# Converting GPS Height into NAVD 88 Elevation with the GEOID96 Geoid Height Model

by

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# Presentation

## A Review of Height Systems

- Ellipsoidal Height (GPS)
- Orthometric Height (MSL)
- Geoid Height

## Datums and Reference Systems

## The GEOID96 and G96SSS Geoid Models

## Results with the GEOID96 Procedure

## Conclusions

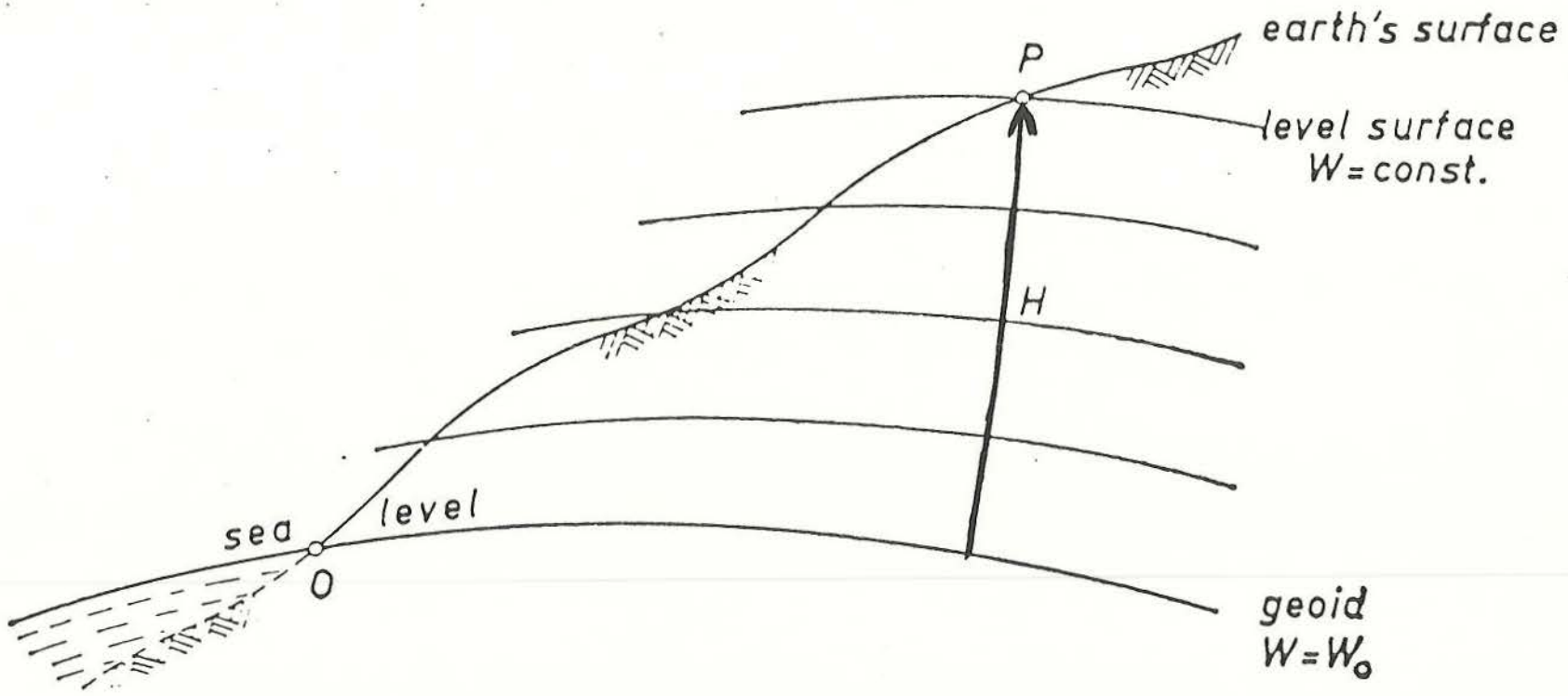
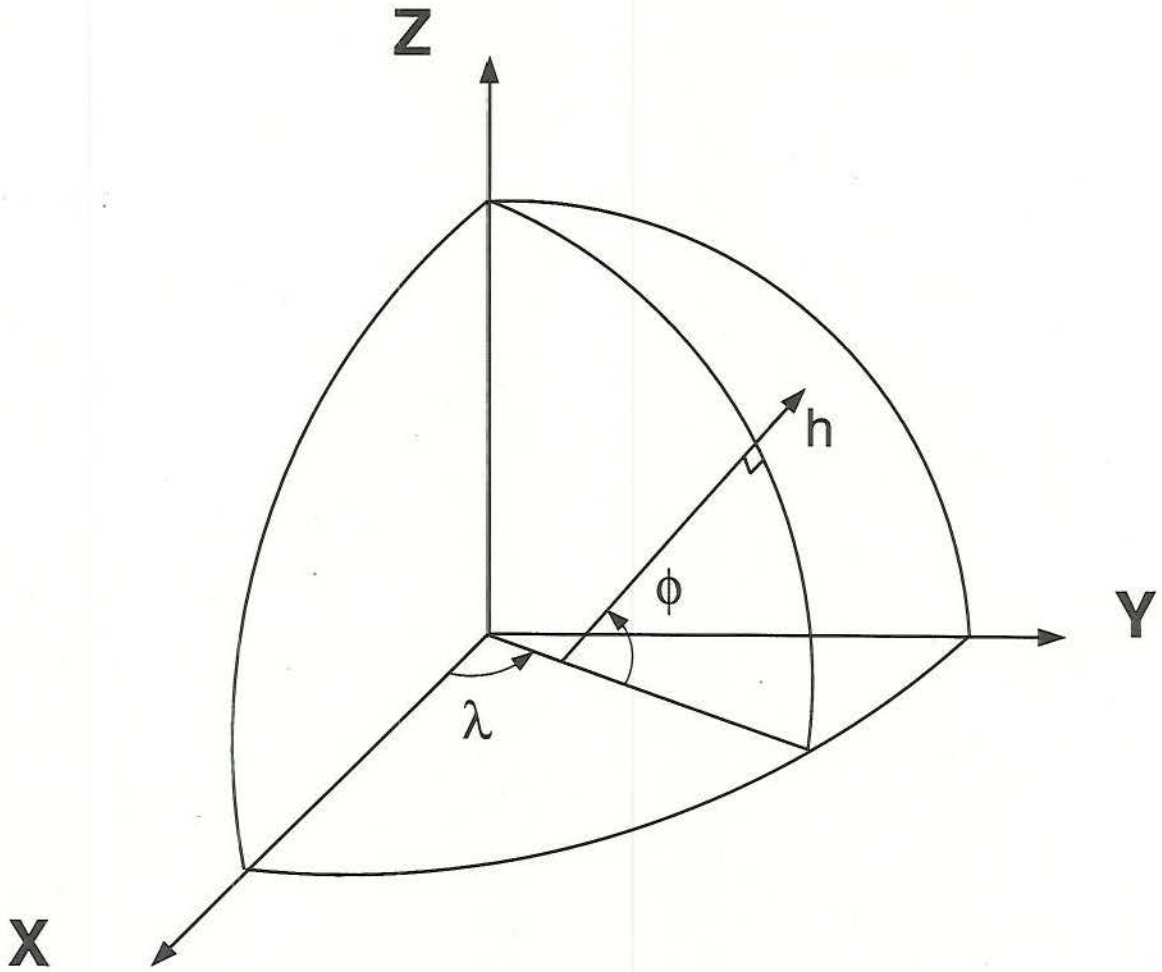
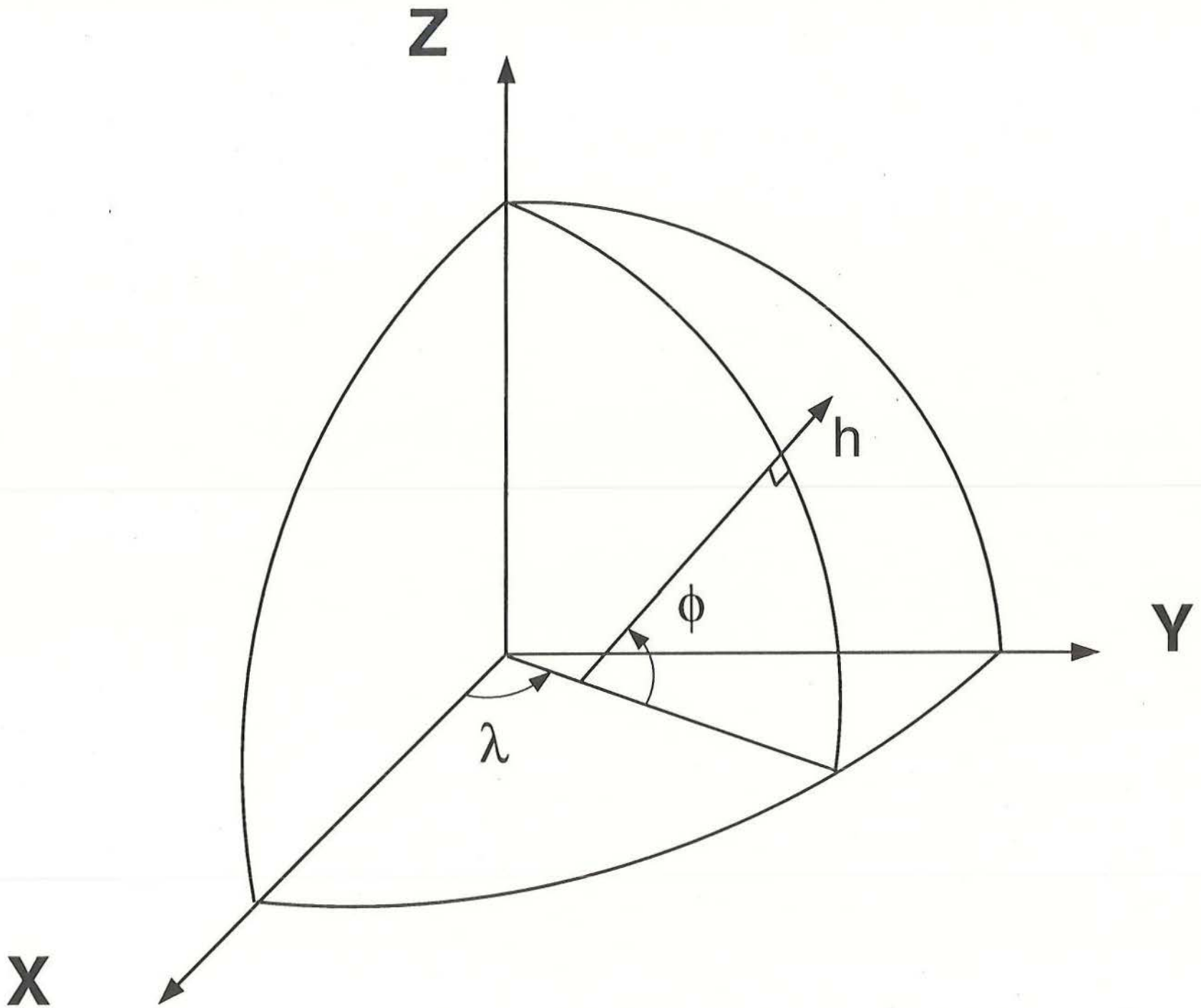
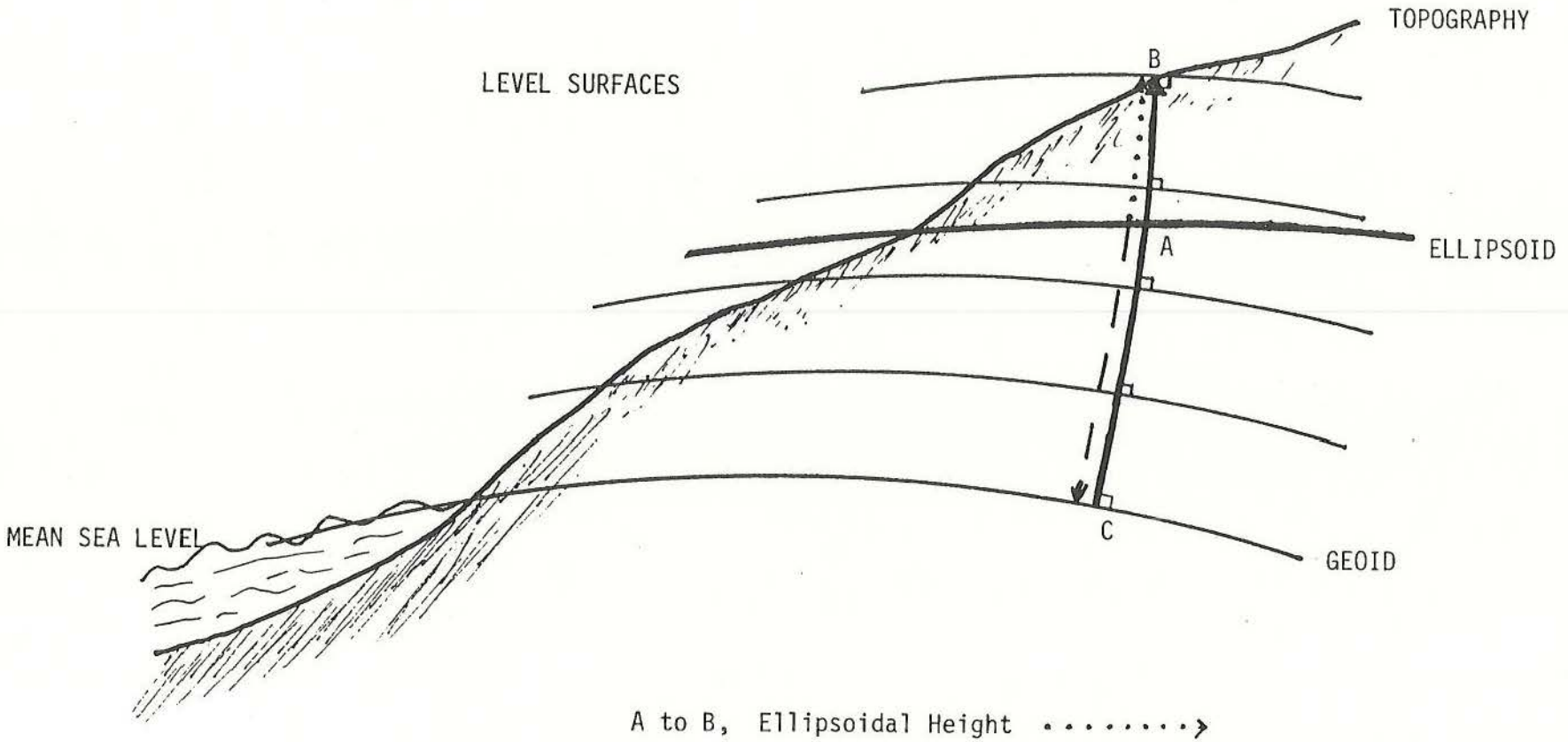



**FIGURE 2-8**

*The orthometric height H.*


Z
h
ϕ
Y
λ
X


Z
h
ϕ
Y
λ
X


TOPOGRAPHY
LEVEL SURFACES
B
A
ELLIPSOID
MEAN SEA LEVEL
C
GEOID
A to B, Ellipsoidal Height
A to C, Geoid Height
C to B, Orthometric Height

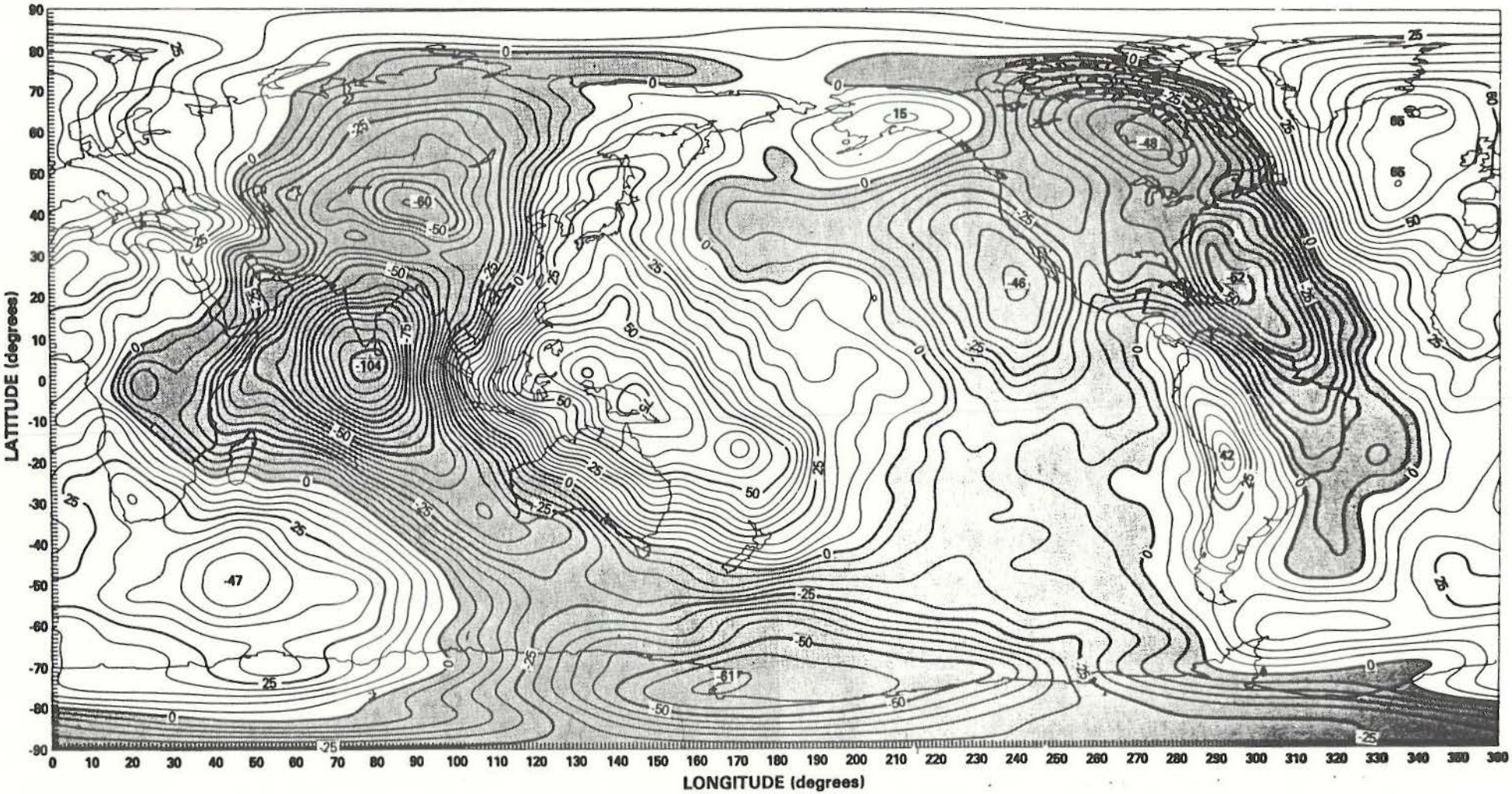
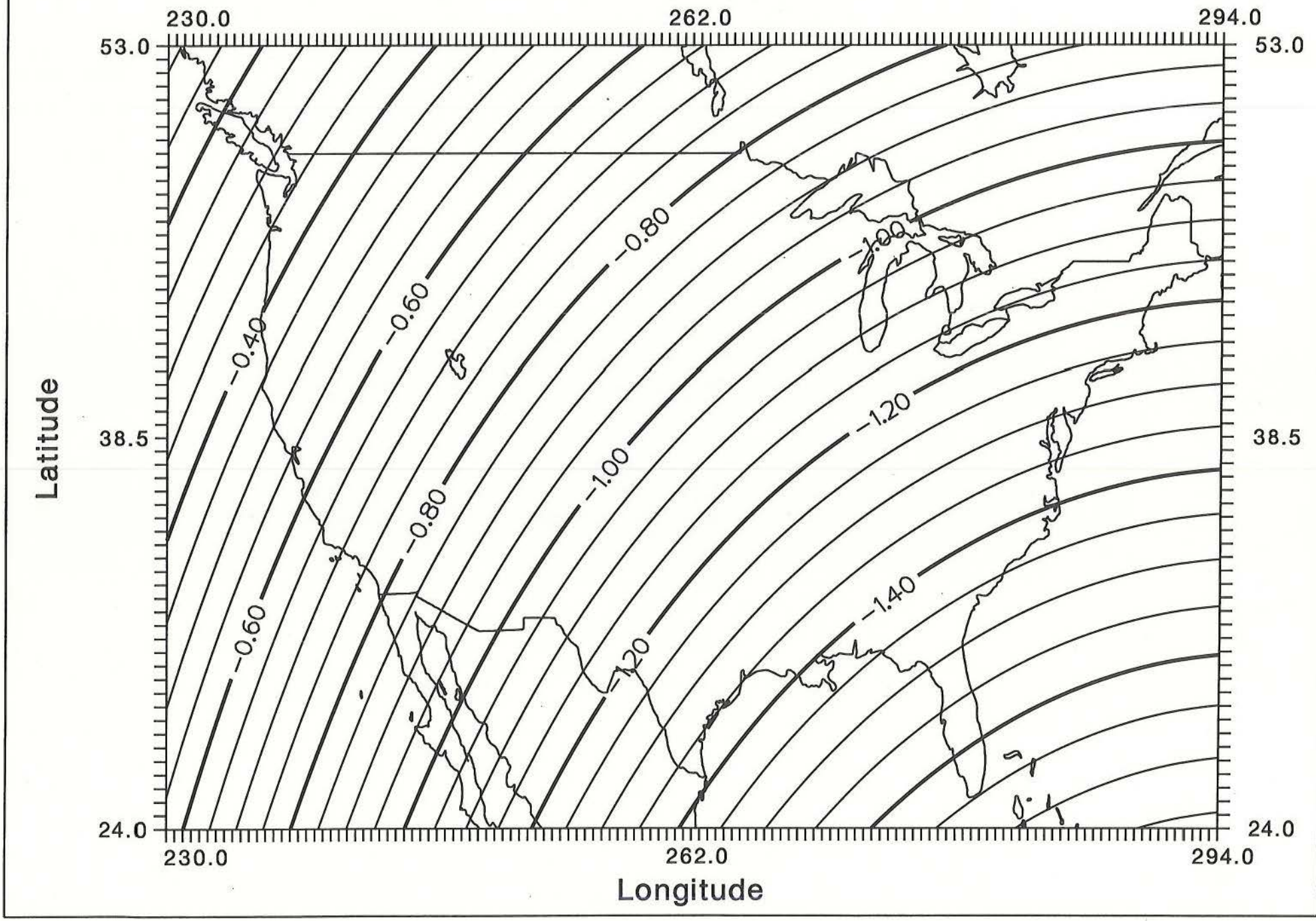
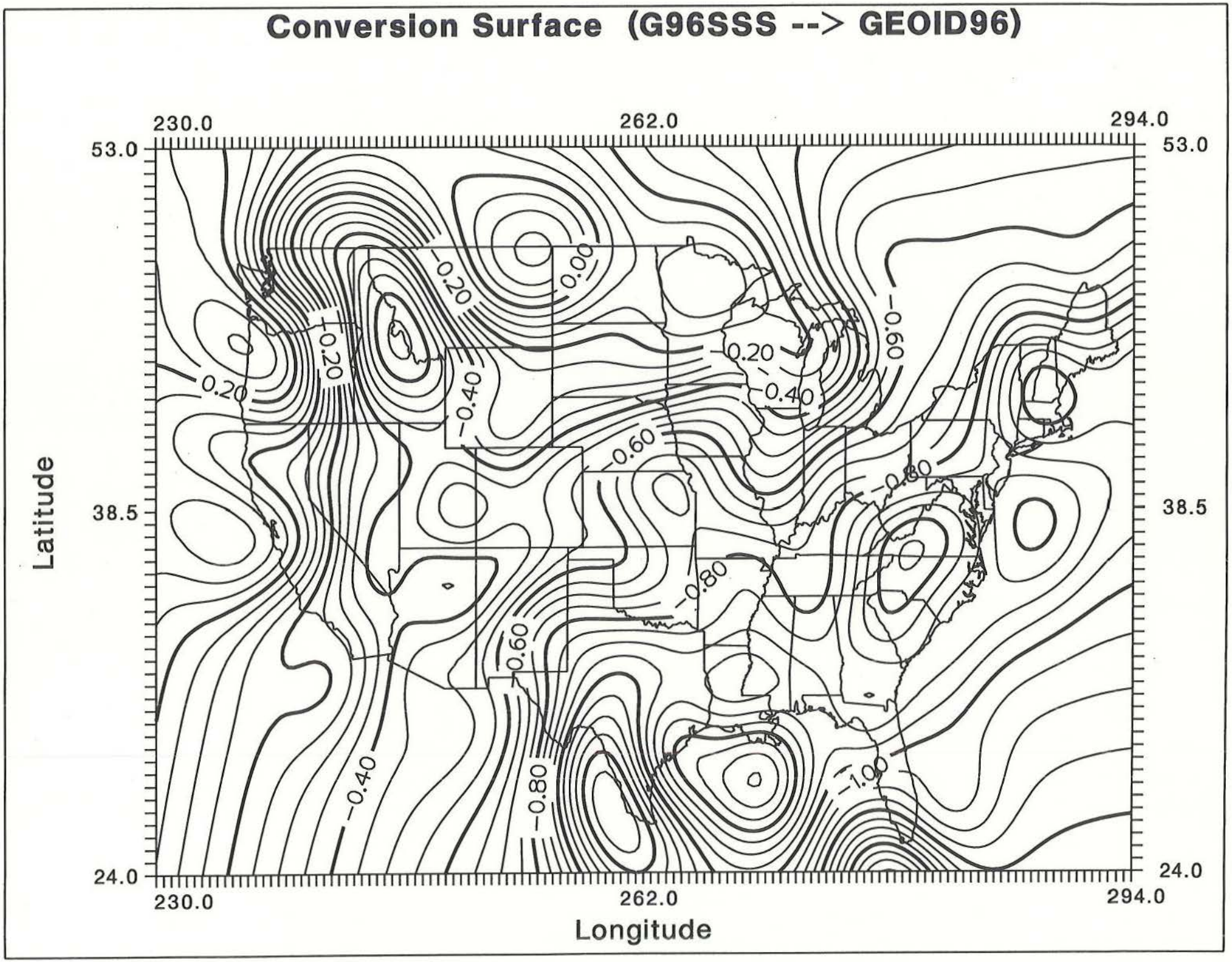



**FIGURE 3.** Geoid surface computed from the GEM 10B model (height in meters above the mean ellipsoid, f = 1/298.257).

# NAD83(86) to ITRF94(96) Ellipsoid Height



# Conversion Surface (G96SSS --> GEOID96)



# Datums and Reference Systems

## Theory

$$h = H + N$$

## Realization of a Datum

$$h_{83} + \delta h = (H_{88} + \delta H) + (N_{SSS} + \delta N)$$

bias

**Height datums can be inconsistent due to measurement error or definitional issues**

## Objective

$$h_{83} = H_{88} + N_{96}$$

**We desire a geoid model (GEOID96) to support direct conversion between NAD 83 GPS heights and NAVD 88 orthometric heights**

```
1        National Geodetic Survey,   Retrieval Date = JUNE 28, 1996
 KK1535 ***********************************************************************
 KK1535  DESIGNATION -  U 392
 KK1535  PID         -  KK1535
 KK1535  STATE/COUNTY-  CO/DENVER
 KK1535  USGS QUAD   -  COMMERCE CITY (1980)
 KK1535
 KK1535  HORZ DATUM  -  NAD 83 (1992)
 KK1535  VERT DATUM  -  NAVD 88
 KK1535
 KK1535  POSITION    -  39 47 49.04816(N)     104 52 54.36555(W)      ADJUSTED
 KK1535  92 minus 83 -       +00.00587              +00.00100        NADCON
 KK1535  83 minus 27 -       -00.04840              +01.91562        NADCON
 KK1535
 KK1535
 KK1535  HEIGHT      -      1596.695  (meters)     5238.49   (feet)  ADJUSTED
 KK1535  88 minus 29 -        +0.888                                  ADJUSTED
 KK1535  DY minus 88 -        -1.501                                  COMPUTED
 KK1535. (NOTE - For assistance in applying shifts see file readme.dat)
 KK1535 ***********************************************************************
 KK1535
 KK1535  LAPLACE CORR-          -5.66                                 DEFLEC93
 KK1535  GEOID HEIGHT-         -17.22                                 GEOID93
 KK1535  ELLIP HEIGHT-        1579.216
 KK1535  X           -  -1,260,596.431
 KK1535  Y           -  -4,743,743.854
 KK1535  Z           -   4,061,700.790
 KK1535  MODELED GRAV-     979,630.2                                 NAVD88
 KK1535
 KK1535
 KK1535  HORZ ORDER  -  FIRST
 KK1535  VERT ORDER  -  FIRST     CLASS 2
 KK1535  ELLP ORDER  -  THIRD     CLASS 1
 KK1535
 KK1535
 KK1535.The horizontal coordinates were established by GPS observations
 KK1535.and adjusted by the National Geodetic Survey in May 1996.
 KK1535
 KK1535.The orthometric height was determined by differential leveling
 KK1535.and adjusted by the National Geodetic Survey in June 1991.
 KK1535
 KK1535.The dynamic height is computed by dividing the NAVD 88
 KK1535.geopotential number by the normal gravity value computed on the
 KK1535.Geodetic Reference System of 1980 (GRS 80) ellipsoid at 45
 KK1535.degrees latitude (G = 980.6199 gals.).
 KK1535
 KK1535.The Laplace correction was computed from DEFLEC93 derived deflections.
 KK1535
 KK1535.The geoid height was determined by GEOID93.
 KK1535
 KK1535.The ellipsoidal height was determined by GPS observations
 KK1535.and referenced to NAD 83.
 KK1535
 KK1535.The X, Y, and Z were computed from the position and the ellipsoidal ht.
 KK1535
 KK1535.The modeled gravity was interpolated from observed gravity values.
 KK1535
 KK1535;                    North         East       Scale          Converg.
 KK1535;SPC CO C     -  1,715,764.06  3,173,717.93  1.00000963     +0 23 23.7 sFT
```

```
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 KK1535  MODELED GRAV-     979,630.2                                  NAVD88
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 KK1535
 KK1535  HORZ ORDER  -  FIRST
 KK1535  VERT ORDER  -  FIRST      CLASS 2
 KK1535  ELLP ORDER  -  THIRD      CLASS 1
 KK1535
```

-17.22

1596.695 = -17.479

25.9 cm !!

# Transformation from NAD 83 (86) to ITRF94(1996.0)

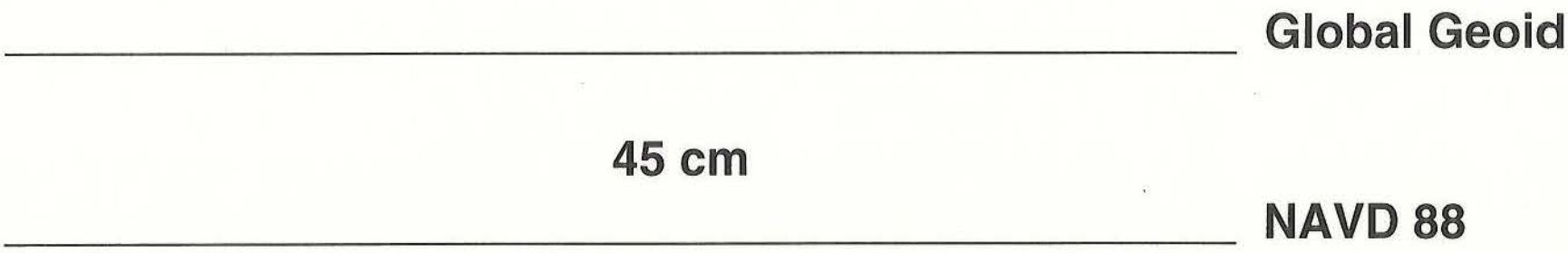
7 parameter Helmert transformation
8 points common to both reference systems
RMS of fit: 13 mm.

$\Delta X$ = -0.9738 $\pm$ 0.0261 meters
$\Delta Y$ = +1.9453 $\pm$ 0.0215 meters
$\Delta Z$ = +0.5486 $\pm$ 0.0221 meters

$\omega_X$ = -0.0276 $\pm$ 0.0009 arc sec
$\omega_Y$ = -0.0101 $\pm$ 0.0008 arc sec
$\omega_Z$ = -0.0114 $\pm$ 0.0007 arc sec

scale = -0.0078 $\pm$ 0.0026 ppm

# NAVD 88 Offset from the Global Geoid




**NAVD 88 datum realized by a *single* datum point (Father Point, Quebec)**

**Selection minimized recompilation of national mapping products**

**No guarantee that NAVD 88 corresponds to a GRS-80 level surface**

**Tests with G96SSS and ITRF94 GPS benchmarks find offset**

**Tests do not find evidence of a tilt across the U.S.**

# Datums and Reference Systems, II

$N_{SSS}$ -- G96SSS geoid height model
geocentric, gravimeteric

## Strategy

$$N_{96} = N_{SSS} + (\delta H - \delta h)$$

Apply the datum biases to the gravimetric geoid model, and obtain GEOID96

Biases obtained by models and by NAD 83 GPS on NAVD 88 leveled benchmarks

## GEOID96

Relative to a non-geocentric ellipsoid
Biased relative to a geocentric ellipsoid
Includes height error from GPS network

## Result

$$h_{83} = H_{88} + N_{96}$$

# GEOID96 Refinements Over GEOID93

Denser grid, 2 arc-minute spacing

Incorporates NAD 83 GPS heights on NAVD 88 benchmarks

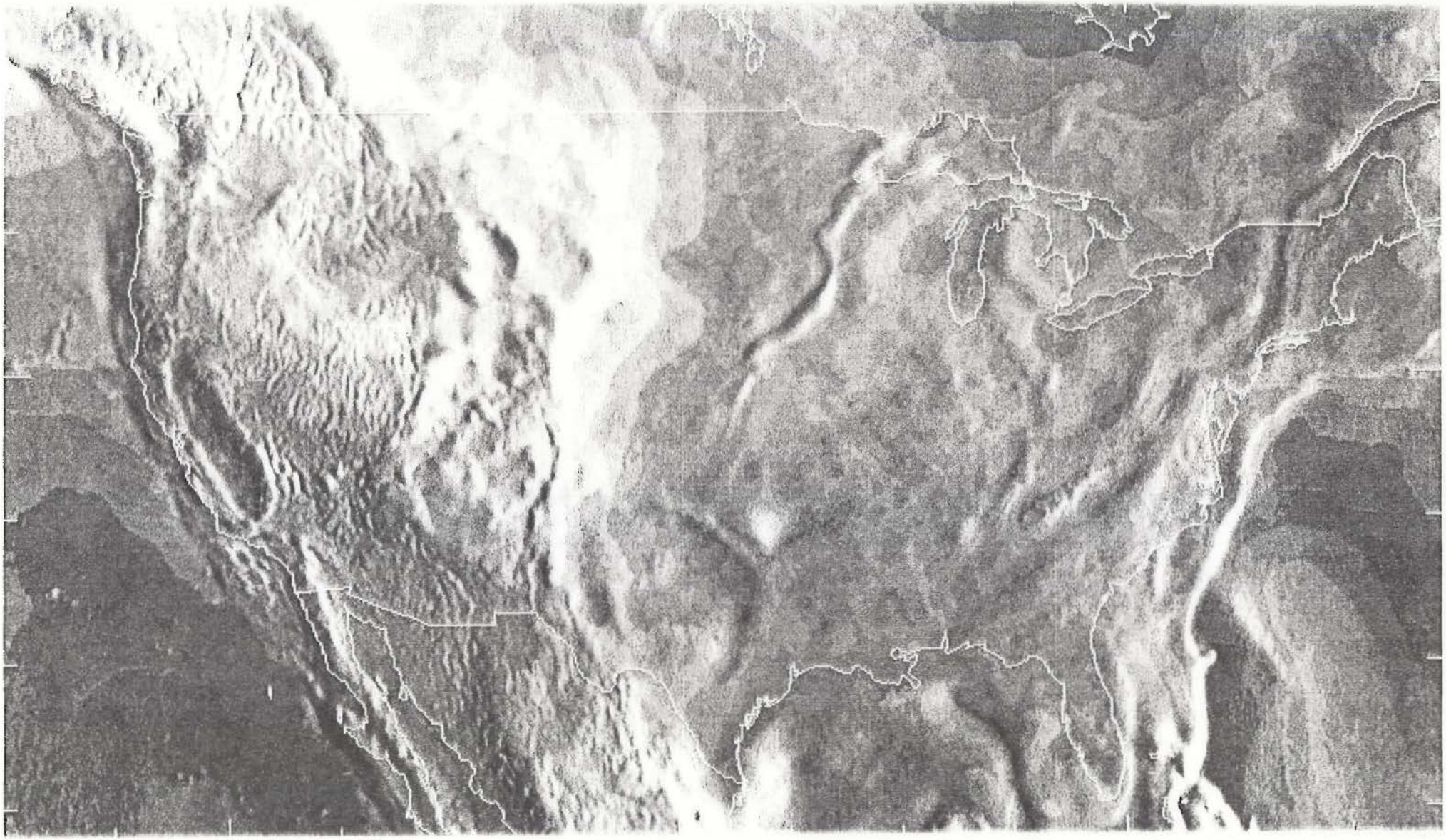
"Spherical" 1-D FFT Stokes integration procedure

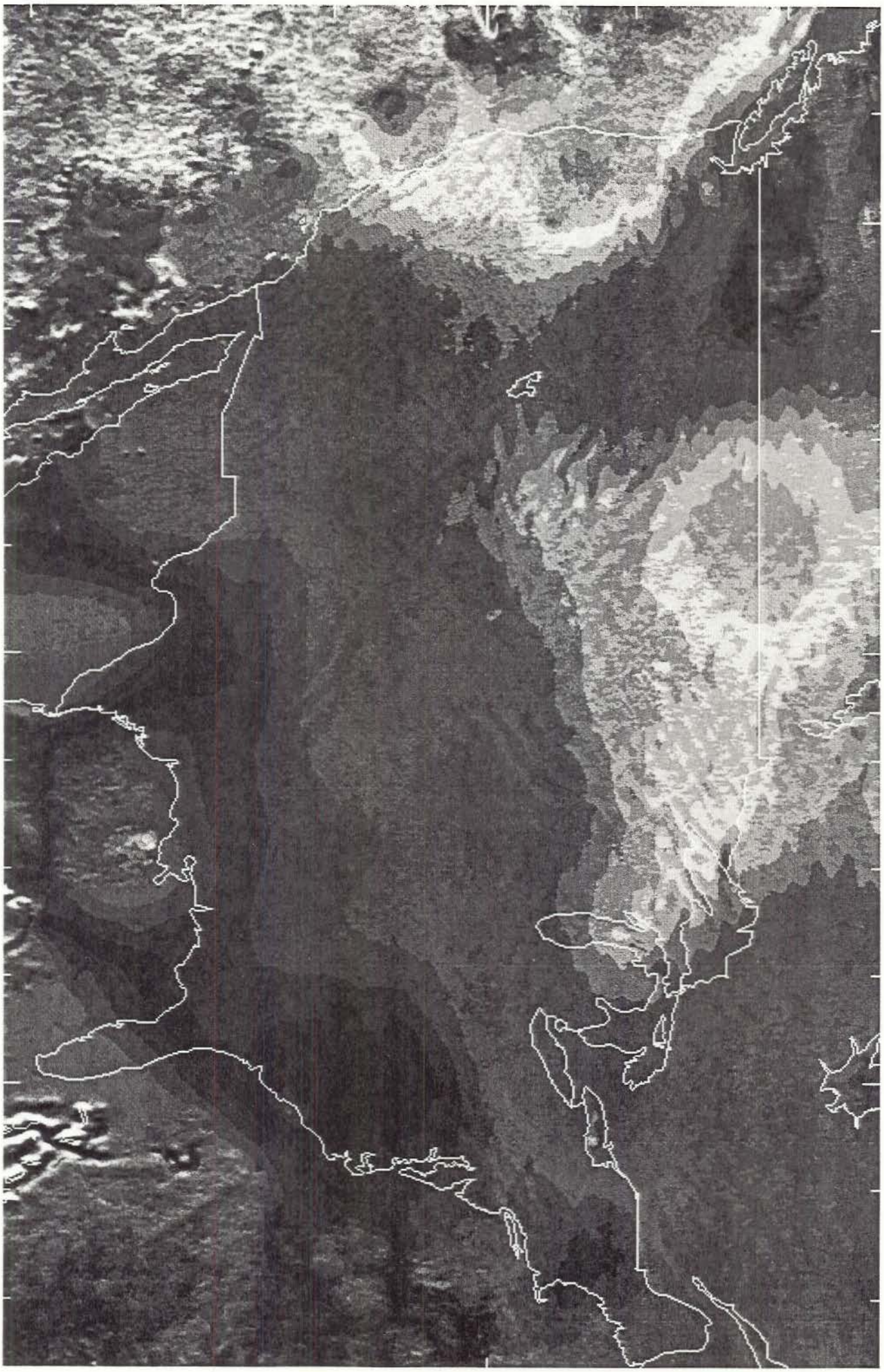
Offshore gravity derived from altimetry (Sandwell/Smith)

Additional gravity data from our contributors

New digital terrain data in the Canadian Rockies

New GSFC/NIMA (DMA) global model, EGM96





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KK1535  DESIGNATION -  U 392
KK1535  PID         -  KK1535
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KK1535  USGS QUAD   -  COMMERCE CITY (1980)
KK1535
KK1535                          *CURRENT SURVEY CONTROL
KK1535
KK1535* NAD 83(1992)-  39 47 49.04816(N)     104 52 54.36555(W)      ADJUSTED
KK1535* NAVD 88     -      1596.695  (meters)     5238.49   (feet)   ADJUSTED
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KK1535  X           -  -1,260,596.431 (meters)                        COMP
KK1535  Y           -  -4,743,743.854 (meters)                        COMP
KK1535  Z           -   4,061,700.790 (meters)                        COMP
KK1535  LAPLACE CORR-          -5.53  (seconds)                       DEFLEC96
KK1535  ELLIP HEIGHT-        1579.216 (meters)                        GPS OBS
KK1535  GEOID HEIGHT-         -17.45  (meters)                        GEOID96
KK1535  DYNAMIC HT  -        1595.194 (meters)    5233.57   (feet)   COMP
KK1535  MODELED GRAV-     979,630.2   (mgal)                         NAVD 88
KK1535
KK1535  HORZ ORDER  -  FIRST
KK1535  VERT ORDER  -  FIRST     CLASS II
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KK1535.The orthometric height was determined by differential leveling
KK1535.and adjusted by the National Geodetic Survey in June 1991.
KK1535
```

# Future Geoid Research

Molodensky theory and height anomaly to geoid conversion
Spectral decomposition of the terrain correction integral
Development of geoid theory rigorous at the 1 cm level

Combination of marine and altimetry derived gravity data
Investigation of surface rock density data

-- and --

Resolution of GPS network height discrepencies

# Conclusions

**The scientific model G96SSS is geocentric in ITRF94(1996.0)**

**NAD 83 non-geocentricity affects GPS heights 0.28 to 1.64 m**

**NAVD 88 is offset about 45 cm from global mean sea level**

**These effects are incorporated into GEOID96**

**Gaussian noise in the GPS ellipsoid heights is 5.5 cm RMS.**

**Correlated error GEOID96/GPS is 2.5 cm, random over 50 km**

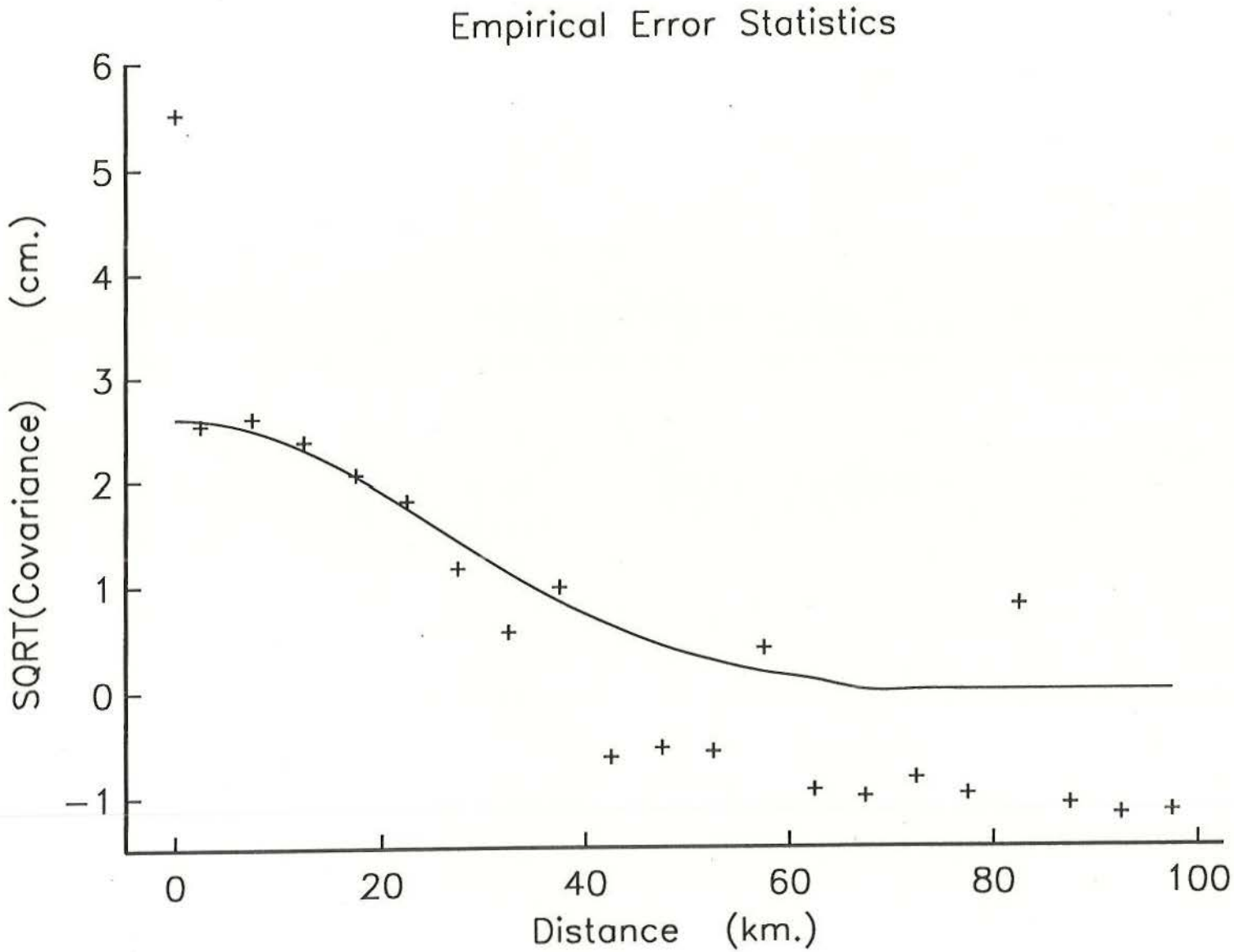
**Anticipate successful GPS leveling with fewer survey connections to existing NAVD 88 benchmarks**

# GEOID96 Accuracy Implications

Users of GPS pseudorange (even with differential correctors) can apply GEOID96 ($H_{88} = h_{83} - N_{96}$), and not introduce significant errors in the transformation.

Users of GPS carrier phase (geodetic surveying) still need to tie to local orthometric height control points, and apply GEOID96 differences ($\Delta H_{88} = \Delta h_{83} - \Delta N_{96}$).

Better results in cases with poor control point distribution.


Empirical Error Statistics
SQRT(Covariance) (cm.)
6
5
4
3
2
1
0
−1
0
20
40
60
80
100
Distance (km.)

# GEOID96 Availability

Information Services Branch
National Geodetic Survey, NOAA, N/NGS12
1315 East-West Highway, Station 9202
Silver Spring, MD 20910-3282
Phone: 301-713-3242
Fax: 301-713-4172

CD-ROM and 3.5" diskettes

Anonymous FTP: ftp.ngs.noaa.gov
World Wide Web: http://www.ngs.noaa.gov
Bulletin Board: 301-713-4181 and 4182 (8-N-1)