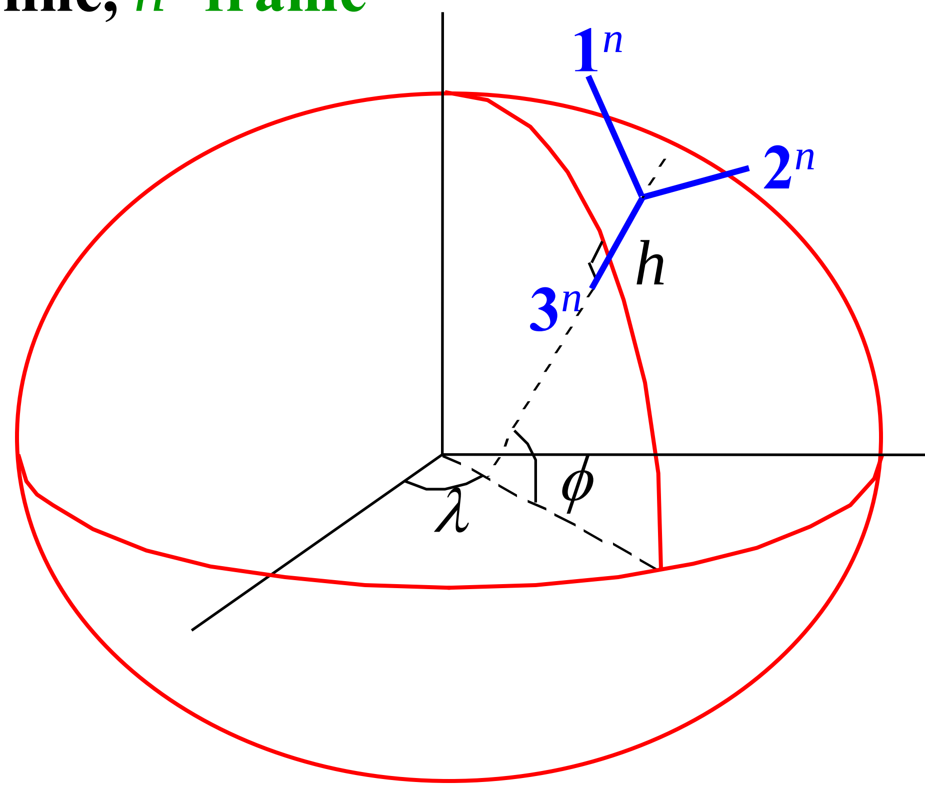
$$= -\boldsymbol{a}_{\text{cent}}^e + \boldsymbol{q}^e$$

← **defines $q$** $\quad\Rightarrow\quad \bar{\boldsymbol{g}}^e = \boldsymbol{q}^e - \boldsymbol{a}^e$

# Navigation Frame

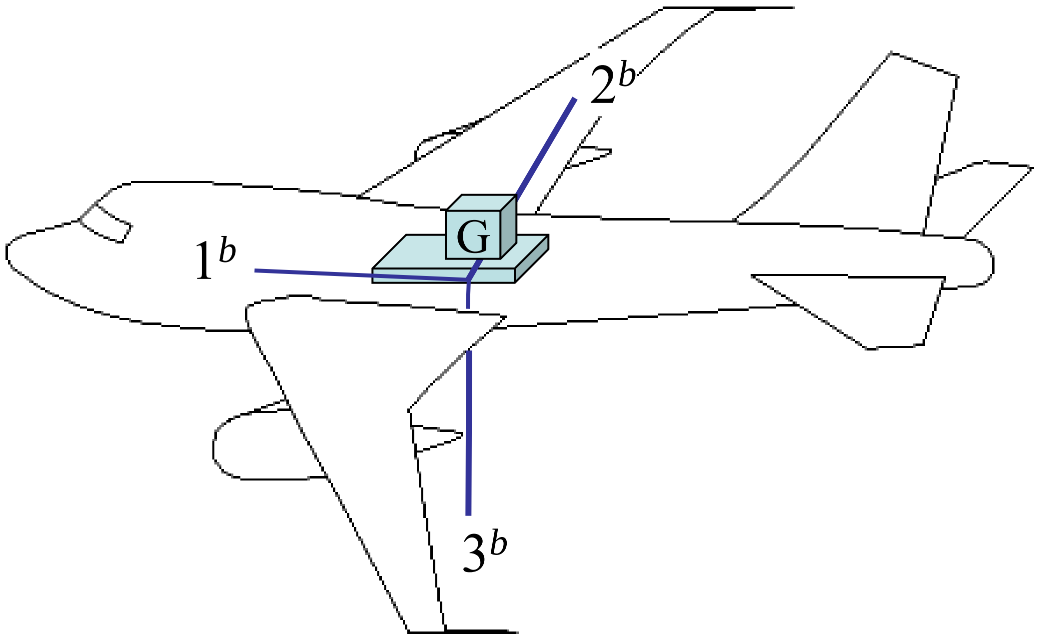
- **Usually, north-east-down (NED)-frame, or *n*-frame**
  - **moves with the vehicle – *not* used for coordinates of the vehicle**
  - **used as reference for velocity and orientation of the vehicle; and, gravity**
- **Alternative: vertical along plumb line, $n'$-frame**
  - **conventional reference for terrestrial gravity**
  - **no “horizontal” gravity components**



# Body Frame

- **Axes are defined by principal axes of the vehicle: forward (1), to-the-right (2), and through-the-floor (3)**




- **Gravimeter (G) measurements are made either:**
  - **in the *b*-frame – strapdown system; gyro data provide orientation**
  - **in the *n*-, *n*′-frames – platform is stabilized using IMUs***

*** inertial measurement units**

# Moving-Base Gravimetry – Strapdown Mechanization

in *i*-frame

**Gravitational Vector in *n*-frame** ←

$$\boldsymbol{g}^n = \mathbf{C}_i^n \left( \ddot{\boldsymbol{x}}^i - \mathbf{C}_b^i \boldsymbol{a}^b \right)$$

**transformation from inertial frame to *n*-frame** ← **GNSS**

**accelerometers**

**transformation from body frame to inertial frame** ← **gyros**

$\boldsymbol{a}^b$ – **inertial accelerations measured by accelerometers in body frame**

$\ddot{\boldsymbol{x}}^i$ – **kinematic accelerations obtained from GNSS-derived positions, *x*, in *i*-frame**

$$\mathbf{C}_i^n = \begin{pmatrix} -\sin\phi\cos(\lambda+\omega_E t) & -\sin\phi\sin(\lambda+\omega_E t) & \cos\phi \\ -\sin(\lambda+\omega_E t) & \cos(\lambda+\omega_E t) & 0 \\ -\cos\phi\cos(\lambda+\omega_E t) & -\cos\phi\sin(\lambda+\omega_E t) & -\sin\phi \end{pmatrix}$$

– **transformation obtained from GNSS-derived positions, $\phi$, $\lambda$**

– **lever-arm effects are assumed to be applied**

# Moving-Base Gravimetry – Stabilized Mechanization

**Gravitational Vector in *n*-frame** ←

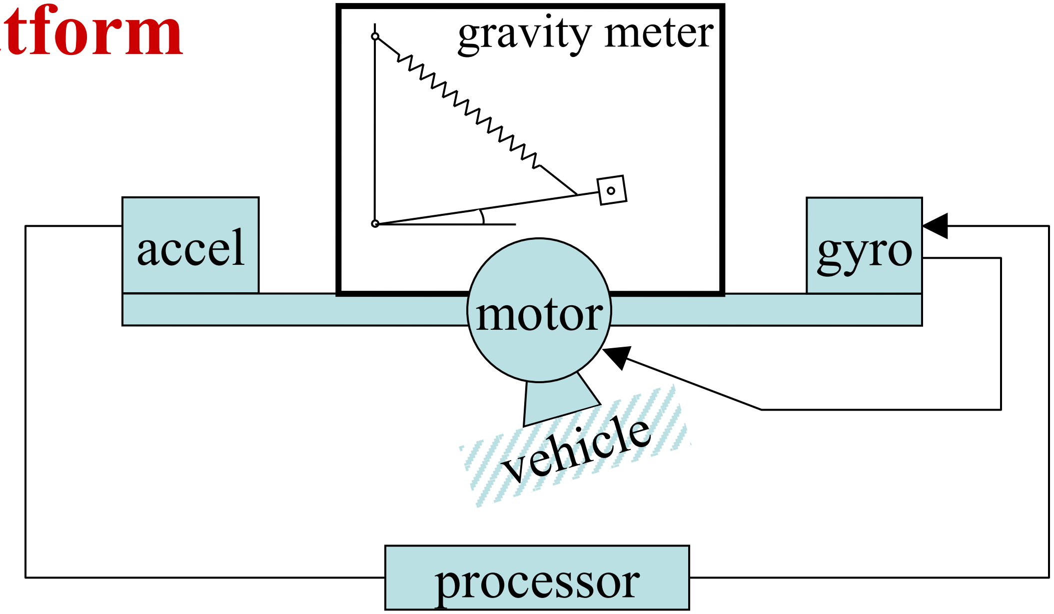
$$\mathbf{g}^n = \mathbf{C}_i^n \ddot{\mathbf{x}}^i - \mathbf{C}_p^n \mathbf{a}^p$$

- gyros → platform (*p*-) frame
- GNSS → transformation from inertial frame to *n*-frame
- tilt of platform relative to *n*-frame
- accelerometers: vertical plus 2 horizontal

- **Inertial accelerations, $\mathbf{a}^p$, from accelerometers in platform frame**
- **Mechanizations**
  - **two-axis damped platform** – level ($n'$-frame) alignment using gyro-driven gimballed platform in the short term and **mean zero output** of horizontal accelerometers in the long term
    - ◦ adequate for benign dynamics
  - **Schuler-tuned inertial stabilized platform** – alignment to ***n*-frame** based on inertial /GNSS navigation solution and gyro-driven platform stabilization
    - ◦ ideally, $\mathbf{C}_p^n = \mathbf{I}$; but note, *n*-frame differs from $n'$-frame by deflection of the vertical
    - ◦ better for more dynamic environments

# Two-Axis Stabilized Platform

- **Schematic for one axis**




  - **gryoscope maintains direction in space, and commands torque motor to correct deviation of platform orientation due to non-level vehicle**
  - **horizontal accelerometer, through processor, ensures that gryoscope reference direction is precessed to account for Earth rotation and curvature**
    - **zero acceleration implies level orientation (without horizontal specific forces!)**
    - **ad hoc damping of platform by processor**
    - **corrects gyro drift, but is subject to accelerometer bias**

- **Schuler-tuned three-axis stabilization: more accurate IMUs and *n*-frame stabilization (using navigation solution velocity in *n*-frame)**

# Scalar Moving-Base Gravimetry

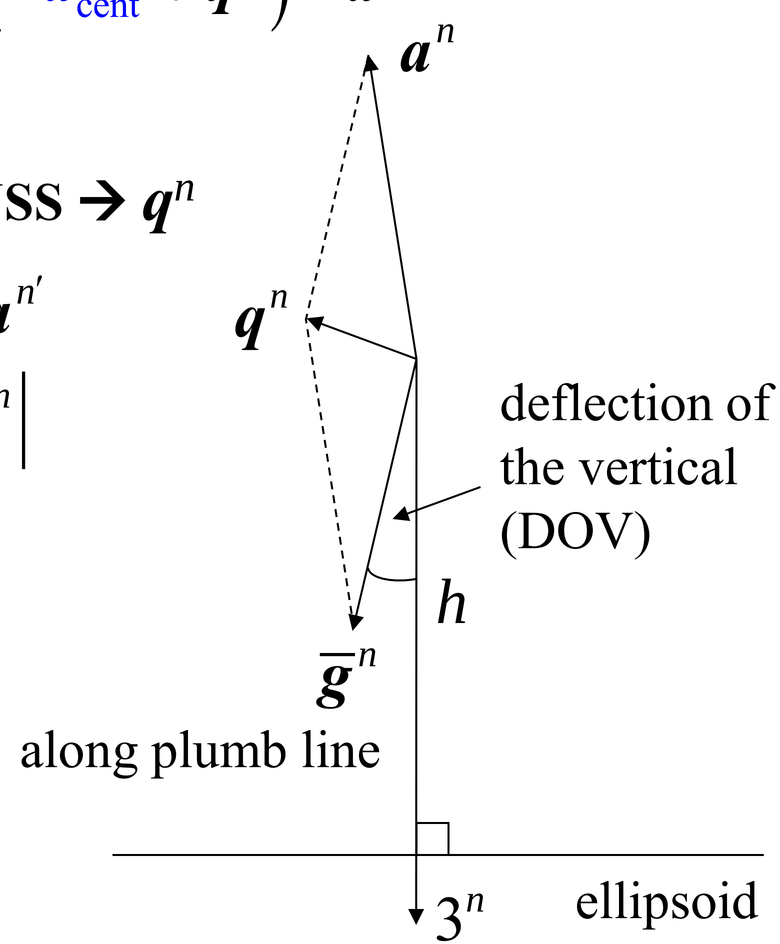
- **Determine the magnitude of *gravity* – the plumb line component**
  - **consistent with ground-based measurement (recall gravimeter is leveled)**
  - **gravitation vector:** $\boldsymbol{g}^n = C_i^n \ddot{\boldsymbol{x}}^i - C_b^n \boldsymbol{a}^b = \left(-\boldsymbol{a}_{\text{cent}}^n + \boldsymbol{q}^n\right) - \boldsymbol{a}^n$
  - **gravity vector:** $\bar{\boldsymbol{g}}^n = \boldsymbol{g}^n + \boldsymbol{a}_{\text{cent}}^n$

$$\boxed{\bar{\boldsymbol{g}}^n = \boldsymbol{q}^n - \boldsymbol{a}^n}$$

**GNSS → $\boldsymbol{q}^n$**

  - **this holds in any frame! e.g.,** $\bar{\boldsymbol{g}}^{n'} = \boldsymbol{q}^{n'} - \boldsymbol{a}^{n'}$
  - **note: straight and level flight** $\rightarrow \left|\boldsymbol{q}^n\right| \ll \left|\boldsymbol{a}^n\right|$
  - **thus:** $\bar{g} = \left|\bar{\boldsymbol{g}}^n\right|$**, but note:** $\bar{g} \neq \bar{g}_3^n$
    - ***n*-frame mechanization does not account for the deflection of the vertical**
  - **since $n'$-frame is aligned to plumb line,**

$$\bar{g} = \bar{g}_3^{n'}$$



# Unconstrained Scalar Gravimetry

- **One Option:** $\bar{g} = \left|\bar{\boldsymbol{g}}^n\right| = \left|\boldsymbol{q}^n - \boldsymbol{a}^n\right|$
  - **unconstrained in the sense that the frame for vectors is arbitrary (*n*-frame is used for illustration)**
  - **also known as strapdown inertial scalar gravimetry (SISG)**
  - $\boldsymbol{q}^n = \mathrm{C}_i^n \ddot{\boldsymbol{x}}^i + \boldsymbol{a}_{\text{cent}}^n$ **obtained exclusively from GNSS**
  - $\boldsymbol{a}^n = \mathrm{C}_b^n \boldsymbol{a}^b$ **requires orientation of *b*-frame (relative to *n*-frame)**
- **Requires comparable accuracy in all accelerometers and precision gyros if platform is arbitrary (e.g., strapdown)**
- **Calgary group demonstrated good results (e.g., Glennie and Schwarz 1999); see also (Czompo and Ferguson 1995)**

# Rotation-Invariant Scalar Gravimetry (RISG)

- **Another option to get $\bar{g}$: based on total specific force from gravimeter and orthogonal accelerometers**

$$a^2 = \left(a_1^p\right)^2 + \left(a_2^p\right)^2 + \left(a_3^p\right)^2 \quad \left( = \left(a_1^{n'}\right)^2 + \left(a_2^{n'}\right)^2 + \left(a_3^{n'}\right)^2 \right)$$

- **Then** $$a_3^{n'} = \sqrt{a^2 - \left(a_1^{n'}\right)^2 - \left(a_2^{n'}\right)^2}$$

$$= \sqrt{a^2 - \left(q_1^{n'} - \bar{g}_1^{n'}\right)^2 - \left(q_2^{n'} - \bar{g}_2^{n'}\right)^2}$$

$$= \sqrt{a^2 - \left(q_1^{n'}\right)^2 - \left(q_2^{n'}\right)^2} \qquad \textbf{since} \quad \bar{g}_1^{n'} = 0 = \bar{g}_2^{n'}$$

$$\bar{g} \equiv \bar{g}_3^{n'} \approx q_3^n - \sqrt{a^2 - \left(q_1^n\right)^2 - \left(q_2^n\right)^2}$$ **neglecting the DOV, $q^{n'} = q^n$**

- **Platform orientation is not specifically needed for $a^2$**
  - **however, errors in $q^n$, being squared, tend to bias the result (Olesen 2003)**

# RISG Approach in More Detail (1)

- **Define Earth-fixed velocity vector in the *n*-frame**

$$\boldsymbol{v}^n = \mathbf{C}_e^n \dot{\boldsymbol{x}}^e = \begin{pmatrix} v_N \\ v_E \\ v_D \end{pmatrix} = \begin{pmatrix} \dot{\phi}(M+h) \\ \dot{\lambda}(N+h)\cos\phi \\ -\dot{h} \end{pmatrix} \qquad \mathbf{C}_e^n = \begin{pmatrix} -\sin\phi\cos\lambda & -\sin\phi\sin\lambda & \cos\phi \\ -\sin\lambda & \cos\lambda & 0 \\ -\cos\phi\cos\lambda & -\cos\phi\sin\lambda & -\sin\phi \end{pmatrix}$$

- **It can be shown (Appendix A) that**

$$\boldsymbol{q}^n = \frac{d\boldsymbol{v}^n}{dt} + \left(\boldsymbol{\Omega}_{ie}^n + \boldsymbol{\Omega}_{in}^n\right)\boldsymbol{v}^n \qquad \boldsymbol{\Omega}_{ie}^n + \boldsymbol{\Omega}_{in}^n = \begin{pmatrix} 0 & \left(\dot{\lambda}+2\omega_e\right)\sin\phi & -\dot{\phi} \\ -\left(\dot{\lambda}+2\omega_e\right)\sin\phi & 0 & -\left(\dot{\lambda}+2\omega_e\right)\cos\phi \\ \dot{\phi} & \left(\dot{\lambda}+2\omega_e\right)\cos\phi & 0 \end{pmatrix}$$

- **Thus, strictly from GNSS, the third component is**

$$q_3^n = -\ddot{h} + 2\omega_e v_E \cos\phi + \frac{v_N^2}{M+h} + \frac{v_E^2}{N+h}$$

# RISG Approach in More Detail (2)

- **Inertial acceleration includes tilt error if platform is not level**

$$a_3^{n'} = \left( \mathbf{C}_p^{n'} \boldsymbol{a}^p \right)_3 = a_3^p - \delta a_{\text{tilt}} \quad \left( = \sqrt{a^2 - \left(q_1^{n'}\right)^2 - \left(q_2^{n'}\right)^2} \right)$$

- **Kinematic acceleration (from GNSS) includes neglect of DOV**

$$q_3^{n'} = \left( \mathbf{C}_n^{n'} \boldsymbol{q}^n \right)_3 = q_3^n - \delta q_{\text{DOV}}$$

- **Third component of $\overline{\boldsymbol{g}}^{n'} = \boldsymbol{q}^{n'} - \boldsymbol{a}^{n'}$, along plumb line,**

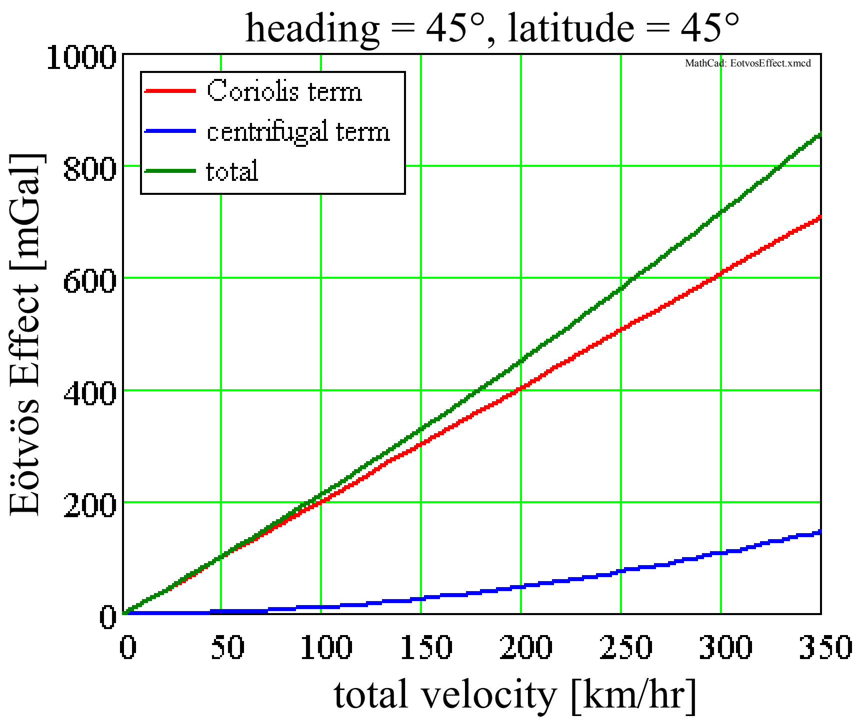
$$\overline{g} = \underbrace{-a_3^p}_{\text{gravimeter}} - \ddot{h} + \underbrace{2\omega_E v_E \cos\phi + \frac{v_N^2}{M+h} + \frac{v_E^2}{(N+h)}}_{\delta g_{\text{Eötvös}}} + \delta a_{\text{tilt}} - \delta q_{\text{DOV}}$$

# Eötvös Effect

Loránd Eötvös
1848 - 1919

- **Exact in *n*-frame (note: $v_{N,E}$ at altitude!)**

$$\delta g_{\text{Eötvös}} = 2\omega_E v_E \cos\phi + \frac{v_N^2}{M+h} + \frac{v_E^2}{(N+h)}$$




- **Approximations**
  - **spherical:**

$$\delta g_{\text{Eötvös}} \approx 2\omega_E v_E \cos\phi + \frac{v^2}{R+h}$$

  - **first-order ellipsoidal (Harlan 1968):**

$$\delta g_{\text{Eötvös}} \approx \frac{v^2}{a}\left(1+\frac{h}{a} - f\left(1-\cos^2\phi\left(3-2\sin^2\alpha\right)\right)\right) + 2v\omega_E \sin\alpha\cos\phi\left(1+\frac{h}{a}\right) + O\left(f^2\right)$$

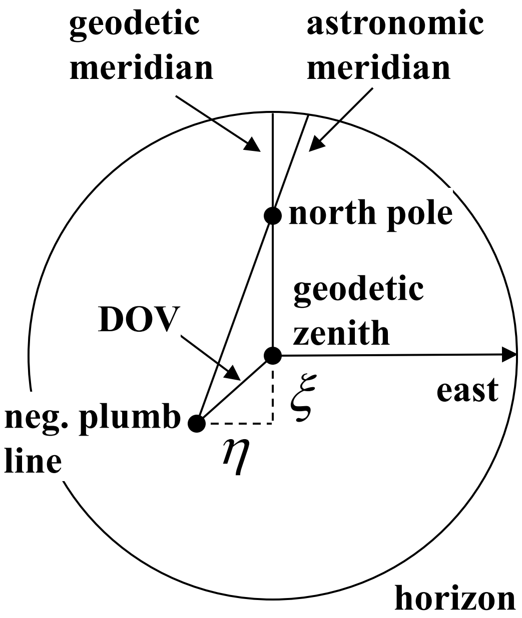
***a* = ellipsoid semi-major axis; $\alpha$ = azimuth; *v* = ground speed!**

# DOV Error in Kinematic Acceleration

- **DOV components define the small angles between the *n*- and *n′*-frames**

$$\mathbf{C}_n^{n'} = \mathbf{R}_1(-\eta)\mathbf{R}_2(\xi) = \begin{pmatrix} 1 & 0 & -\xi \\ 0 & 1 & -\eta \\ \xi & \eta & 1 \end{pmatrix}$$

  – **ignore rotation about 3-axis**




- **DOV error**

$$\delta q_{\text{DOV}} = q_3^n - \left(\mathbf{C}_n^{n'}\boldsymbol{q}^n\right)_3 = -\xi q_1^n - \eta q_2^n$$

- **Assume rms(DOV) = 10 arcsec, $q_{1,2} = 10^4$ mGal**

$$\text{rms}\left(\delta q_{\text{DOV}}\right) = 0.7 \text{ mGal}$$

- **This error is correctable, e.g., using EGM2008 deflection model**

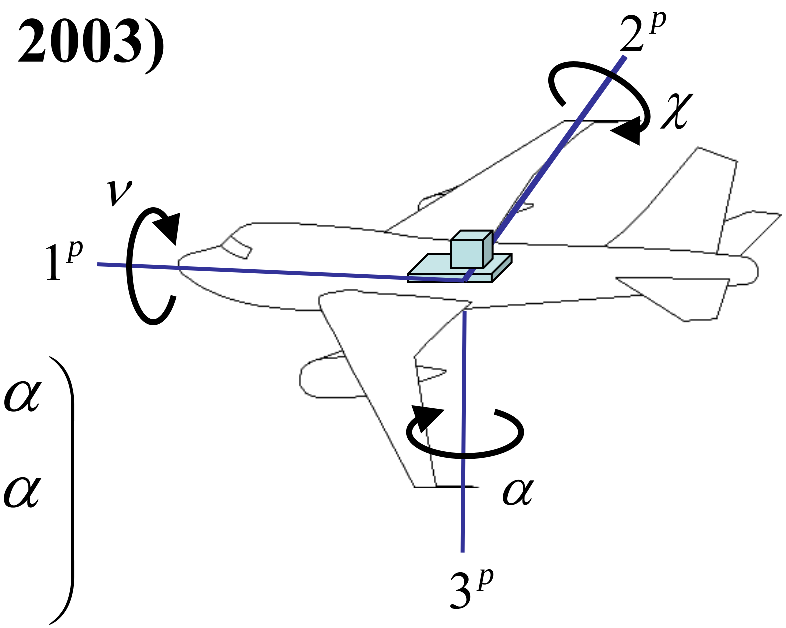
# Tilt Error (1)

- **One way to compute tilt error (p. 3.27)**

$$\delta a_{\text{tilt}} = a_3^p - \sqrt{a^2 - \left(q_1^n\right)^2 - \left(q_2^n\right)^2} \qquad \textbf{(neglecting DOV)}$$

  - **random errors in $q_1^n, q_2^n$ are squared and can cause bias (rectification error)**

- **Better model for the tilt error (Olesen 2003)**




  - **define orientation angles, $\nu$, $\chi$, $\alpha$**
  - **assume $\nu$, $\chi$ are small; $\alpha$ is arbitrary**

$$\mathbf{C}_p^n \approx \begin{pmatrix} \cos\alpha & -\sin\alpha & \chi\cos\alpha + \nu\sin\alpha \\ \sin\alpha & \cos\alpha & \chi\sin\alpha - \nu\cos\alpha \\ -\chi & \nu & 1 \end{pmatrix}$$

  - **thus, approximate platform stabilization is required!**

# Tilt Error (2)

- **From** $\bar{\boldsymbol{g}}^n = \boldsymbol{q}^n - \mathbf{C}_p^n \boldsymbol{a}^p$,

$$\begin{pmatrix} \bar{g}_1^n \\ \bar{g}_2^n \end{pmatrix} = \begin{pmatrix} q_1^n - a_1^p \cos\alpha + a_2^p \sin\alpha - (\chi\cos\alpha + \nu\sin\alpha) a_3^p \\ q_2^n - a_1^p \sin\alpha - a_2^p \cos\alpha - (\chi\sin\alpha - \nu\cos\alpha) a_3^p \end{pmatrix}$$

- **If** $\bar{g}_1^n = 0 = \bar{g}_2^n$ **(neglecting DOV is second-order effect on tilt error)**

$$\chi = \frac{1}{a_3^p}\left(q_1^n \cos\alpha + q_2^n \sin\alpha - a_1^p\right)$$

$$\nu = \frac{1}{a_3^p}\left(q_1^n \sin\alpha - q_2^n \cos\alpha + a_2^p\right)$$

**Tilt angles are computed from accelerometers, GNSS, and azimuth**

- **Third component of** $\delta\boldsymbol{a}_{\text{tilt}} = \boldsymbol{a}^p - \mathbf{C}_p^n \boldsymbol{a}^p$

$$\delta a_{\text{tilt}} \approx a_1^p \chi - a_2^p \nu$$

**(first-order approximation)**

# Tilt Error (3)

- **Tilt error can be written as**

$$\delta a_{\text{tilt}} \approx \frac{q_1^w - a_1^p}{a_3^p} a_1^p + \frac{q_2^w - a_2^p}{a_3^p} a_2^p \qquad \begin{aligned} q_1^w &= q_1^n \cos\alpha + q_2^n \sin\alpha \\ q_2^w &= -q_1^n \sin\alpha + q_2^n \cos\alpha \end{aligned}$$

  – **where $q_1^w, q_2^w$ are kinematic accelerations in the "wander-azimuth" frame**

- **Taking differentials of the specific force components,**

$$\delta\left(\delta a_{\text{tilt}}\right) = \frac{\left(q_1^w - a_1^p\right) - a_1^p}{a_3^p} \delta a_1^p + \frac{\left(q_2^w - a_2^p\right) - a_2^p}{a_3^p} \delta a_2^p - \frac{\left(q_1^w - a_1^p\right) a_1^p + \left(q_2^w - a_2^p\right) a_2^p}{\left(a_3^p\right)^2} \delta a_3^p$$

- **Assume** $\left(q_{1,2}^w - a_{1,2}^p\right) = a_3^p \cdot O\left(\nu,\chi\right) \square\ 10^4 \text{ mGal}, \quad a_3^p \approx 10^6 \text{ mGal}, \quad a_{1,2}^p = O\left(10^4 \text{ mGal}\right)$

  – **error in vertical accelerometer (gravimeter) is second-order for tilt correction**

  – **error in horizontal accels. can be 100 worse than tilt correction accuracy**

- **In practice, tilt correction is subjected to appropriate filters; see (Olesen 2003)**

# Lever-Arm Effect

- **Apply to kinematic acceleration derived from GNSS tracking**
  - **assume gravimeter is at center of mass of vehicle**
- **In the inertial frame:** $\boldsymbol{x}^i_{\text{antenna}} = \boldsymbol{x}^i_{\text{gravimeter}} + \boldsymbol{b}^i$
  - **where** $\boldsymbol{b}^i = \mathrm{C}^i_b \boldsymbol{b}^b$, $\boldsymbol{b}^b$ **= fixed antenna offset relative to gravimeter**

    $\mathrm{C}^i_b$ **– obtained from gyro data**
- **Numerical differentiation:** $\ddot{\boldsymbol{x}}^i_{\text{gravimeter}} = \dfrac{d^2}{dt^2}\left(\boldsymbol{x}^i_{\text{antenna}} - \boldsymbol{b}^i\right)$
  - **extract relevant component in particular frame**
    - ***n*-frame: vertical component of** $\mathrm{C}^n_i \ddot{\boldsymbol{x}}^i_{\text{gravimeter}}$ ➔ $\ddot{h}$

# Scalar Gravimetry Equation

- **Final equation for the gravity anomaly at altitude point, $P_a'$**

$$\Delta g_{P_a'} = f - \ddot{h} + \delta g_{\text{Eötvös}} + \delta a_{\text{tilt}} - \delta q_{\text{DOV}} - \gamma_{Q_a'} - \left( f_0 - g_0 \right)$$

  - **where $f = -a_3^p$ is the gravity meter reading**
  - **where $\gamma_{Q_a'}$ is normal gravity at the normal height of $P_a'$ above the ellipsoid**
  - **where $f_0 - g_0$ is the initial offset of the gravimeter reading from true gravity**
  - **gravimeter measurement, $f$, includes various inherent instrument corrections**
  - **tilt error depends on accuracy of platform accelerometers**
  - **exact and sufficiently approximate formulas exist for normal gravity at $Q_a'$**
  - **accuracy in $\ddot{h}$ must be commensurate with gravimeter accuracy**

# IV. Overview of Airborne Gravimetry Systems

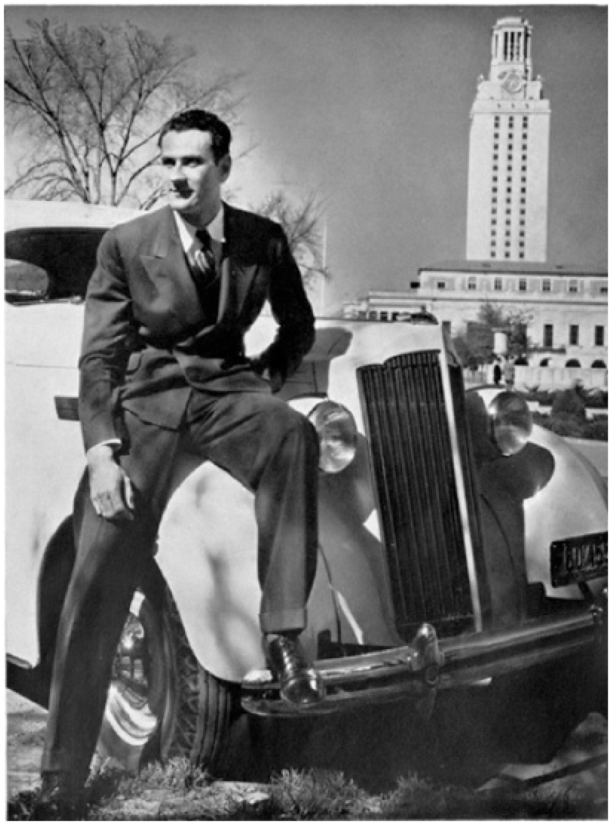
- **LaCoste/Romberg sea-air gravimeters**
- **Other Airborne gravimeters**

# Airborne Gravimeters

- **Instrumentation overview of scalar airborne gravimetry**
  - **LaCoste-Romberg instruments dominate the field**
  - **many other instrument types in operation or being tested**
- **All gravimeters are single-axis accelerometers**
  - **mechanical spring accelerometers (vertical spring, horizontal beam)**
    - **manual or automatic (force-rebalance) nulling**
  - **torsion wire (horizontal beam, no nulling)**
  - **electromagnetic spring (force-rebalance)**
  - **vibrating string accelerometers**

# Lucien J.B. LaCoste (1908-1995)

**“The gravity meters Lucien B. LaCoste invented revolutionized geodesy and gave scientists the ability to precisely measure variations in Earth's gravity from land, water, and space” J.C. Harrison (1996)***



At University of Texas, Austin

- **Inventor**
- **Scientist**
- **Teacher**
- **Entrepreneur**

*https://web.archive.org/web/20080527061634/http://www.agu.org/sci_soc/lacoste.html
*Earth in Space Vol. 8, No. 9, May 1996, pp. 12-13. © 1996 American Geophysical Union*;
see also (Harrison 1995).

# LaCoste-Romberg Air-Sea Model S Gravimeter

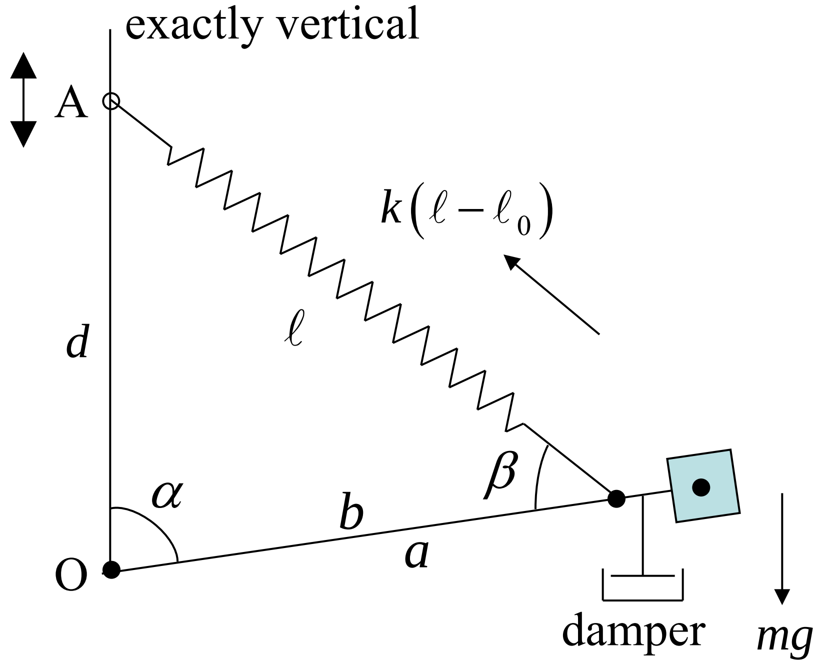
- **Beam is in equilibrium if torque(spring) = torque($mg$)**

$$k\left(\ell-\ell_0\right)b\sin\beta = mga\sin\alpha$$

   - **law of sines:** $\ell\sin\beta = d\sin\alpha$
   - **zero-length spring:** $\ell_0 = 0$

$\Rightarrow \quad kbd = mga$

   - **independent of $\alpha$ → equilibrium at any beam position for a given $g$**
   - **independent of $\ell$ → no change in spring length could accommodate a change in $g$**

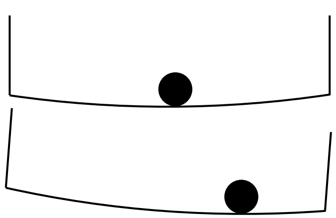


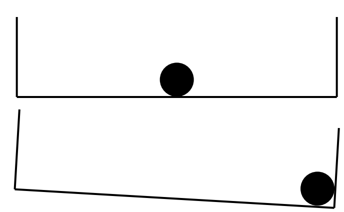

- **From (Valliant 1992):**

**finite sensitivity** | **infinite sensitivity**

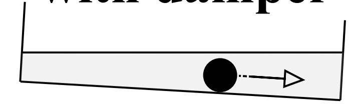
$g$

$g+\Delta g$





**with damper**



**→ measure beam velocity!**

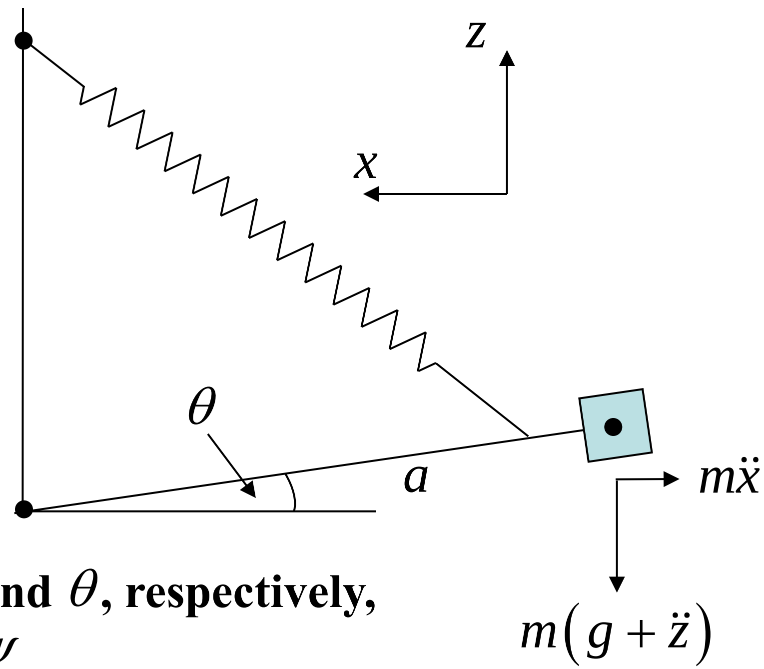
# (Inherent) Cross-Coupling Effect

- **Horizontal accelerations couple into the vertical movement of horizontal beam gravimeters that are not nulled**
  - **total torque on beam due to external accelerations:**

$$T = ma\left(\ddot{x}\sin\theta + \left(g + \ddot{z}\right)\cos\theta\right)$$

  - **it can be shown (LaCoste and Harrison 1961) that the cross-coupling error is**

$$\varepsilon = \frac{1}{2}\ddot{x}_1\theta_1\cos\psi$$



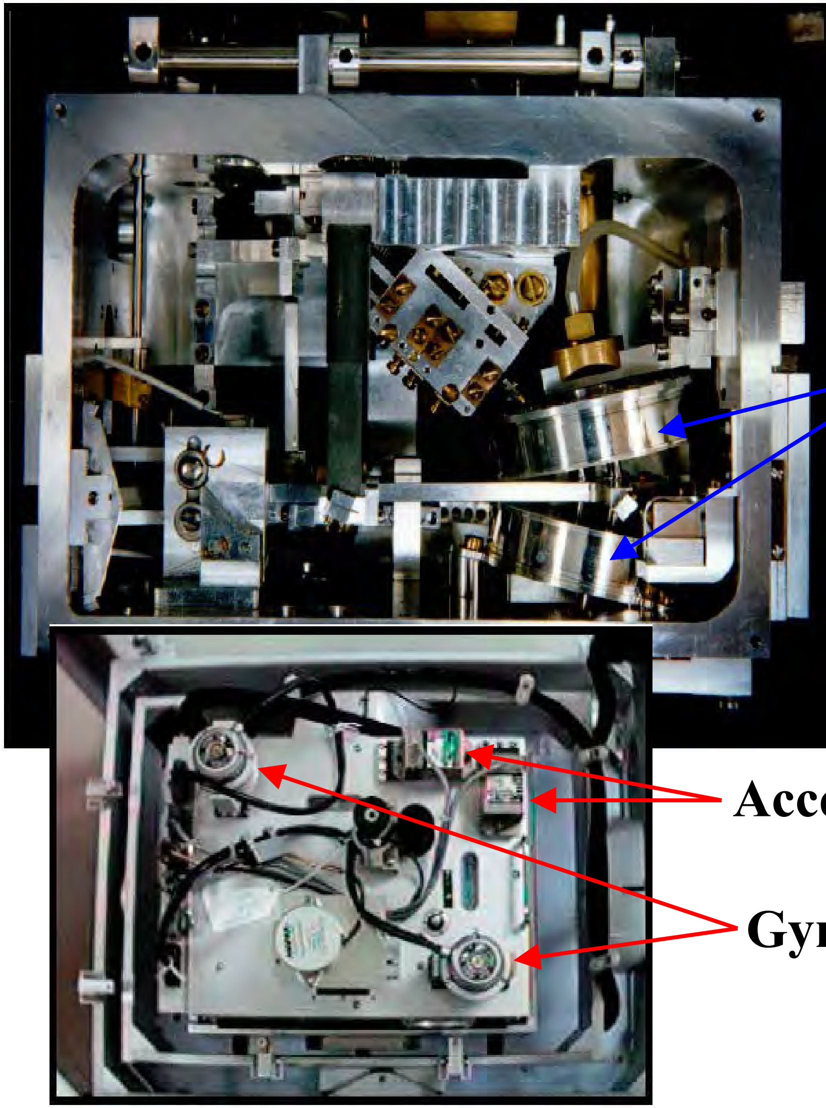

**where $\ddot{x}_1, \theta_1$ are amplitudes of components of $\ddot{x}$ and $\theta$, respectively, that have the same period and phase difference, $\psi$**

  - **e.g.,** $\theta_1 = 1°, \quad \ddot{x}_1 = 0.1 \text{ m/s}^2 \quad \Rightarrow \quad \varepsilon = 90 \text{ mGal}$

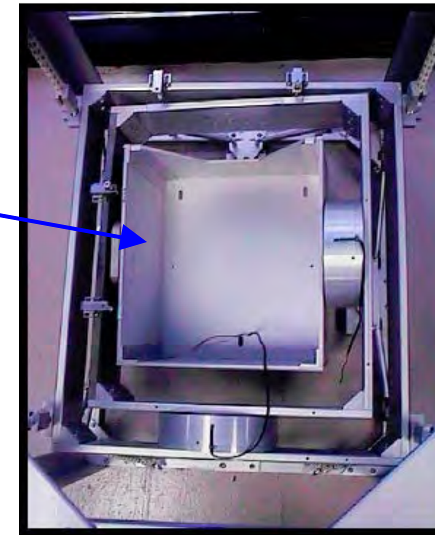
- **There is no cross-coupling effect for**
  - **force-rebalance gravimeters**
  - **vertical-spring gravimeters**

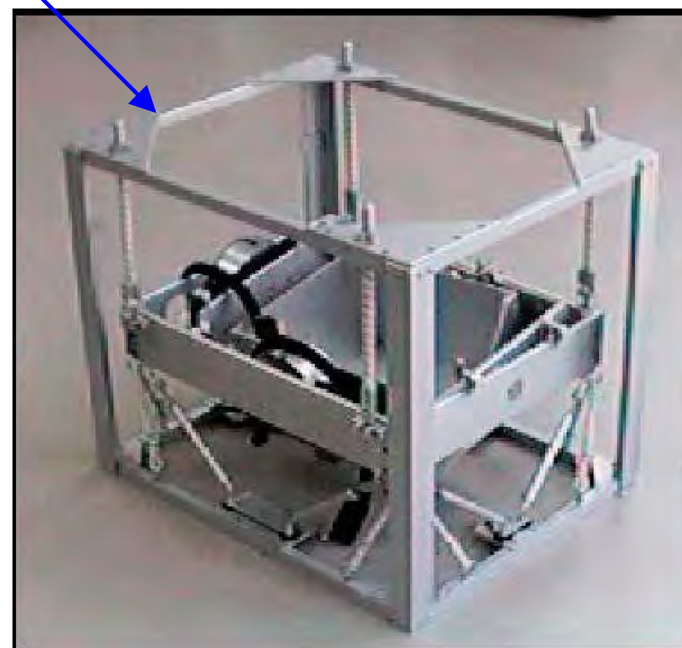
# LaCoste-Romberg Model S Sensor and Platform

Interior Side View




view of top lid

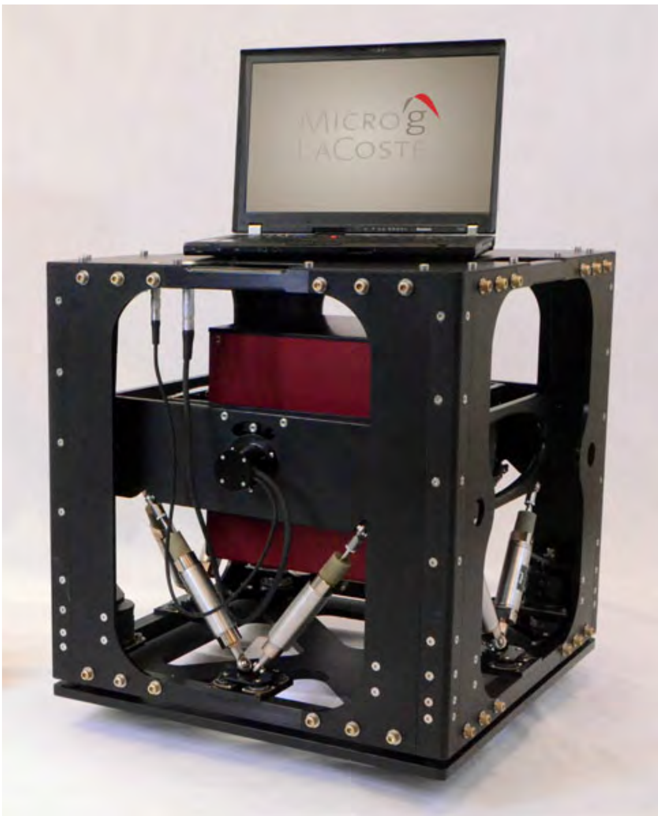



From: Instruction Manual, LaCoste and Romberg Model "S" Air-Sea Dynamic Gravimeter, 1998; with permission

# LaCoste/Romberg TAGS-6

## (Turn-key Airborne Gravity System)



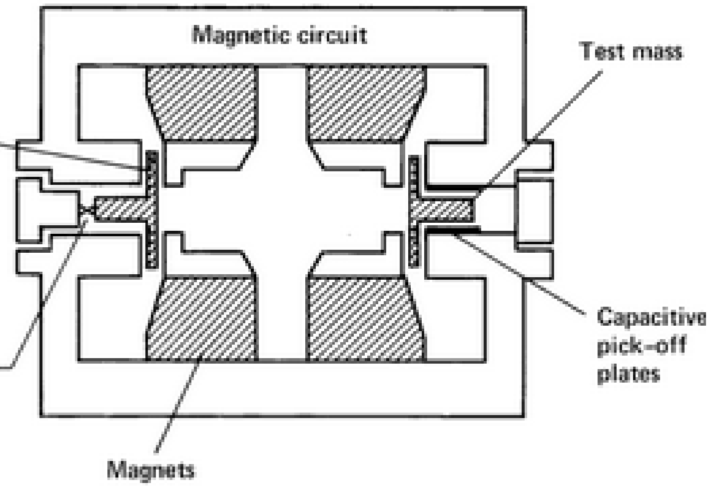
http://www.microglacoste.com/tags-6.php

**SPECIFICATIONS**

| COMPONENT | VARIABLE | SPECIFICATIONS |
|---|---|---|
| SENSOR | WORLDWIDE RANGE: | 20,000 milliGal |
| | DYNAMIC RANGE: | ±500,000 milliGal |
| | DRIFT: | 3 milliGal per month or less |
| | TEMPERATURE SETPOINT: | 45° to 65°C |
| STABILIZED PLATFORM | PLATFORM PITCH: | ± 25 degrees |
| | PLATFORM ROLL: | ± 35 degrees |
| | CONTROL: | |
| | Period | 4 to 4.5 Minutes |
| | Damping | 0.707 of critical |
| CONTROL SYSTEM | RECORDING RATE: | 20 Hz |
| | SERIAL OUTPUT: | RS-232 |
| | ADDITIONAL I/O: | Sensor Temperature |
| SYSTEM PERFORMANCE | DYNAMIC RANGE: | 25,000,000 |
| | STATIC REPEATABILITY: | 0.02 milliGal in 2 min |
| | DYNAMIC REPEATABILITY: | 0.75 milliGal in 2 min |
| MISCELLANEOUS | OPERATING TEMPERATURE: | 5° to 50°C |
| | STORAGE TEMPERATURE: | -10° to 50°C |
| | POWER EQUIPMENTS: | 75W @ 27°C Nominal<br>300W Peak<br>80-265VAC, 47 – 63Hz |
| | DIMENSIONS: | 58.4 x 53.3 x 55.9 cm<br>(not including electronics) |

# BGM-3 Gravimeter

- **Bell Aerospace (now Lockheed Martin)**
  - **Model XI pendulous force-rebalance accelerometer**
  - **current needed to keep test mass in null position is proportional to acceleration**




(Seiff and Knight 1992); see also (Bell and Watts 1986)

**Fugro ➔ WHOI**

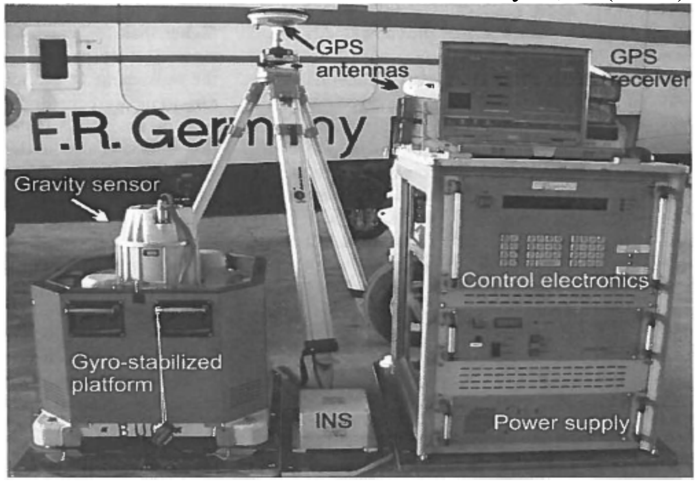
https://www.unols.org/sites/default/files/Gravimeter_Kinsey.pdf

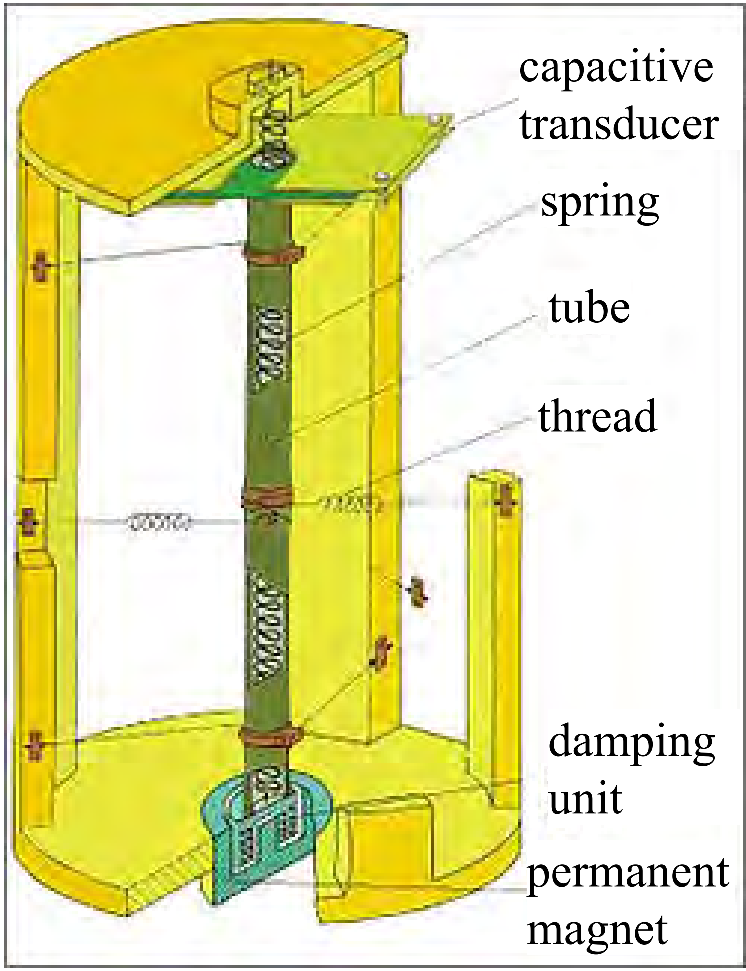
**Two BGM-3 gravimeters installed on the USCG ship *Healy***

# Sea Gravimeter KSS31, Bodenseewerk Geosystem GmbH

- Gravity sensor based on Askania vertical-spring gravimeter
  - Federal Institute for Geosciences and Natural Resources (BGR)
  - force rebalance feedback system
  - highly damped output, ~ 3 minute average
  - sensor on a gyro-stabilized platform
  - also used for fixed wing and helicopter gravimetry

Heyde, J. (2010)



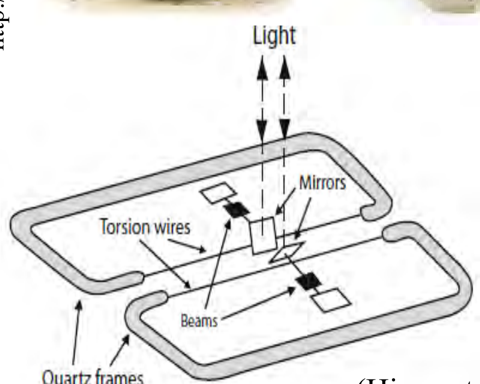

•http://www.bgr.bund.de/DE/Themen/MarineRohstoffforschung/Meeresforschung/Geraete/Gravimeter/gravimeter_inhalt.html
•http://www.bgr.bund.de/EN/Themen/GG_Geophysik/Aerogeophysik/Aerogravimetrie/aerogravimetrie_node_en.html

# Chekan-A Gravimeter

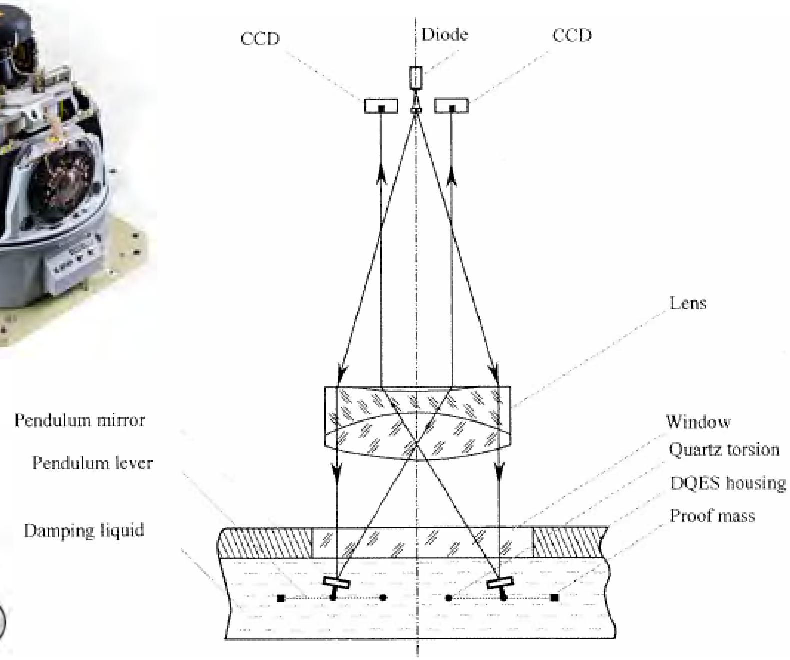
- **Air-Sea gravimeter; CSRI* Elektropribor, St. Petersburg, Russia**
  - gravity sensed by deflection of pendulum hinged on quartz torsion wire in viscous fluid
  - pendulum deflection: 0.3–1.5 ″/mGal; e.g., ±1° → ±10 Gal total range (0.36 ″/mGal)
  - cross-coupling effect minimized by double-beam reverted pendulums
  - evolutionary modifications: Chekan AM, "Shelf" (Krasnov et al. 2014)

*Central Scientific & Research Institute

http://www.gravionic.com/gravimetry.html




(Hinze et al. 2013)




(Krasnov et al. 2008)

**Installation in test aircraft**


(Stelkens-Kobsch 2005)

# Airborne Gravimetry on Airship Platform


(http://rosaerosystems.com/airships/obj17)

**Relative gravimeter Chekan in a cabin of AU**

- **test flight January 2014**
- **reported in IAG Commission 2 Travaux 2015**

# Airborne Gravimeter GT-2A

- **Gravimeter system designed by *Gravimetric Technologies* (Russia)**
  - **vertical accelerometer of axial design with a test mass on spring suspension**
  - **photoelectric position pickup**
  - **moving-coil force feedback transducer**
  - **three-axis gyro-stabilized platform**
  - **large dynamic range**

| | |
|---|---|
| Measurement range | 9.75 to 9.85 m/sec² |
| Dynamic range | > +/- 1,000 Gals |
| Drift per day (corrected) | < 0.1 mGals |
| RMS error in gravity anomaly estimation | |
| (static mode up to 12 hours on bench) | |
| RMS error | 0.6 mGals (+/- 1 LSD*) |
| Attitude limits | |
| roll | +/- 45° |
| pitch | +/- 45° |
| Operating temp | +5°C to +50°C |
| Power | |
| operating | 150 W at 27Vdc |
| standby | 50 W at 27Vdc |
| Weight (with base) | 153.5 kg |
| Dimensions console | 400 x 400 x 600 mm |
| Dimensions base | 600 x 300 mm |
| Service life | 30,000 hours |
| Error in gravity anomaly estimation (RMS) | |
| 0.01 Hz cut-off | 0.6 mGals (+/- 1 LSD*) |

*Least Significant Digit

Specifications subject to change

(Gabell et al. 2004)

http://eongeosciences.com/wp-content/uploads/2015/01/GT_2A.pdf
(Canadian Micro Gravity)

# Sander Geophysics Ltd. AirGrav System

- "Purpose-built" airborne gravimeter designed for airborne environment, not modified sea gravimeter
  - Honeywell inertial navigation grade accelerometers (Annecchione et al. 2006, Sinkiewicz et al. 1997)
  - Schuler-tuned (three axis) inertially stabilized platform
  - three-accelerometer system; vertical accelerometer used as gravimeter
  - advertize gravimetry on topography-draped profiles
  - demonstrated success in vector gravimetry
  - comparison tests over Canadian Rockies between AirGrav and GT-1A (Studinger et al. 2008)


http://www.sgl.com/news/Sander%20Geophysics%20-%20Antarctica.pdf