COMPUTING GPS-DERIVED ORTHOMETRIC HEIGHTS WITH THE GEOID90 GEOID HEIGHT MODEL

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ABSTRACT

The Global Positioning System (GPS) has revolutionized the surveying of horizontal geodetic networks. However, vertical geodetic network surveying has not yet been able to take advantage of GPS technology. When GPS data are processed, they yield heights above a smooth ellipsoid surface. Vertical control, on the other hand, is based on orthometric height. These are heights above a complex, equipotential surface called the geoid. To accurately convert ellipsoidal heights into orthometric heights, one must know the relationship of the geoid to the ellipsoid. Before now, geoid height in the United States was computed from spherical harmonic models possessing 0.5 degree resolution, or about 50 km. They do not embody the finescale structure of the geoid, and have provided only limited success in obtaining GPS-derived orthometric heights.

Recent advances in processing methodologies now enable efficient computation of continental-scale, high resolution, geoid height models. At the National Geodetic Survey, we have developed GEOID90: a high-resolution geoid height model for the conterminous United States. GEOID90 was computed using nearly 1.5 million terrestrial and ship gravity data. It is modeled on a 37 x 3/ grid (about 5 km resolution), and spans 24 °N to 53 °N and 66 °W to 125°W. The increase in resolution has greatly improved the accuracy of GPS-derived orthometric heights. Comparisons between vertical bench marks and GPS surveys of county and statewide extent, demonstrate accuracy from a 10 cm RMS level (at 100 km distances) down to a 1 cm RMS (at 10 km distances) . This quantum leap in accuracy extends the utility of GPS technology and supports a wider variety of surveying and mapping requirements.

INTRODUCTION

The impact of the Global Positioning System (GPS) on control network surveying can hardly be overstated. In a short span of time, the National Geodetic Survey (NGS) has adopted differential GPS technology for horizontal geodetic surveys, completely replacing conventional surveying techniques. The superb length accuracy, coupled with greater efficiency and increased productivity in the field, has revolutionized our field operations.

However, one sector of geodetic surveying has remained much the same. That sector is vertical control surveying. GPS is a three-dimensional system, and certainly provides height information. But the heights produced by GPS measurements orthometric height, H, use the simple equation

$$H = h - N$$

where N is the geoid height. Geoid heights relative to the GRS 80 ellipsoid range from +75 to -104 meters worldwide. Within the conterminous United States, geoid heights will range from -5 to -50 meters. The negative values of geoid height express the fact that the geoid is below the ellipsoid, as portrayed in figure 2.

THE GEOID90 GEOID HEIGHT MODEL

The technique for computing the GEOID90 geoid height model uses gravity data to compute high frequency corrections to an existing model of the Earth's geoid heights. Figure 3 displays a shaded relief image of the OSU89B model, computed by Rapp and Pavlis (1990) at The Ohio State University.
This model is a global model and can resolve features down to 0.5° (about 50 km) in horizontal extent. By means of a Fast Fourier Transform (FFT) algorithm, I computed the high frequency geoid component using nearly 1.5 million point gravity data in the NGS data base. The high frequency geoid grid was combined with OSU89B geoid heights and a correction derived from digital terrain data to yield the final geoid heights. The GEOID90 regional geoid height model has a grid spacing of 3' (about 5 km); a major increase in resolution over that of OSU89B.

GEOID90 is depicted by a shaded relief image in figure 4. The geoid heights are referred to the Geodetic Reference System 1980 (GRS 80) ellipsoid, and range from a low of -53 meters in the Atlantic Ocean to a high of -5 meters in the Rocky Mountains. At this high resolution, both topographic and geologic features become clearly evident. The Rocky Mountains are prominent, the Sacramento Valley in California is clearly visible, and one may discern the outline of the continental shelf in the Atlantic Ocean and the Gulf of Mexico. Note that GEOID90 geoid height is not topographic elevation. The range is only a few tens of meters. GEOID90 portrays gravity variation caused by topographic relief and by density variations in the Earth's crust. One striking feature is the Mid-continent Gravity High, running south from Lake Superior. This elongated structure is the expression of the high density igneous formations associated with a rift. Although the terrain in Iowa is quite flat, these formations cause a 4 to 5 meter "step" in the geoid. A practitioner who is unaware of such a feature may wrongly question the accuracy of an elevation data set or the correct function of a GPS system.

USING GEOID90

GEOID90 consists of two parts: one or more binary files of geoid height values on a regular grid, and a menu-type program for interpolation from the grid files. One may enter NAD 83 geodetic latitude and longitude for a point at a time from the keyboard, or one may designate a file of points to be processed. Mechanically, the operation is simple.

In using GEOID90, one must remember that GEOID90 behaves very much like GPS. That is, they both provide very accurate height differences; whereas, the height of a single point is much less accurate. In a related problem, we know that the NGVD 29 vertical datum is not exactly a mean sea level datum. And, it is highly likely that the NAVD 88 datum is not exactly at mean sea level either. In short, GEOID90 geoid height, Ngg ', GPS ellipsoidal height, h**gpg** ; and NAVD 88 orthometric height, Hgg , are all subject to height bias, or a datum ambiguity. So we may write

$$h_{GPS} + \delta h_{GPS} = (H_{88} + \delta H_{88}) + (N_{90} + \delta N_{90}) \text{ ,}$$

where the "5" indicates a height bias. But, these height biases will cancel when we consider local height differences

$$\Delta h_{GPS} = \Delta H_{88} + \Delta N_{90} \text{ ,}$$

where the "A" indicates a height difference between two stations. Therefore, GPS together with GEOID90 functions like a level rod and not like a tide gauge. They will not deliver an accurate orthometric height by themselves. They must be connected to existing vertical control. Now, it is certainly possible to convert individual GPS vectors into sets of ellipsoidal height differences. Ah, and to subtract the associated geoid height differences, AN, to get a collection of orthometric height differences, AH. But the process is cumbersome. And, one is still faced with the prospect of a least squares adjustment of the Aff's. Is there an easier way?

When one adjusts a set of GPS vectors, one must fix one or more points. Instead of fixing the ellipsoidal height of an existing GPS control point, fix the orthometric height of an existing vertical bench mark. This has the effect of moving the three-dimensional GPS network up or down to coincide with the local vertical datum, modified by the local geoid height model. If this is done, the GPS ellipsoidal heights will absorb the height biases from the vertical datum and the geoid. Formally, we may write

$$h_{GPS} + \delta h_{bias} = H_{88} + N_{90} \text{ ,}$$

where Sh**bias** denotes an ellipsoidal height bias for the adjustment. In practice, one is usually interested in orthometric height rather than ellipsoidal height. So, a biased set of ellipsoidal heights would not be a problem.

There is an added benefit to such a "fixed bench mark" adjustment. In many cases, this approach will work equally well for NAVD 88 heights or NGVD 29 heights. In the case of fixing an NGVD 29 height, a somewhat different height bias will be absorbed into the adjusted ellipsoidal heights. A contour diagram of vertical datum differences between a preliminary version of NAVD 88 and published NGVD 29 heights can be found in Zilkoski (1991, these proceedings). That figure displays vertical datum variation of 2 meters from Florida to the Rocky Mountains. Even if GEOID90 was a perfect model, a requirement to obtain NGVD 29 orthometric heights through a GPS/geoid system would motivate the use of a fixed bench mark height approach.

In closing this section, it must be stressed that a single vertical bench mark connection is dangerous. The mark may have been disturbed or misidentified, and GPS vector noise in the vertical component at that point will affect all heights. Just as one should not depend upon a single vertical bench mark in conventional leveling, one should not rely upon one lone bench mark in GPS height computation.
One should connect into several bench marks and check the misclosures at those bench marks. If an isolated error is identified, its source may be in the GPS or the leveling, but not in GEOID90. GEOID90 error is evident as long wavelength, two part per million tilts over areas of a few hundred kilometers. More detail on the diagnosis of height errors can be found in Zilkoski (1990).

AN EXAMPLE

The following is a brief example demonstrating that GPS when combined with a high resolution geoid height model can function as a leveling system. More detail can be found in Milbert (1991).

The Prince William County, Virginia, GPS project is a second-order class I survey to establish 83 control stations and azimuth marks in support of a geographical information system. The project was observed in 1986, using two to six Trimble receivers per session, two sessions per day, with over 29 days of observation. In the survey, 12 stations were existing bench marks or were connected to bench marks by means of short level ties. The area covered is about 40 km on each side.

Using the fixed bench mark height approach, I computed orthometric heights from GPS and compared them to the NGVD 29 heights at the 12 bench marks. The upper part of figure 5 is a perspective plot of the orthometric height differences obtained when using OSU89B geoid heights. The large systematic trend is not due to GPS or leveling error; it is model error in OSU89B. The rms scatter of the height discrepancies is 123 mm.

The computation is repeated with the same GPS data and GEOID90. The bottom half of figure 5 displays the new results. The large systematic trend seen with OSU89B is no longer evident with GEOID90. The two small bumps on the error surface are very local. They are probably due to random error in the vertical component of the GPS data. The rms scatter of these height discrepancies is now only 16 mm.

CONCLUSION

Based on many comparison with GPS and leveling, GEOID90 is seen to provide 10 cm accuracy (one sigma) between points spaced at 100 km and 1 cm accuracy between points spaced at 10 km. In some locations, long wavelength errors up to two-parts-per-million may occur. An exact accuracy assessment is difficult due to the random error in the vertical component of the GPS data.

Research is underway at NGS to improve geoid height modeling procedures. One important approach is to exploit the theory of integrated geodesy, which can combine

GPS leveling and gravity data in a simultaneous least squares adjustment. This research, in conjunction with the greater availability of high precision GPS surveys, will likely yield a significant upgrade to our geoid model by 1995.

AVAILABILITY

The GEOID90 geoid height grid occupies almost 3 Mb in binary format. It is distributed in three overlapping subsets:

Eastern U.S.	(24°N-50°N, 270°E-294°E, 66°W-90°W)
Central U.S.	(24°N-50°N, 253°E-277°E, 83°W-107°W)
Western U.S.	(24°N-50°N, 235°E-259°E, 101°W-125°W)

Each file, plus a menu-driven interpolation program (Program GEOID), is available on a separate 1.2 Mb, DOS format, high density, floppy disk (5 **1/4**" or 3 **1/2**") . Two ancillary disks are also available. One disk contains geoid heights for Hawaii, Puerto Rico, the Virgin Islands, and the conterminous United States computed from the OSU89B model, and some utility programs which perform formatting. The other disk contains OSU89B geoid heights for Alaska, and the source code for the utility programs. These disks may be obtained from:

National Geodetic Information Center
N/CG174, Rockwall Building, Room 26
National Geodetic Survey, NOAA
11400 Rockville Pike
Rockville, Maryland 20852
301-443-8631

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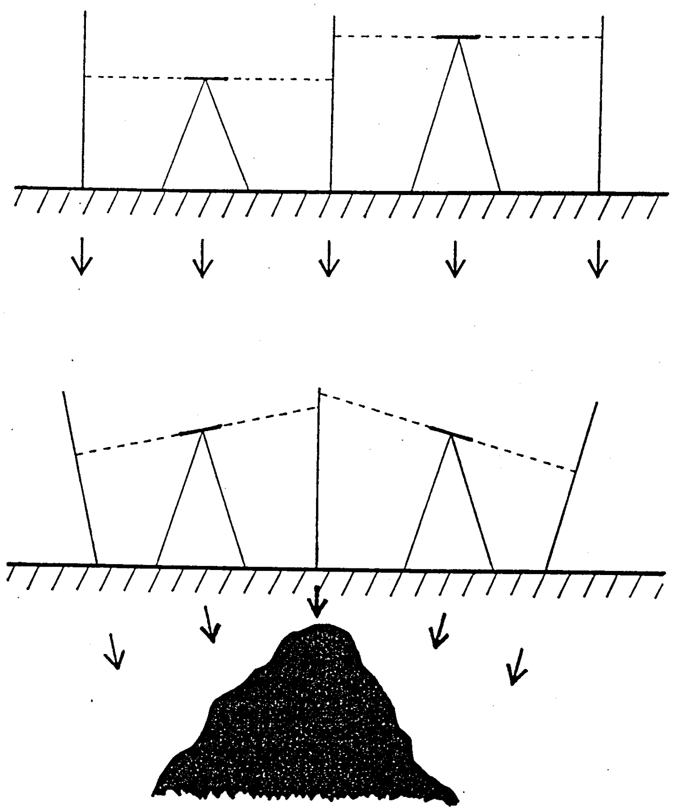

FIGURE 1--Leveling and Local Gravity Variation

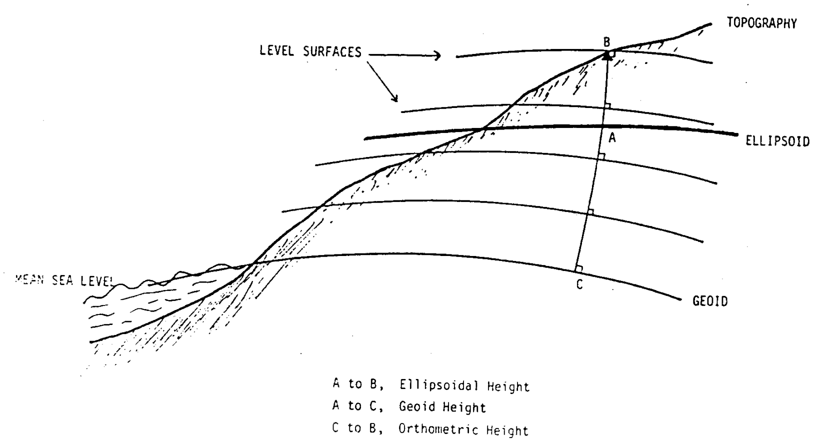

FIGURE 2--Types of Heights

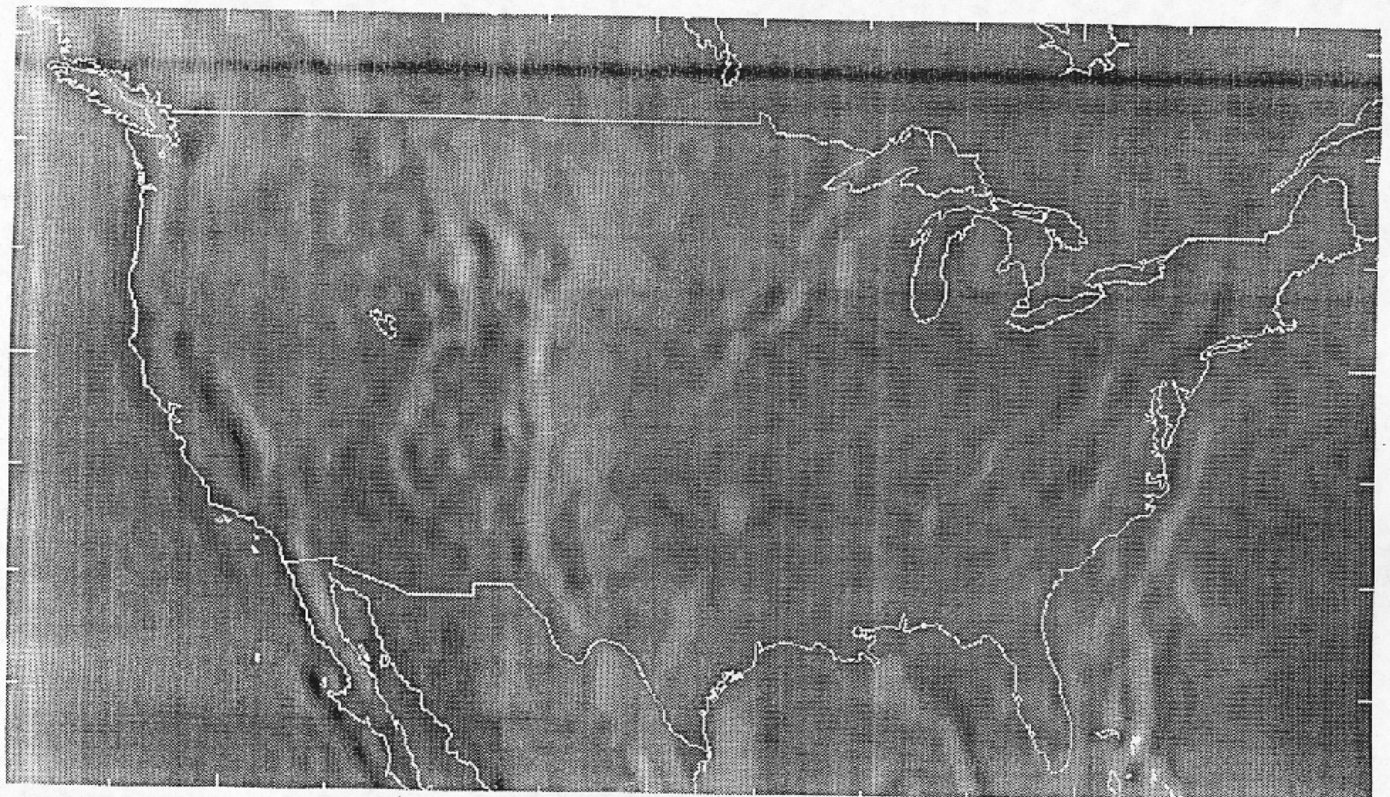

FIGURE 3--OSU89B Geoid Heights

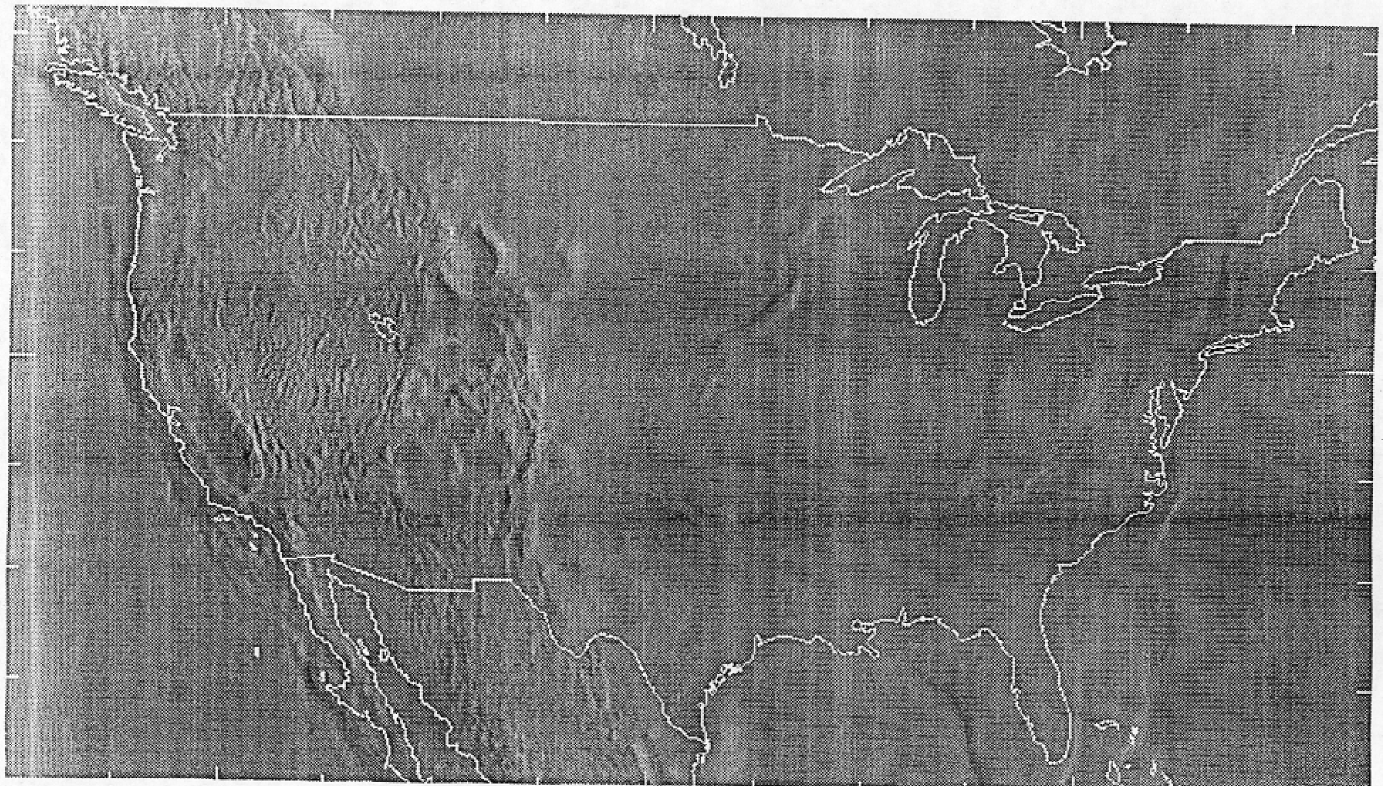

FIGURE 4--GEOID90 Geoid Heights

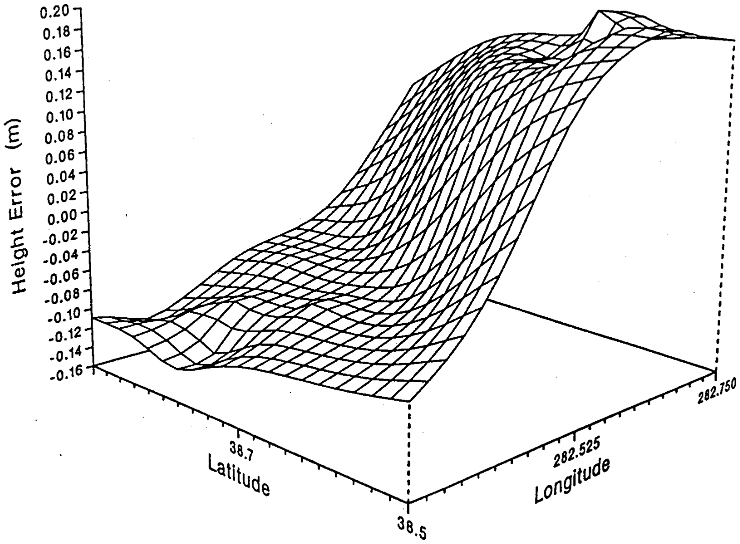

FIGURE 5a--Height Error using GPS, Leveling, and OSU89B Geoid Height

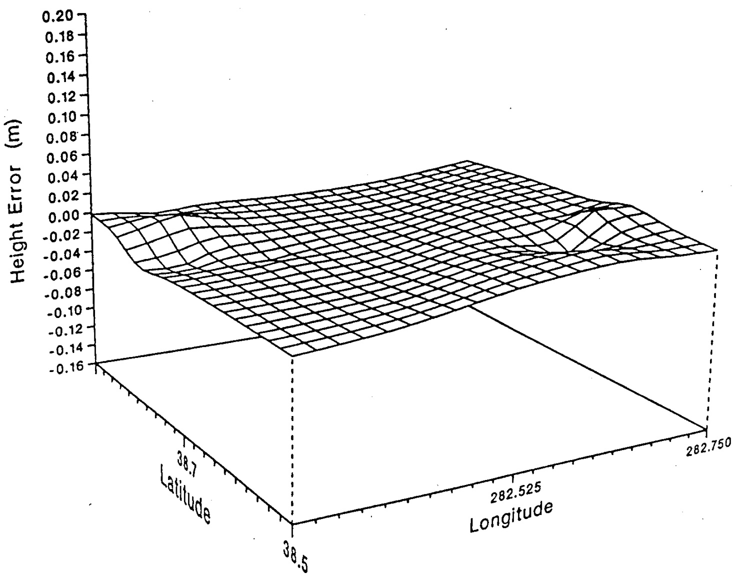

FIGURE 5b--Height Error using GPS, Leveling, and GEOID90 Geoid Height