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FUNDAMENTALS OF THE STATE PLANE COORDINATE SYSTEMS

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Introduction

Although the State Plane Coordinate Systems (henceforth referred to as SPCS) have been in existence for about 40 years, the number of surveyors and engineers who employ these systems on a day-to-day basis is very limited. Furthermore, their number is augmented only slightly by those in the profession who occasionally employ or have a working knowledge of the systems. This is a deplorable situation in a country such as the United States, which leads the world in the practical application of technological advancements. Regardless of any personal preferences or philosophy, the SPCS fall in this category. The main point then resolves into a question - why does this condition exist? Over the years numerous answers have been given. Some are justified; lack of available control points are a good example, and perhaps the geodetic community did not offer much help in the past although such is not the case today, but many answers seem more of an escape mechanism than valid reasoning. Most of us hesitate when facing something new, but when we realize that what we are facing is nothing more than a new front on an old house, with only a very few minor modifications to the interior, our fears soon dissipate. This analogy applies to the SPCS, for these systems are simply an adaptation of the method of latitudes and departures which has been in use in one form or another since the beginning of the surveying profession. The intent here is to conclusively prove this point through a primer approach. To do this, in the first half of the paper reasons for employing the systems will be outlined, pertinent terms will be defined in plane surveying parlance, a simple example will be carried out in step-by-step sequences without any refinements made to the horizontal ground measurements, and throughout this first section the emphasis will be placed on the simplicity of the procedures and their similarity to long-used practices. In the second section, the reduction to sea level and for the scale distortions will be discussed and simply illustrated. The example employed in the first section will be recomputed, with refinements made to the distances to take into account the reductions to sea level and for scale. In the conclusions the differences in lengths and bearings resulting from the various approaches will be examined and the significance of these differences discussed.

Why Employ the State Plane Coordinate Systems?

Employing the SPCS cannot, of course, improve the accuracy of the surveys. However, they can assure that greater benefits result from a survey, since any point in the survey can be redetermined to the same accuracy as the original positioning. Obviously this attribute alone is a very good reason for employment since no monument can ever be considered legally lost once its position is related to the SPCS. Other important considerations are that all surveys are correlated to a single reference framework, thus achieving consistency in overlapping or bordering mapping projects; few, if any blunders will remain undiscovered if two or more previously coordinated points are used to control the surveys and often the sources and locations of the blunders are indicated, and the final data are presented in more or less the same format regardless of the field practices utilized.

The SPCS can be utilized in the computation of any type of survey data. It makes no difference if the lengths are obtained through stadia (tachymetric) methods or the bearings are the result of compass observations or the field operations are carried out under geodetic specifications. As a matter of fact, the SPCS can be employed by those surveyors who prefer to plot their survey data rather than perform any computations. Some may question this last statement, yet it logically follows that if the relationship of a coordinated point with such a survey is known, the locations of the plotted points on the SPCS are obviously also known.

No one should assume, however, that the SPCS are a panacea for all land surveying problems. For example, in most instances at this time the fact that a corner monument can be proved to be displaced carries little weight when the proof is SPCS related and there is counter-evidence offered which involved the contentions of an individual, even when these contentions are based on 30-year old recollections. Hopefully, the time is near, however, when it will be recognized that surveying is a branch of applied mathematics and hence that many solutions to boundary problems can be resolved positively through mathematics utilizing field-obtained survey data.

This transition is already taking place as an increasing number of real estate attorneys are turning to plane coordinate systems as an aid in the solution of some problems. The real value of the SPCS is not in resolving problems resulting from past practices, for here it is only in a few special cases that the systems are of value; but such will not be the case in the future when the most important and valuable tracts of land will be referenced to a coordinated survey network. Furthermore, city and regional planners are beginning to understand the advantages of coordinated systems with the result a number of ordinances have been enacted in several areas which require the use of SPCS under certain conditions.

Perhaps the most important consideration, since it relates to cost, is the time element involved. In the matter of computations, the additional time required for the SPCS has been found to involve at most a few additional minutes, and certainly would not exceed one hour in a survey containing 100 points. It must be emphasized and will be shown that the computational effort involved to place a survey on the SPCS requires little effort. However, making the connections to coordinated points can be an expensive field exercise, especially when the control points are some distance from the survey area. It is here, and not in the computation phase, where costs can increase significantly.

What are the solutions to this problem? Obviously closer-spaced, good quality control is one answer and possibly the only answer to those surveyors using tapes (chains). A cooperative approach as outlined in the paper "Use of Control for Land Surveys" may be the most economical solution in many areas where control is sparse. In larger, more heavily populated sections, the cost of a good, closely spaced survey system can be easily justified, but seldom understood by the powers to be, even when evidence can be presented that the costs involved will be returned several fold over the years. It can be successfully argued that an easily usable survey system is as important to the orderly development of a community as any other publicly furnished service such as water, streets and sewer facilities. However, it is not a visible asset and often it takes a concerted effort by all concerned (surveyors, lawyers, planners, etc.), over a long period of time to obtain the necessary funds. Even when the effort is successful, means must be found to maintain and extend the system; for a system without monuments has no value and as a municipality grows, the system must grow with it.

It would be very simple to expand this dialogue, but the primary intent is to explain how to use the SPCS and we will now proceed in that direction.

Definitions

It has been long thought that among the reasons many surveyors have shown little interest in the SPCS is terminology. A land surveyor, for example, generally would use the term "chained" when he measures a distance, regardless whether a tape or a chain was employed. The geodesist, on the other hand, would refer to a measurement obtained by taping methods as being "taped." Both, of course, mean the same thing. As another example, the land surveyor considers a survey to be in the "meridian" when it is referenced to north through solar or astronomic observations. The geodesist prefers the word "orient." Here too, the words may seem different but they are identical in meaning, except that the geodesist (in the United States) considers south, rather than north, as being the origin. A further discussion in this matter - north or south as the origin for referencing a survey will be given later.

To ease the transition to SPCS, definitions of a few of the more pertinent terms in land surveying parlance follow:

Lambert Projection = the SPCS used in those states whose major dimensions are primarily east to west. The Long Island zone of the New York SPCS, the north zone of Florida system and zone 10 in Alaska (Aleutian Islands) are also on this projection.

Transverse Mercator Projection = the SPCS used in those states extending primarily north to south.

Zone = a subdivision of the SPCS within a State. With one exception, these subdivisions follow county boundaries. In order to maintain a maximum scale reduction in the center of a particular system at 1:10,000, the limits cannot exceed 158 miles north to south for the Lambert projection and east to west for the transverse Mercator projection. The Lambert projection bands can be extended around the world and the transverse Mercator zones to within a few degrees of the north pole. The published tables, of course, provide some overlap outside a zone or beyond a State boundary but not to the extend as just noted.

There are a small number of zones where for one reason or another the maximum range is larger than 158 miles and therefore the scale reduction is worse than 1:10,000 at their centers. In many more instances the areas included within the zones are less than 158 miles wide in the pertinent dimension and in these cases the scale reductions at the center are better than 1:10,000.

X = the plane coordinate value in feet perpendicular to the "Y" or north-south axis (center) of the system. In land surveying terms, "X" would be the Easting or "E" and consist of C ± X' where "C" is a constant of sufficient numerical size to keep the "X" values positive. The definition of "X'" follows.

X' = the distance in feet east or west of the "Y" or north-south axis of the system. When east of the "Y" axis the values are added to "C" and subtracted from "C" when west of the "Y" axis. This value may be considered the Departure measured from the "Y" axis.

Y = the plane coordinate value in feet along the "Y" or north-south axis, north from the origin to its intersection with the "X" coordinate. In plane surveying terminology, this quantity may be considered the Northing or "N", or the Latitude measured from the origin.

C.M. = Central Meridian = the meridian (Longitude) or "Y" (north-south) axis usually located near the center of the plane coordinate system which separates the positive and negative "X'" quantities.

In a local system, it is the meridian through the point selected as the origin.

θ = theta = the so called mapping angle in the Lambert projection. This value is applied to grid azimuths to obtain very close approximations to geodetic azimuths or bearings. The angle is considered positive when the point is east of the "Y" axis and negative west of this axis. When the sign of "θ" is positive, it is always added to the grid azimuths or to those bearings in the N-E and S-W quadrants and subtracted from bearings in the S-E and N-W quadrants. When the sign of "θ" is negative, it is applied the opposite to that just described. On those occasions when grid azimuths are to be determined from geodetic azimuths, the sign of "θ" is considered opposite to that shown and applied in that fashion.

$\Delta\alpha$ = delta alpha = the so called mapping angle in the transverse Mercator projection. The remainder of the definition is exactly the same as that given for the "θ" angle, except, of course, that "$\Delta\alpha$" replaces "θ".

This term is also used to describe the difference between forward and backward geodetic azimuths. It is often used as well in plane surveying to define the difference (convergence) between "true" and grid or plane bearings which occurs at all points not on the same meridian as the origin in a local system.

Scale Factor = a multiplier which when applied to horizontal measured distances produce distances at the average elevation of the points involved which have been corrected for the distortions due to the projecting of a line measured over a curved surface (the earth) onto a plane through the means of a particular map projection. A single scale factor can only be used over a finite range, but rarely will this range be exceeded in surveys envisioned in this paper.

In the Lambert projection, the scale factor is a function of latitude. For the transverse Mercator projection the scale factor is based on the distances in feet (X') that the points are east or west of the central meridian. Values of the scale factors are given for each minute of latitude in the projection tables for those states which use the Lambert projection and at 5,000 feet intervals in the tables compiled for those states where the transverse Mercator projection is employed.

Sea Level Factor = a multiplier which reduced horizontal distances at the mean elevations of the points to the sea level reference surface. Such distances are referred to as geodetic distances. The multiplier is derived from the mean elevation of the two points involved or, as is the case on many occasions, a single multiplier for an entire survey may be employed.

When combined with the scale factor into a single multiplier, the resulting reduced horizontal distances are grid distances. It is these values which should be used in computing surveys on the SPCS. This combined factor can be used to derive adjusted ground level lengths or to rise the State plane coordinates to the mean elevation of the points. Furthermore, the mean scale factor has also been eliminated in these instances and the distances computed from these coordinates are essentially ground level values.

NOTE: All SPCS are at the sea level reference (except the Michigan Lambert system which is compiled at an elevation of 800 feet above sea level.

Azimuth = the compass direction between two points expressed as the total number of degrees, minutes, and seconds taken clockwise from the origin. In the United States, South is considered the origin and hence azimuths between 0°-90° are in the SW quadrant, 90°-180° the NW quadrant, 180°-270° the NE quadrant and 270°-360° (0°) the SE quadrant.

By applying 180°00'00" to any geodetic or grid azimuth published by the NGS or from other sources where South was used as the origin the azimuths are now referenced to north as the origin. The sign conventions usually used in plane surveying computations where bearing are employed are then identical.

Bearing = the compass direction between two points expressed in degrees, minutes, and seconds within a particular quadrant, that is NE, SE, SW, and NW. The relationship of azimuths and bearings will be discussed later.

Symbols also used in paper:

ϕ = phi = latitude
λ = lambda = longitude
Δ = delta = difference
α = alpha = geodetic azimuth
αg = alpha sub g = grid azimuth
αa = alpha sub a = astronomic azimuth
S = geodetic distance
Sg = grid distance
ΔX = Departure
ΔY = Latitude

Computations - Part I

In this segment computations will be made of a simulated survey involving a tract of about 40 acres where it is assumed the corners are related to the U.S. Public Land surveys. The area selected is near Eau Claire, Wisconsin, and the center of the Central Zone of the Wisconsin SPCS. This area was chosen because the scale factor for the latitude of Eau Claire is very near the maximum scale factor which is less than one. Thus, when the scale factor is combined with the sea level factor, the multiplier will produce the maximum change in the measured distances for the elevation of the site. These calculations will not be made in Part I of the paper.

The sketch of the surveys shown by Figure 1. One of the control points MT TOM (CofEC) is a first-order station of the national network which was originally established by the City of Eau Claire as part of its control system. The other control station, Point K, is assumed to have been established by some responsible agency and is located about midway between the quarter corner and the section corner of the NE quarter of S16 T27N R9W. Point A is the center of the Section and the tract is the SW quarter of the NE quarter of the Section.

Bearings Versus Azimuths: When one uses bearings, the compass direction between two points is absolutely defined in as far as the quadrant is concerned and in describing property this is undoubtedly a primary consideration. However, when employed in calculations, the use of bearings is somewhat awkward since azimuth angles measured clockwise in the same order as the survey computations are progressing are not always added, nor are similarily measured counter-clockwise angles always subtracted even when the resulting bearings fall in the same quadrant. The awkwardness remains when deflection angles are observed and one must always keep his wit's keen to assure that the correct bearings are derived.

Such is not the case with azimuths since clockwise measured angles are added and counterclockwise angles are subtracted in almost every case. Deflection angles measured to the Right are added in most instances and those to the Left subtracted. The exceptions to the rules governing the application of angles enter when the route of the traverse computations is counter to the direction (clockwise, counterclockwise, deflection right or left) that the angles were measured. On these occasions, the angles are applied with the opposite sign. When deflection angles are employed, 180° is always added to the resultant azimuth. Back azimuths must be taken into account, of course. Turning to the following example:

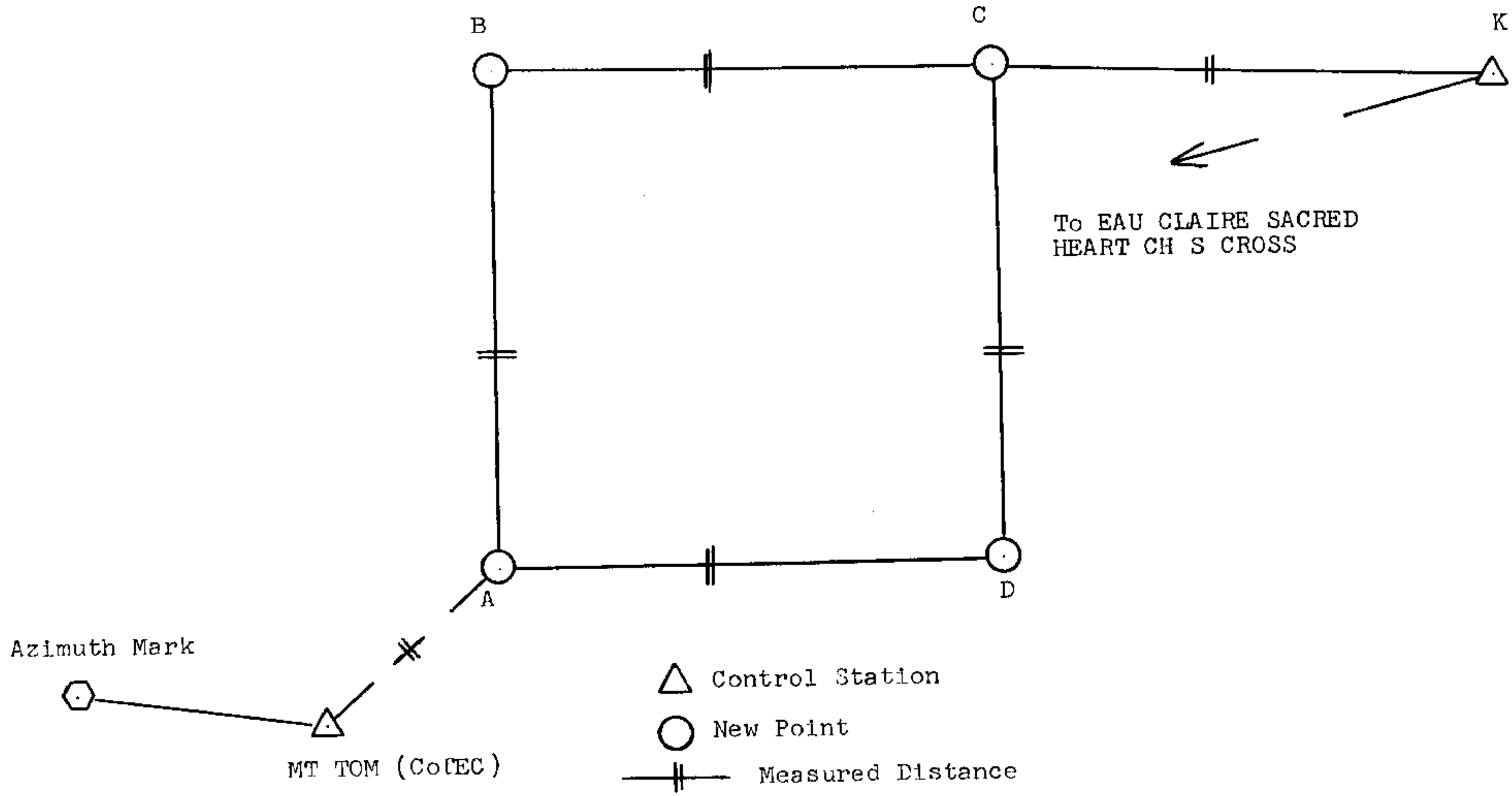
B
C
K
To EAU CLAIRE SACRED
HEART CH S CROSS
A
D
Azimuth Mark
MT TOM (CofEC)
Control Station
New Point
Measured Distance

Figure 1

From	To	Bearing	Azimuth (from south)
1	2	N 13° 34' W	166° 26'
$\angle$		44 20 R	44 20 R
		180 00	180 00
1	3	S 30 46 W	30 46
3	1	N 30 46 E	210 46
$\angle$		1 15 L	1 15 L
		180 00	180 00
3	4	S 29 31 W	29 31
4	3	N 29 31 E	209 31
$\angle$		300 04 *	300 04*
			180 00
		329 35	
4	5	N 30 25 W	149 35

* Azimuth angle or clockwise measured angle

To the uninitiated the computation of the bearings seem unduly complicated, although there are very simple rules that land surveyors and other users have devised to arrive at the correct results. The computation of the azimuths, on the other hand, is rather straightforward.

With respect to whether Bearings or Azimuths should be used in SPCS computations, the only answer that can be given, is that the procedure employed is a personal matter since the end results are identical. To transform the azimuths given on NGS issued data sheets to bearings, the following rules apply:

- NE quadrant - subtract 180° from the published azimuth
- SE quadrant - subtract the published azimuth from 360°
- SW quadrant - the published azimuth and bearing are identical
- NW quadrant - subtract the published azimuth from 180°

If the published azimuths are transformed to azimuths with north as the origin, the rules as given for the NE and SW quadrants and for the SE and NW quadrants are interchanged. Examples follow:

Azimuth*	Azimuth**	Bearing
38° 25' 40"	218° 25' 40"	S 38° 25' 40"W
147 10 25	327 10 25	N 32 49 35 W
262 53 15	82 53 15	N 82 53 15 E
322 46 55	142 46 55	S 37 13 05 E

* From south

** From north

Some of the computations which follow will be processed using bearings while others will employ azimuths. Corresponding quantities will also be noted. This is being done to more fully acquaint land surveyors with the azimuth concept. The simulated angles used in the computations are a mixture of deflection and azimuth angles.

Observed Angles and Measured Distances: The angles and distances as given in Figure 2 are assumed to be field observations with the distances being reduced to the horizontal:

From	To	Angle	Distance (in feet)
MT TOM (CofEC)	Az Mk		
		126° 03' 40"*	
MT TOM (CofEC)	A		1103.34
A	MT TOM (CofEC)		
		43 47 35 L	
A	B		1321.21
		89 54 00 *	
A	D		1319.70
B	A		
		89 54 30 R	
B	C		1314.99
C	D		1320.69
		89 53 55 L	
C	B		
		0 00 30 R	
C	K		1314.10
D	A		
		89 54 35 *	
D	C		
K	C		
		344 10 25 *	
K	EAU CLAIRE SACRED HEART CH S. CROSS		

* Azimuth Angles = Clockwise Angles

Figure 2

Local Coordinate Systems: For many engineering projects where the surveys will have little future use or their use would be limited to monitoring structures or to control extensions to the projects local coordinate systems are entirely satisfactory and, in fact, are generally recommended. In those areas where national network control does not exist, similar systems have been successfully employed to establish control networks. As a matter of fact, prior to the development of the SPCS, numerous cities and counties established such systems which are still being used today.

Since the areas encompassed by these reference frameworks are relatively small, scale factors can be ignored and unless there are significant differences in elevation, the reduction of the lengths to some elevation reference surface can also be neglected. For example, if an area can be placed within a circle not exceeding 40 miles in diameter; and the origin of the system is selected near the center of the circle, the error in the coordinates at the extremities would not exceed 1:60,000. Where the maximum difference in elevation over an entire area is not greater than 400 feet, no length would be in error by more than 1:50,000.

All land surveyors and those engineers, who practice surveying, are familiar with such local systems and the primary purpose in bringing them into discussions concerned with the SPCS is to show the similarities of the two procedures. Two examples will be given both involving the tract defined by the corners A-B-C-D (See Sketch Figure 1). The first example will be controlled by the bearing A-B which is assumed to be an observed value. Bearings will be used in this example. In the second problem, the same area and observational data will be used with the exception that the bearing A-B will be changed by 20". Azimuths based on south as the origin will be used. These examples will serve to introduce the azimuth concept and also to show that errors in loop-type traverses closing on themselves can go undetected. In this regard, most surveyors are aware that the proportional part accuracy of a length is obtained by dividing the difference into the total length, but there are probably some who are not aware that errors or changes in angles, bearings or azimuths as they effect the resulting latitudes and departures can also be expressed in the same fashion. To do this, simply divide the error or change in angular measure into 206265, (actually for all intents and purposes 200,000 will suffice). For the example given here, the bearing A-B was changed by 20"; and therefore, the square root of the sums of the squares of the changes in the latitudes and departures ($\sqrt{\Delta L^2 + \Delta D^2}$) will correspond to 20/206265 or 1:10626 of the lengths involved (See Figure 6).

Figures 3 and 4 are the computation of the preliminary and corrected bearings and azimuths for the two examples under discussion. The assumed observed bearing between A-B is N 0°06'10"E or an azimuth of 180°06'10" from the south. For the second example, the azimuth A-B changed by +20" (180°06'30" or N 0°06'30"E) will orient the survey. The angles are from Figure 2. Note that the angle at C which is indicated by a double asterick ** in both examples is applied with the opposite sign. This was necessary because the deflection angle was observed from D to B and the application in this computation is B to D. In addition, the clockwise observed angles at A and D are subtracted in both examples because the routes of the bearing and azimuth computation are opposite to the direction of the angle measurements.

STATION FROM	STATION TO	PRELIMINARY bearing ° ′ ″	CORRECTION FOR CLOSURE ′ ″	CORRECTED bearing ° ′ ″
A	B	N 0 06 10 E		N 0 06 10 E*
B	A	S 0 06 10 W		
∠		89 54 30 R		
B	C	S89 59 20 E	-2	S89 59 18 E
C	B	N89 59 20 W		
∠		89 53 55 L	**	
C	D	S 0 05 25 E	-5	S 0 05 20 E
D	C	N 0 05 25 W		
∠	-	89 54 35		
D	A	N90 00 00 W	-8	N89 59 52 W
A	D	S270 00 00 E		
∠	-	89 54 00		
A	B	N 0 06 00 E	+10	N 0 06 10 E*
*Observed	bearing		10"/4 = 2".5	per angle
		Figure 3		

STATION FROM	STATION TO	PRELIMINARY azimuth ° ′ ″	CORRECTION FOR CLOSURE ′ ″	CORRECTED azimuth ° ′ ″
A	B	180 06 30		180 06 30 *
B	A	0 06 30		
∠		89 54 30 R		
B	C	270 01 00	+2	270 01 02
C	B	90 01 00		
∠		89 53 55 L	**	
C	D	359 54 55	+5	359 55 00
D	C	179 54 55		
∠	-	89 54 35		
D	A	90 00 20	+8	90 00 28
A	D	270 00 20		
∠	-	89 54 00		
A	B	180 06 20	+10	180 06 30 *
* Observed	azimuth from	south	10"/4 = 2".5	per angle
		Figure 4		

The addition and subtraction of angles on mechanical and many electronic desk top calculators can be cumbersome if one is not aware of a very simple procedure. This procedure involves adding 40 or 940 to the minutes and/or seconds of an angle when the addition or subtraction of this angle with another produces values for the minutes and/or seconds in excess of 60 or less than zero. Examples follow:

	A	B	C
(1)	39° 40' 52"	394052	39040052
	+25 25 10	+256550	+25965950
	65 06 02	650602=65°06'02"	65006002=65°06'02"
(2)	103° 14' 25"	1031425	103014025
	-27 13 40	-271380	-27013980
	76 00 45	760045=76°00'45"	76000045=76°00'45"
(3)	96° 39' 44"	963944	96039044
	+10 20 52	+106092	+10960992
	107 00 36	1070036	107000036
	-26 12 42	-265282	-26952982
	80 47 54	804754=80°47'54"	80047054=80°47'54"

A = Problems with answers

B = Solution of problems using the 40 concept. Place first angle in calculator without spaces between units. Place second angle in keyboard without spaces and observe whether the sum of the minutes or seconds will exceed 60 or be less than zero. If either is the case, change the particular unit in the keyboard by adding 40 and perform the addition or subtraction. This process is continued until the required additions and subtractions of the angles are completed.

C = Solution of the problems using the 940 practice. The procedure is identical to B except that spaces are left between the degrees, minutes, and seconds.

NOTE: When using many electronic calculators, it will be necessary to examine the angles and add either 40 or 940 to those minutes and seconds, which when added or subtracted to entered quantities, will exceed 60 or be less than zero prior to placing them in the keyboard.

It is thought the computation of the bearings and azimuths in Figures 3 and 4 follow practices which are familiar to all surveyors and no detailed explanation will be given here. The distribution of the angular closures is another matter however, and some discussion seems necessary.

In these particular examples and for all examples given in this paper, the distribution of the angular closures will be made on the basis of equal weight, i.e. each angle in a route will receive the same correction based on the closure divided by the number of angle points. The actual procedure is an accumulative process with the corrections being made to the bearings and azimuths rather than to the individual angles. If the angles had been corrected, the end results would be, of course, identical.

Other distribution methods are justified on occasion, but one must be aware that traverses can be easily pulled in one direction or another and this is bound to occur when larger corrections are placed on some angles than on others. As a matter of fact, it is not a very difficult matter to eliminate one of the position closure components (either that related to the Northing or Y or the Easting or X) by juggling the corrections to the angles. However, it is not suggested that anyone adopt this practice.

Some may prefer not to correct the angular closures, leaving this to the balancing process. There is really nothing wrong with this practice except that in most tables of standards and specifications the positional closures in traverses are defined as being those obtained after the angle closures have been distributed. If this criterion is of no concern, then the procedure is acceptable.

The computation of the Latitudes and Departures, Northings (y's), Eastings (y's), the balancing of the closures and the calculations of the final distances, bearings and azimuths are given in Figures 5 and 6. In normal practice, the preliminary values would simply be changed to reflect the adjusted quantities. This method will not be followed in this paper purely in the interest of clarity. The Compass Rule will be used in all examples for balancing (adjusting) purposes. This rule is known throughout most of the world as the Bowditch Rule after Dr. Nathaniel Bowditch, who first stated it; and it certainly should be known in the country of his birth by the same name. All further references to this adjusting procedure in this paper will be made in this context, i.e. the Bowditch Rule.

The Bowditch Rule presupposes that the angles and distances are of equal quality and this seems a fair assumption when transits and tapes (or theodolites and EDM) are used. There are other balancing procedures, the Transit Rule for example, which was devised for use in those instances where the angles are considered superior to the distances, and numerous individual rules which balance out a survey in irregular fashions known by some as the "Gosh by Golly" rules.

No inference should be drawn that the use of these non-conventional methods for balancing surveys is incorrect on all or, in fact, any occasion. There are undoubtedly numerous cir-

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	True Bearing	Distance	SIN Bearing / DISTANCE / COS Bearing	Latitude	Departure	Local Plane Coordinates Northing (y)	Local Plane Coordinates Easting (x)
	° ' "	Feet		Feet	Feet	Feet	Feet
A					**	5000.00	5000.00
			+0.0017938	+ 1321.21			
	N 0 06 10E	1321.21			*	6321.21	5002.37
			+0.9999984		+ 2.37	- 0.08	+ 0.07
B					**	6321.13	5002.44
			+1.0000000	- 0.27			
	S 89 59 18E	1314.99			*	6320.94	6317.36
			-0.0002036		+1314.99	- 0.15	+ 0.14
C					**	6320.79	6317.50
			+0.0015514	-1320.69			
	S 0 05 20E	1320.69			*	5000.25	6319.41
			-0.9999988		+ 2.05	- 0.22	+ 0.22
D					**	5000.03	6319.63
			-1.0000000	+ 0.05			
	N 89 59 52W	1319.70			*	5000.30	4999.71
			+0.0000388		-1319.70	- 0.30	+ 0.29
A	Sum of Lengths	5276.59			**	5000.00	5000.00

USCOMM-DC 8270

Closure = $\Delta N = \Delta y = -0.30$ $\Delta E = \Delta x = +0.29$ $\Delta S = \sqrt{(0.30)^2+(0.29)^2} = 0.42$

$0.42/5276.59 = 1:12,563$

N Factor= $-0.30/5276.59 = -0.05686$ per 1000 feet

E Factor= $+0.29/5276.59 = +0.05496$ per 1000 feet

* = Preliminary Coordinates

** = Final Coordinates

Figure 5

cumstances when these procedures are the only solution to a particular problem. However, when such is not the case, then the Bowditch Rule is more mathematically sound where modern surveying practices have been employed than the Transit Rule and its use is recommended generally.

The computations shown on Figure 5 will be followed throughout the paper except that in one instance (Figures 25 and 27) the SPCS values will be used as the NGS would employ them. This simply means that the Latitude and Departure columns and the Y's (Northings) and X's (Eastings) will be interchanged. The reason for doing this is to further assure those with little experience in using the SPCS that no matter how we interchange equivalent quantities, the final results are identical.

For the examples given on Figures 5 and 6, Point A was assigned a value of 5000.00 ft. in both the Northing (y) and Easting (x). Actually constants of 1000 ft. would have been sufficient.

Some may not be acquainted with the Bowditch Rule balancing procedure and the following is a brief description. First divide the closure in the Northing or "y" by the sum of the distances. This provides a factor, usually expressed as per some number of feet (1000 feet in this case) which is multiplied by the length between two points to obtain the correction to the preliminary value for the Northing. The method recommended by the NGS is to use an accumulative process and correct the preliminary Northings, rather than make single multiplication to correct each individual Latitude. Here too, it must be emphasized that this is a personal preference since the final results will be the same. As example, the correction to the preliminary Northing for point C would be computed in the following manner:
$(-0.05686 \times 1.32121) + (-0.05686 \times 1.31499) = -0.15$ feet
The Eastings (x's) would be balanced in the same manner.

Adjusted Latitudes, Departures, Distances and Bearings (Azimuths)

From	To	Latitude	Departure	Distance	Bearing Azimuth
A	B	+1321.13	+2.44	1321.13	N 0°06'21"E 180 06 21
B	C	- 0.34	+1315.06	1315.06	S89 59 07 E 270 00 53
C	D	-1320.76	+2.13	1320.76	S 0 05 33 E 359 54 27
D	A	- 0.03	-1319.63	1319.63	S89 59 55 W 89 59 55

Figure 5 (Cont.)

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	True Azimuth	Distance	SIN AZIMUTH / COS AZIMUTH	Latitude	Departure	Local Plane Coordinates	
						Northing (y)	Easting (x)
	° ' "	Feet		Feet	Feet	Feet	Feet
A					**	5000.00	5000.00
			-0.0018908	+1321.21			
	180 06 30	1321.21			*	6321.21	5002.50
			-0.9999982		+ 2.50	- 0.08	+ 0.07
B					**	6321.13	5002.57
			-1.0000000	- 0.40			
	270 01 02	1314.99			*	6320.81	6317.49
			+0.0003006		+ 1314.99	- 0.15	+ 0.14
C					**	6320.66	6317.63
			-0.0014544	-1320.59			
	359 55 00	1320.69			*	5000.12	6319.41
			+0.9999989		+ 1.92	- 0.22	+ 0.22
D					**	4999.90	6319.63
			+1.0000000	+ 0.18			
	90 00 28	1319.70			*	5000.30	4999.71
			-0.0001357		-1319.70	- 0.30	+ 0.29
A	Sum of Lengths	5276.59			**	5000.00	5000.00

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Closure = $\Delta N = \Delta y = -0.30$ $\Delta E = \Delta x = +0.29$ $\Delta S = \sqrt{(0.30)^2 + (0.29)^2} = 0.42$
0.42/5276.59 = 1:12,563

N Factor = -0.30/5276.59 = -0.05686 per 1000 feet

E Factor = +0.29/5276.59 = +0.05496 per 1000 feet

* = Preliminary Coordinates

** = Final Coordinates

Figure 6

The adjusted bearings (Figure 5) were obtained using the Tangent or the Cotangent (whichever value is less than one) as derived by the formula Cotangent = Latitude/Departure or Tangent = Departure/Latitude. If these quantities are to be determined using sines or cosines through the formulas Sine (bearing) = Departure/Distance or Cosine = Latitude/Distance, always use the formula where the smaller of the two values (Departures or Latitudes) is involved. For example: the adjusted bearing C-D = Sine (bearing)= +2.13/1320.76 = 0.0016127 and the bearing = S 0° 05' 33" E which checks the value given in Figure 5. Now, using the Cosine 1320.76/1320.76 = 1.0000000 or S 0° 00' 00" E. These differences are due to the number of significant numbers involved. To those who may be interested, a check of a tables of sines and cosines will provide considerable insight to this problem. For an angle of 0° 05', it takes a change of about 5 in the sixth decimal place of the sine to change the angle one second, yet it takes only a change of 7 in the ninth decimal place of the cosine to achieve the same result.

Adjusted Latitudes, Departures, Distances and Azimuths (Bearings)

From	To	Latitude	Departure	Distance	Azimuth Bearing
A	B	+1321.13	+2.57	1321.13	180°06'41" N 0 06 41 E
B	C	- 0.47	+1315.06	1315.06	270 01 14 S89 58 46 E
C	D	-1320.76	+2.00	1320.76	359 54 48 S 0 05 12 E
D	A	+ 0.10	-1319.63	1319.63	90 00 16 N89 59 44 W

Figure 6 (Cont.)

Figure 6 is the computation of the local plane coordinates using the bearing between A-B as employed in Figure 5 converted to an azimuth with south as the origin and changed by +20". All the calculations are carried out in the same fashion as for the first computation (Figure 5). However, it is necessary to change the signs of the sines and cosines of the azimuths to obtain the same result. This is due to the fact that the quadrants have been swung through 180° because the azimuths are defined from the south. If the azimuths are defined from the north, as will be done in a future example, it will not be necessary to change the signs. The Latitudes are, of course, the cosines of the azimuths times the corresponding distances and the Departures are the sines of the azimuths times the same distances.

Note that although the azimuth (bearing) A-B was changed by 20", the positional closures and corrections to the Latitudes and Departures, and hence to the coordinates, are identical to those obtained in the computation given on Figure 5. However, the Departures between A-B and C-D and the Latitudes between B-C and D-A differ by 0.13 ft. in the two computations with resulting changes in the Northing of B, Northing and Easting of C and Easting of D. These differences in the Latitudes and Departures amount to about 1:10,000 and as noted previously, were known in advance. Note also that the adjusted azimuths and bearings in the two computations amount to about 20". This change as well was known in advance.

To determine the azimuths from Latitudes and Departures, the same formulas as used for deriving the bearings (p. 18) are employed except the values found in the SE and NW quadrants are subtracted from 180° and 180° is added to the bearing determined to be in the NE quadrant. The bearing and azimuth in the SW quadrant are identical. This subject was discussed in some detail earlier in the section entitled "Bearings Versus Azimuths" (pp.7-10).

Although there were several reasons for carrying out the two computations (Figures 5 and 6), among the more important is the need to accentuate the major weakness of loop-type traverses which close upon themselves. That is, errors of a constant or proportional part nature are almost impossible to uncover. In this case, the initial bearing was changed, yet the closures are identical; if the tape (chain) was used in all measurements in such a manner that a porportional part or constant error was introduced, the closures would also remain unchanged. These problems are inherent to loop traverses, closing on themselves and the use of theodolites, EDM, and SPCS can do little to alleviate occurrences of such blunders.

Care in all phases of the field operations and office computations is the only solution to these problems, and even when the greatest care is exercised, blunders can go undetected. Having a good control network available will substantially reduce the chances for this type of blunder to go undiscovered. However, some errors, interchanging of distances when points are on line, for example, are very difficult or impossible to discover under any circumstances and a constant vigil must be maintained especially in recording field data to be sure that the opportunities for blunders of this type be kept to a minimum.

Convergence: It is sometimes required that the correction due to convergence of the meridians be computed at a point in a local system. This correction is applied to all grid bearings or azimuths at the point to obtain corresponding true or geodetic quantities. The formula is as follows:

$\Delta''\alpha = \text{Sine}\ \phi_m\ (\lambda_o - \lambda_p)$ where ϕ_m is the mean latitude of the origin and the point involved and $(\lambda_o - \lambda_p)$ is the difference in seconds between the longitudes of the origin and the particular point. These values can be scaled from a good map to sufficient accuracy. However, for most surveys, the following formula is adequate: $\Delta\alpha$ (Approx.) = Tan ϕ (Departure $_{o-p}$)/102. The latitude (ϕ) need not be more accurate than one or two minutes (1-2 miles) of the mean value and a table of 4 place tangents is entirely satisfactory even for the more distant points from the origin of a local system. As an example:

Origin	$\phi = 44°\ 49'\ 06''$	Point Q	$\phi = 44°\ 51'\ 25''$
	$\lambda = 91\ \ 28\ \ 58$		$\lambda = 91\ \ 22\ \ 20$

Grid Azimuth POINT Q-POINT R = 243° 45' 51" Grid Bearing = N63° 45' 51"S

Sine Mean $\phi = \phi_m =$ Sine 44° 50!3 = 0.705 $\Delta\lambda$ = +6' 38" = +398"

$\Delta\alpha$ = (0.705)(+398") = +281" =+0° 04' 41"

or

Departure Origin to Point Q = +28934 ft. Tanϕ where ϕ = 44° 50'= 0.994

$\Delta\alpha$ = 0.994 (+28934)/102 = +282" which agrees within 1" of the more precise computation.

POINT Q-POINT R = 243° 45' 51" = Grid or plane azimuth

$\Delta\alpha$ = + 0 04 41

POINT Q-POINT R = 243 50 32 = Geodetic or true azimuth

N 63 50 32S = True bearing

To complete the first section of this paper, a computation of the entire survey illustrated by Figure 1 will be made using the data as tabulated in Figure 2. Wisconsin Central zone plane coordinates for MT TOM (CofEC)and Point K will be used to control the survey. Although these coordinates are referenced to sea level, no corrections to the coordinates to place them at the elevation of the site or to reduce the measured lengths to sea level or for scale distortion will be made in this computation. Before continuing, it is thought a brief discussion is now in order regarding the control data issued by the National Geodetic Survey.

Control Data: Figures 7 and 8 are examples of one of the formats used by the National Geodetic Survey to publish data. Newer type and former formats and other data issued by the NGS are discussed in some detail in the paper "National Geodetic Survey Data-Availability-Explanation-Application." This paper is available from the NGS without charge.

MAY 1964

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COAST AND GEODETIC SURVEY
WASHINGTON D.C.

HORIZONTAL CONTROL DATA

by the
Coast and Geodetic Survey
NORTH AMERICAN 1927 DATUM

QUAD 440911 STATION 1003
WIS
LATITUDE 44°30' TO 45°00'
LONGITUDE 91°00' TO 91°30'
DIAGRAM NL 15-12 EAU CLAIRE

MT. TOM (City of Eau Claire) (Eau Claire County, Wis., C.A.S., 1934)--This station is in the east part of the City of Eau Claire on land owned by the City of Eau Claire. It is on a locally prominent hill near the center of sec. 16, T. 27 N., R. 9 W. The mark is set in the center of a narrow ridge which runs in a southwesterly direction from the main part of the mound. It is about 10 feet from the south slope of the ridge, 10 feet from the north slope of the ridge, 70 feet northeast of the southwest end of the ridge, 27-1/2 feet south of a 10-inch oak tree, and projects 4 inches.

Note: This station was established by the Eau Claire City engineers and remarked with standard disks by C.A.S.

Reference mark no.1 is northeast of the station on the northeast slope of the hill, 27 feet north of a 16-inch oak tree, 30 feet east of an oak tree, and projects 4 inches.

Reference mark no.2 is southwest of the station on the south edge of the ridge near the foot path, 30 feet northeast of the southwest end of the ridge, and projects 9 inches.

Azimuth mark is west of the station, in front of 1422 West Summit Street in the sidewalk on the north side of the street, 6 inches north of the south edge of the sidewalk, 34 feet west of the west edge of the walk on the west side of Spring Street.

MT. SIMON and MT. WASHINGTON are visible from the ground at this station.

A 60-foot tower at THOMPSON is visible from the ground at this station.

A light 74 feet at NELS comes into view 40 feet above this station.

A light 47 feet at ZICH comes into view 15 feet above this station.

Surface, underground, reference and azimuth marks are standard bronze disks set as described in notes 1b, 7a and 11b.

OBJECT	DISTANCE	DIRECTION
MT. WASHINGTON	feet	0°00'00"0
R.M.No.2	68.84	1 24 37
Azimuth mark	approx. 0.25 mile	34 45 49.2
Mr. Simon (City of Eau Claire)	V.G.	67 43 31.8
R.M.No.1	103.70	161 24 57
Eau Claire, St. John's Lutheran Church spire	V.G.	358 06 08.8

Height of telescope above station mark - 64 feet.

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: MT TOM CITY OF EAU CLAIRE YEAR 1934

STATE: Wisconsin LOCALITY: Alma to Tomah to Antigo

FIRST -ORDER Triangulation SOURCE: G-6309 FIELD SKETCH: WIS 19-1,24

GRID DATA	COORDINATES (Feet)	PLANE AZIMUTH θ (OR Δα) ANGLE	MARK
STATE: Wis ZONE: C CODE: 4802	x 1,615,233.86 y 362,611.25	98°52'50" -1 02 47	AZIMUTH MARK
STATE: ZONE: CODE:	x y		

GEODETIC DATA	POSITION	SECONDS IN METERS	ELEVATION
	LATITUDE: 44°49'06"086 NORTH		METERS
	LONGITUDE: 91 28 58.263 WEST		FEET

TO STATION	GEODETIC AZIMUTH (From south)	DISTANCE LOGARITHM (Meters)	DISTANCE METERS
	FIRST-ORDER		
THOMPSON	17°34'58"72	4.269 6364	18,605.29
MT WASHINGTON	63 04 14.83	3.694 5439	4,949.30
NELS	243 45 50.66	3.988 8465	9,746.45
ZICH	299 56 10.92	4.156 9094	14,351.90
	THIRD-ORDER		
EAU CLAIRE STATE TEACHERS COLLEGE STACK	30 38 34	3.435 965	2,728.8
EAU CLAIRE FIRST CONGREG CHURCH TOWER	51 14 04.2	3.366 028	2,322.9
EAU CLAIRE HIGH SCHOOL STACK	54 51 15.5	3.116 141	1,306.6
EAU CLAIRE GILLETTE RUBBER CO S STACK	56 47 18.6	2.839 381	690.8
EAU CLAIRE GILLETTE RUBBER CO N STACK	58 12 10.4	2.832 347	679.7
EAU CLAIRE JUNIOR H S STEEPLE	58 33 03.6	3.358 522	2,283.1
EAU CLAIRE ST JOHNS LUTHERAN CH SPIRE	61 10 22.9	3.121 490	1,322.8
EAU CLAIRE ST PATRICKS CHURCH STEEPLE	68 34 34.8	3.332 733	2,151.5
EAU CLAIRE ST PATRICKS SCHOOL CUPOLA	68 38 51.4	3.351 974	2,248.9
EAU CLAIRE SACRED HEART CHURCH S CROSS	89 13 36.2	3.115 204	1,303.8
EAU CLAIRE SACRED HEART CHURCH N CROSS	89 53 48.0	3.115 234	1,303.9
AZIMUTH MARK	97 50 03.3		
EAU CLAIRE SACRED HEART HOSPITAL STACK	98 04 59.2	3.153 976	1,425.5
EAU CLAIRE CO INSANE ASYLUM TANK	113 37 09.6	3.633 488	4,300.2
ALTOONA MUNICIPAL TANK	296 29 59.7	3.627 513	4,241.4
	SECOND ORDER		
ELEVENTH SRTEET (CITY OF EAU CLAIRE)	83 09 12.4	3.531 566	3,400.7
MT SIMON (CITY OF EAU CLAIRE)	130 47 46.6	3.398 626	2,504.0

Figure

FORM 284 (10-23-56)

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HORIZONTAL CONTROL DATA

by the
Coast and Geodetic Survey
NORTH AMERICAN 1927 DATUM

QUAD 440911 STATIONS 1018,1019
WIS
LATITUDE 44°30' TO 45°00'
LONGITUDE 91°00' TO 91°30'
DIAGRAM NL 15-12 EAU CLAIRE

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: EAU CLAIRE SACRED HEART CH S CROSS YEAR: 1934

STATE: Wisconsin LOCALITY: Alma to Tomah to Antigo

Third -ORDER Triangulation SOURCE: G-6309 FIELD SKETCH: WIS 19-I

GRID DATA	COORDINATES (Feet)	PLANE AZIMUTH θ (OR Δα) ANGLE	MARK
STATE: Wis. ZONE: C CODE: 4802	x 1,610,956.65 y 362,631.67	- 1 03 28	
STATE: ZONE: CODE:	x y		

GEODETIC DATA	POSITION		SECONDS IN METERS	ELEVATION
	LATITUDE: 44°49'05".512 LONGITUDE: 91 29 57.597	NORTH WEST		METERS FEET

TO STATION	GEODETIC AZIMUTH (From south)	DISTANCE LOGARITHM (Meters)	DISTANCE METERS
	THIRD-ORDER		
MT WASHINGTON	(54°24'42".2)	3.582 346	3,822.5
MT SIMON (CITY OF EAU CLAIRE)	(160 17 35.0)	3.244 613	1,756.4
MT TOM (CITY OF EAU CLAIRE)	(269 12 54.4)	3.115 204	1,303.8

SACRED HEART CHURCH, S. CROSS (Eau Claire County,Wis., C.A.S.,1934)--This station is the base of the southerly of twin crosses on twin domes on the Sacred Heart Catholic Church in Eau Claire. The church is a red brick building facing east and has a square brick tower at each the northeast and southeast corners. The top of the tower is an octagonal brick belfry with white stone trimmings. The sides of the belfry are open. The top of the belfry is a vari-colored slate dome with a gold cross at the center of the top. The top of the dome is about 90 feet above the street, and is quite prominent. The building was erected in 1928.

The church is about 1/2 mile north of the business section of town on a high hill overlooking the town, about 0.3 mile east of the Chippewa River and about 0.3 mile north of the Dewey Street bridge over the Eau Claire River, on the west side of Dewey Street and on the prolongation westward of the center line of Division Street. The Chicago, St. Paul, Minneapolis & Omaha Railway runs along the base of the bluff about 200 feet west of the church.

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: EAU CLAIRE ST JOHNS LUTH CH SPIRE YEAR: 1934

STATE: Wisconsin LOCALITY: Alma to Tomah to Antigo

Third -ORDER Triangulation SOURCE: G-6309 FIELD SKETCH: WIS 19-I

GRID DATA	COORDINATES (Feet)	PLANE AZIMUTH θ (OR Δα) ANGLE	MARK
STATE: Wis. ZONE: C CODE: 4802	x 1,611,394.50 y 360,588.64	- 1 03 24	
STATE: ZONE: CODE:	x y		

GEODETIC DATA	POSITION		SECONDS IN METERS	ELEVATION
	LATITUDE: 44°48'45".421 LONGITUDE: 91 29 51.001	NORTH WEST		METERS FEET

TO STATION	GEODETIC AZIMUTH (From south)	DISTANCE LOGARITHM (Meters)	DISTANCE METERS
	THIRD-ORDER		
MT WASHINGTON	(63°45'08".5)	3.559 608	3,627.5
MT SIMON (CITY OF EAU CLAIRE)	(162 02 14.1)	3.378 431	2,390.2
MT TOM (CITY OF EAU CLAIRE)	(241 09 45.7)	3.121 490	1,322.8

ST. JOHN'S LUTHERAN CHURCH, SPIRE (Eau Claire County,Wis., C.A.S.,1934)--This station is the tip of the spire surmounting the St. John's Lutheran Church in Eau Claire. The church is a red brick building with a square red brick tower at the front (west) center over the entrance. There is a circular ornamental window on each side of the tower. Above the tower is an octagonal belfry with a louvered window on each side. Above the belfry is an octagonal green shingle spire. The spire has a ball at the tip and a finial with weather vane above the ball. The spire rises about 125 feet above the street and is quite prominent. The building was erected in 1882.

The church is in the business section of town, about 1300 feet east of the Chippewa River at the mouth of the Eau Claire River and about 1 block south of the Eau Claire River, at the east corner of South Dewey Street and Gibson Street.

Figure 8

Much of the information given on these data sheets is of little interest to most surveyors yet some of the data which are generally ignored are very important and extremely useful. More on this shortly. The primary consideration are the descriptions and the grid data. On occasion, the latitude may be used as an aid in determining the scale factor necessary to reduce measured distances to grid values on the Lambert system and both latitudes and longitude are useful if observations on Polaris are made. Elevations, when given, are helpful in deriving the elevation factors to reduce lengths to sea level, but are not generally recommended as control for leveling projects where a high accuracy in the actual elevation of the points is needed. It is much better to obtain up-to-date leveling data from the NGS on these occasions.

Important information, which is generally overlooked, are the distances and directions to the reference marks and the directions to other marks and objects in the box. To assure that the station monument has not been disturbed, the distances to the references marks and the angle between them, and if practical, the angle between the azimuth mark and one of the RM's should always be measured. Distances should check within 0.03 to 0.05 ft. and angles to ±3' to 5'. If any discrepancies significantly larger than these tolerances are found, the NGS should be advised. Often monuments have been moved for one reason or another by uninformed individuals and after a short period of time, there is little evidence to this effect which can be gained through a casual on site inspection of the monument. As an additional check, if a theodolite is available, and two or more distant marks or objects are visible from the stations, observations should be made between these points, and compared with the information in the box or through computations. In these cases, a check of 5" to 10" is usually considered satisfactory if the points are a mile or more distant. No great effort need be expended in securing these checks. A cloth tape, if nothing else is available, may be used to make the measurements and a single pointing of a transit or theodolite is sufficient to check the angles.

The word "Direction" as used in the descriptions does not mean a compass bearing, but simply refers to a method of observing. All the angles shown in a box of a description are referenced to the initial or 0°00'00".0 value. To obtain the angle between points other than those involving the initial, subtract one direction from the other. For example: From Figure 7 - the angle between the Azimuth and RM No. 1 is 181°24'57" minus 34°45'49" = 146°39'08".

Many surveyors are understandably uncomfortable when the data are issued in a X-Y and grid azimuths referenced from the south format. To these individuals, there is but one answer - record the data in your personal files or change the data sheets to suit your preferences. For example: Figures 9 and 10 are simply the data given in Figures 7 and 8 which have been changed such that the Y's are now Northings and X's are now Eastings, the plane azimuth (Figure 7) is now a plane bearing. An additional quantity, the plane azimuth referenced from the north is also given.

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: MT TOM CITY OF EAU CLAIRE YEAR: 1934

STATE: Wisconsin LOCALITY: Alma to Tomah to Antigo

First -ORDER Triangulation SOURCE: G-6309 FIELD SKETCH: WIS 19-I,24

GRID DATA	COORDINATES (Feet)	PLANE Bearing θ(OR Δα) ANGLE	MARK
STATE: Wis ZONE: C CODE: 4802	N 362,611.25 E1,615,233.86	N 81°07'10"W - 1 02 47 278 52 50	AZIMUTH MARK AZIMUTH MARK (Azimuth from north)
STATE: ZONE: CODE:			

Figure 9

ADJUSTED HORIZONTAL CONTROL DATA

NAME OF STATION: EAU CLAIRE SACRED HEART CH S CROSS YEAR: 1934

STATE: Wisconsin LOCALITY: Alma to Tomah to Antigo

Third -ORDER Triangulation SOURCE: G-6309 FIELD SKETCH: WIS 19-I

GRID DATA	COORDINATES (Feet)	PLANE Azimuth θ(OR Δα) ANGLE	MARK
STATE: Wis. ZONE: C CODE: 4802	N 362,631.67 E1,610,956.65	- 1 03 28	
STATE: ZONE: CODE:			

Figure 10

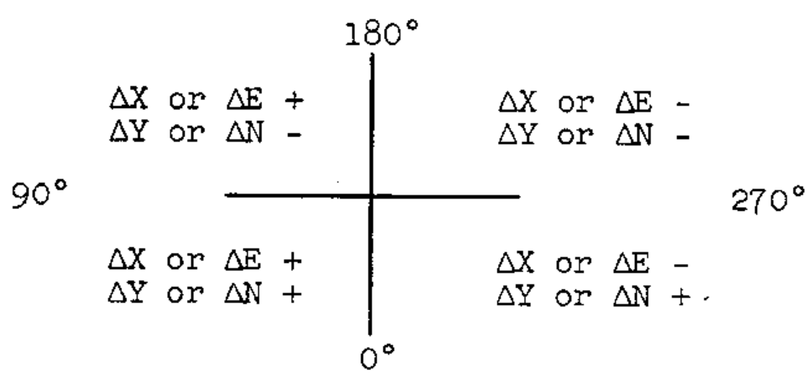
<u>Computation of Grid Azimuths</u>: Grid azimuths referenced from the south may be computed from the following formulas:

(1) Tan grid azimuth $_{1-2}$ = $\frac{X_1 - X_2}{Y_1 - Y_2}$ or Cot. grid azimuth $_{1-2}$ = $\frac{Y_1 - Y_2}{X_1 - X_2}$ or if Northings and Eastings are used

Tan grid azimuth $_{1-2}$ = $\frac{E_1 - E_2}{N_1 - N_2}$ or Cot. grid azimuth $_{1-2}$ = $\frac{N_1 - N_2}{E_1 - E_2}$

Generally, it is best to use the value for the tangent or cotangent which is less than one. The reason for this practice is simply a matter of significant figures as discussed previously.

The signs of differences define the quadrant as shown below. Consider $X_1 - X_2$ as ΔX, $Y_1 - Y_2$ as ΔY, $N_1 - N_2$ as ΔN and $E_1 - E_2$ as ΔE.



To obtain the azimuths apply the computed angles as follows:

In the first quadrant (0°-90°) determine the angle directly from the function, either tangent or cotangent, this is the azimuth. Do the same for the third quadrant (180°-270°) and add 180°. For the second (90°-180°) or fourth quadrant (270°-360°), determine the angles directly from the functions and subtract from 180° or 360°, respectively. In the second and fourth quadrants, the angles from the opposite functions (Cotangent for Tangent or Tangent for Cotangent) may be determined and added to 90° and 270°, respectively. The grid Bearings are simply the angles as computed.

(2) If the geodetic azimuth to a point is known, the grid azimuth may be obtained to within a few seconds by applying the θ angle (Lambert projection) or $\Delta\alpha$ angle (transverse Mercator projection) with the opposite sign from that given. For example from Figure 7:

MT TOM (CofEC) - ALTOONA MUNICIPAL TANK	296°	30'	00"	Geodetic
$-\theta =$	+ 1	02	47	
MT TOM (CofEC) - ALTOONA MUNICIPAL TANK	297	32	47	Grid

(3) On occasion, there may be observations shown in the box of the descriptions to intersected or marked points for which no positions were determined. Yet the points are visible and could serve as good azimuth marks. To determine the grid azimuths to the points, compute the grid azimuth to the initial station, either through coordinates or by the application of the θ or $\Delta\alpha$ angle and add the directions to the points. For example: assume that no position is available for MT SIMON (CofEC) but the azimuth to the point is required. Rather than compute the plane azimuth to MT. WASHINGTON, the initial station, through differences in coordinates the grid value is derived by using the θ angle and the grid azimuth to MT SIMON (CofEC) is determined as follows.

MT TOM (CofEC) - MT WASHINGTON	63°	04'	15"	Geodetic
$-\theta =$	+ 1	02	47	
MT TOM (CofEC) - MT WASHINGTON	64	07	02	Grid
Direction to MT SIMON (CofEC) from box	+67	43	32	
MT TOM (CofEC) - MT SIMON (CofEC)	131	50	34	Grid

If grid azimuths from the north are desired, the formulas given in Item 1 the identical formulas would be used, but the signs would apply to the quadrants as shown in the following diagram. The bearings would be identical to the values as computed from the functions, but the grid azimuths would follow the conventions as previously described.

	0°	
270°	ΔX or ΔE − ΔY or ΔN +	ΔX or ΔE + ΔY or ΔN +
	ΔX or ΔE − ΔY or ΔN −	ΔX or ΔE + ΔY or ΔN −
	180°	90°

Computation of Coordinates Using Ground Level Lengths and SPCS Coordinates as Control: Although there are two routes available to compute the preliminary bearings (MT TOM (CofEC)-A-B-C-K or MT TOM (CofEC)-A-D-C-D) and both contain the same number of angle points, the first name route (Figure 11) was selected. The closure was distributed and the bearings were used to compute the coordinates for the points involved. After the closures were balanced out, the adjusted coordinates were employed to compute the adjusted bearings A-B and C-B necessary to control the second computation (Figure 12). When more than one route is available between known bearings, it is usually best to select for the first computation the route containing the fewest number of angle points, and to continue this practice of the fewest number of angle points for successive routes.

In the first computation (Figure 11) bearings are employed and will be used in determining the Latitudes and Departures in Figure 13. In the second computation (Figure 12) azimuths from the south were determined and used in the determination of the Latitudes and Departures (Figure 14).

The control at Point K involves an observation to EAU CLAIRE SACRED HEART CH S CROSS and it is necessary to compute the bearing from Point K. The coordinates for EAU CLAIRE SACRED HEART CH S CROSS are given in Figures 8 and 10. Wisconsin central zone coordinates for Point K follow:

$N = Y = 364{,}664.01 \qquad E = X = 1{,}618{,}667.78$

Point K - S CROSS $\quad \Delta N = \Delta Y = +2032.34 \quad \Delta E = \Delta X = +7711.13$

Cotangent = +2032.34/+7711.13 = 0.2635593

Point K = S CROSS = $75° \; 14' \; 06''$ Grid Azimuth from South

S 75 14 06 W

The computation and adjustment of the traverses are shown by Figures 13 and 14. No explanations are given since the procedures follow those previously described (pp. 14-18). The closures, of course, are not really indicative of the quality of the measurements since the lengths were not reduced to sea level nor to the grid.

Many surveyors do not wish to be concerned with the sea level and scale factor corrections even when it can be shown that the effort to apply these corrections, is for all intents and purposes, negligible. In these circumstances, one must be prepared to "pay the piper" for the computed coordinates and subsequently the distances and bearings derived from these values may be excessively in error. This is perhaps as good a time as any to put an old adage to rest - the sea level and scale corrections are not taken care of by the balancing procedures when the control consists of two or more points whose coordinates were determined on the SPCS. The comparison of the various computations which will be made later will provide

STATION FROM	STATION TO	PRELIMINARY Bearing ° ′ ″	CORRECTION FOR CLOSURE ′ ″	CORRECTED Bearing ° ′ ″
MT TOM(C of EC)	Azimuth Mark	N 81 07 10 W		N 81 07 10 W
∠		126 03 40 *		
MT TOM(C of EC)	A	N 44 56 30 E	- 3	N 44 56 27 E
A	MT TOM(C of EC)	S 44 56 30 W		
∠		43 47 35 L		
A	B	N 1 08 55 E	- 6	N 1 08 49 E
B	A	S 1 08 55 W		
∠		89 54 30 R		
B	C	S 88 56 35 E	+ 8	S 88 56 43 E
C	B	N 88 56 35 W		
∠		0 00 30 R		
C	K	S 88 56 05 E	+11	S 88 56 16 E
K	C	N 88 56 05 W		
∠		344 10 25 *		
K	EAU CLAIRE	S 75 14 20 W	-14	S 75 14 06 W
	SACRED HEART CH S CROSS			
* Azimuth	Angle		14/5 = -2″.8	per angle
		Figure 11		

STATION FROM	STATION TO	PRELIMINARY AZIMUTH ° ′ ″	CORRECTION FOR CLOSURE ′ ″	CORRECTED AZIMUTH ° ′ ″
A	B	181 08 28		181 08 28
∠		89 54 00 *		
A	D	271 02 28	+11	271 02 39
D	A	91 02 28		
∠		89 54 35 *		
D	C	180 57 03	+23	180 57 26
C	D	0 57 03		
∠		89 53 55 L		
C	B	271 03 08	+34	271 03 42
Azimuths	from South		34/3 = + 11.	33 per angle
* Azimuth	Angle			
		Figure 12		

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	Plane Bearing	Distance	SIN Bearing / COS Bearing	Latitude	Departure	GRID COORDINATES Northing (Y)	Easting(X)
	° ' "	Feet		Feet	Feet	Feet	Feet
MT TOM (CofEC)					**	362,611.25	1,615,233.86
			+0.7063762	+ 780.98			
	N 44 56 27 E	1103.34			*	363,392.23	1,616,013.23
			+0.7078366		+ 779.37	- 0.13	- 0.12
A					**	363,392.10	1,616,013.11
			+0.0200166	+1320.95			
	N 1 08 49 E	1321.21			*	364,713.18	1,616,039.68
			+0.9997996		+ 26.45	- 0.29	- 0.26
B					**	364,712.89	1,616,039.42
			+0.9998306	- 24.21			
	S 88 56 43 E	1314.99			*	364,688.97	1,617,354.45
			-0.0184073		+1314.77	- 0.44	- 0.40
C					**	364,688.53	1,617,354.05
			+0.9998282	- 24.36			
	S 88 56 16E	1314.10			*	364,664.61	1,618,668.32
			-0.0185382		+1313.87	- 0.60	- 0.54
K	Sum of Lengths	5053.64			**	364,664.01	1,618,667.78

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Closure $= \Delta N = \Delta Y = -0.60$ $\quad \Delta E = \Delta X = -0.54$ $\quad \Delta S = \sqrt{(0.60)^2 + (0.54)^2} = 0.81$

$0.81/5053.64 = 1:6,239$

N or Y Factor $= -0.60/5053.64 = -0.1187$ per 1000 feet * Preliminary Coordinates

E or X Factor $= -0.54/5053.64 = -0.1069$ per 1000 feet ** Adjusted Coordinates

Figure 13

some further evidence in this regard.

Finally, in a strict sense, the computed coordinates are not SPCS values, nor is it a simple matter to convert them exactly to the State systems except by recomputation. It is, therefore, strongly recommended that when this type of computation is carried out that a full and clear explanation be placed on all relevant documents.

Adjusted Latitudes, Departures, Distances and Bearings (Azimuths)

From	To	Latitude	Departure	Distance	Bearing Azimuth
MT TOM (CofEC)	A	+780.85	+779.25	1103.16	N 44°56'28"E 224 56 28
A	B	+1320.79	+ 26.31	1321.05	N 1 08 28 E 181 08 28
B	C	- 24.36	+1314.63	1314.86	S 88 56 18 E 271 03 42
C	K	- 24.52	+1313.73	1313.96	S 88 55 51 E 271 04 09

Figure 13 (Cont.)

Computations - Part II

In this part of the paper brief discussions of the sea level and scale factor corrections will be made, two computations of the example will be carried out, the area of the tract A-B-C-D will be determined from all five computations and finally the mapping angles for both the Lambert and transverse Mercator projections will be derived.

Sea Level Correction: Except for a very few places in the United States, Death Valley for one, Figure 15 is representative of the relationship of the ground distance "D" with the distance at sea level "S" which is the geodetic length. It may surprise some, but any length measurement is at the mean elevation of the points involved and not directly between the points even after the length has been reduced to the horizontal.

Although most surveyors and engineers usually require that the distances obtained from a survey be at ground level, it would be impossible in a country the size of the United States with elevations ranging from about 280 feet below sea level to 20,000 feet above sea level to have a single elevation reference surface other than sea level. It is true that in the Michigan Lambert System the SPCS coordinates are computed at 800 feet above sea level, but this is the only exception.

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	Distance	SIN AZIMUTH / COS AZIMUTH	Latitude	Departure	GRID COORDINATES Northing (Y)	Easting (X)
	° ' "	Feet		Feet	Feet	Feet	Feet
A					**	363,392.10	1,616,013.11
			-0.9998339	- 24.05			
	271 02 39	1319.70			*	363,368.05	1,617,332.59
			+0.0182231		+1319.48	- 0.01	- 0.30
D					**	363,368.04	1,617,332.29
			-0.0167059	+1320.51			
	180 57 26	1320.69			*	364,688.56	1,617,354.65
			-0.9998604		+ 22.06	- 0.03	- 0.60
C	Sum of Lengths	2640.39			**	364,688.53	1,617,354.05

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Closure $\Delta N = \Delta Y = -0.03$ $\Delta E = \Delta X = -0.60$ $\sqrt{(0.03)^2 + (0.60)^2} = 0.60$ $0.60/2640.39 = 1:4,401$

N or Y Factor = -0.03/2640.39 = -0.01136 per 1000 feet * Preliminary Coordinates

E or X Factor = -0.60/2640.39 = -0.2272 per 1000 feet ** Adjusted Coordinates

Adjusted Latitudes, Departures, Distances and Azimuths (Bearings)

From	To	Latitude	Departure	Distance	Azimuth/ Bearing
A	D	- 24.06	+1319.18	1319.40	271° 02' 42" S 88 57 18 E
D	C	+1320.49	+ 21.76	1320.67	180 56 39 N 0 56 39 E

Figure 14

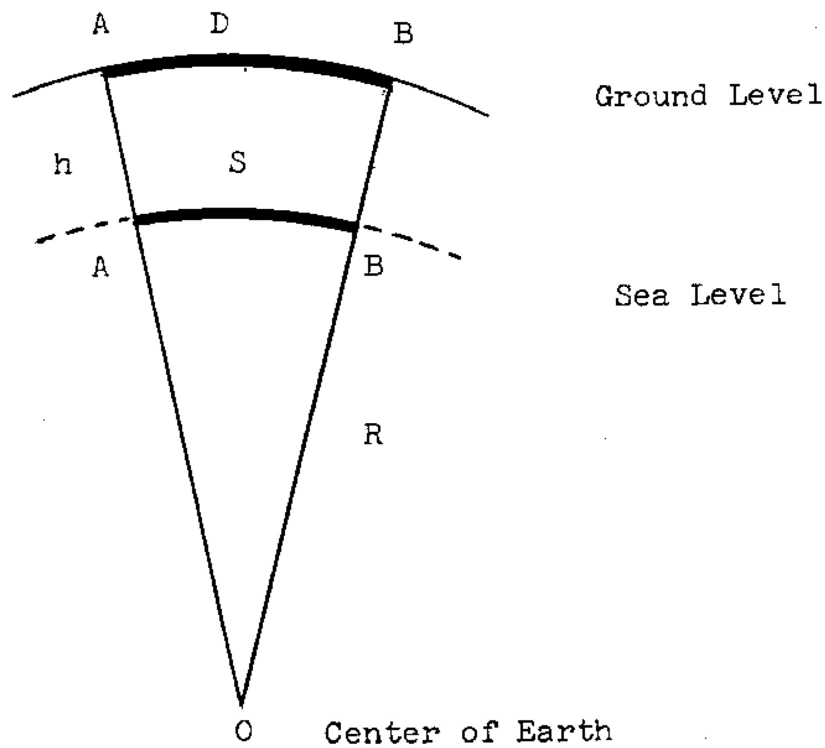


Figure 15

Although the average elevation for Michigan approximates 800 feet, the range is from about 570 feet on the shores of Lake Erie to almost 2000 feet in the northwestern part of the State. Thus, in the more precise surveys, distances must be corrected to this reference surface and, in most cases, corrections for scale are required.

The correction to sea level, to sufficient precision for any survey that might be undertaken by the average surveyor is a relatively simple matter. Although the most precise computation is not a particularlyinvolved exercise, there seems little need to go into details in this paper. If anyone believes that a greater precision is necessary, each length can be corrected for the mean elevation of the two points using the formula that follows.

Returning to Figure 15, an average value for the radius (R) of the earth at the mean latitude of the contiguous United States is about 20,906,000 feet. If "h" is the mean elevation of A and B, then "S" the sea level (geodetic) distance can be determined from the horizontal distance "D" through the following formula: $S = D \times (20{,}906{,}000 / [20{,}906{,}000 + h])$. "D" and "S" are in feet. The proof of this formula is given in many surveying publications and will not be repeated here. Special Publication No. 235 "The State Coordinate Systems" is one such publication.

The factor 20,906,000/(20,906,000+h) can usually be determined for the average elevation of a project. To clarify this point somewhat, for each 21 feet difference in the mean elevation the lengths would be changed 1:1,000,000. Thus, if the total difference in elevation in a project is 1000 feet with the result that the mean value would vary from the extremes by ± 500, the errors introduced into the lengths would not be more than 1:41,000. An example follows:

D = 23,435.26 ft.   h = 2626 ft.

S = 23,435.26 (20,906,000/20,908,626) = 23432.32 ft.

Sea Level Factor (20,906,000/20,908,626)= 0.99987441

Later it will be shown how the sea level factor and scale factor can be combined into a single multiplier.

Scale Factors: One of the reasons, repeated time after time, for not using the SPCS is the scale factor problem. Admittedly, it is difficult to easily understand that ground measured distances must be corrected often by rather significant amounts to account for the distortions in a map projection. Furthermore, the fact that the scale factor may either increase or decrease the measurements adds to the confusion. Although the areas where the scale factor are greater or less than one can be shown graphically, all other proofs involve rather complicated mathematics. It would make little sense to attempt to give the proofs in this paper and hopefully once it is shown just how simple the computations can be made and applied to the lengths, the scale factors will be accepted.

Figure 16 represents the range of the scale factors in the Lambert projection. The factors are functions of latitude and are less than one between the standard parallels and greater than one north and south of these parallels. The maximum scale factor less than one occurs about midway between the two standard parallels. In general, two thirds (2/3) of a zone is contained within the standard parallels and one sixth (1/6) to the north and south of the parallels. As noted in the definitions, the Lambert projection was selected for those states which extend east to west primarily. For examples: North Carolina and Tennessee each utilize but a single zone and a state the size of Texas has only five zones.

The projection tables for each state contain a sketch with various scale ratio for particular sections noted. When more than one zone is required the counties included in each zone are also shown on the sketch. Figure 18 is the sketch included in the tables for the State of Wisconsin and is representative of sketches contained in the tables for those states which employ the Lambert projection. Note that the scale ratios are tabulated north to south, indicative that the scale factors are functions of the latitudes.

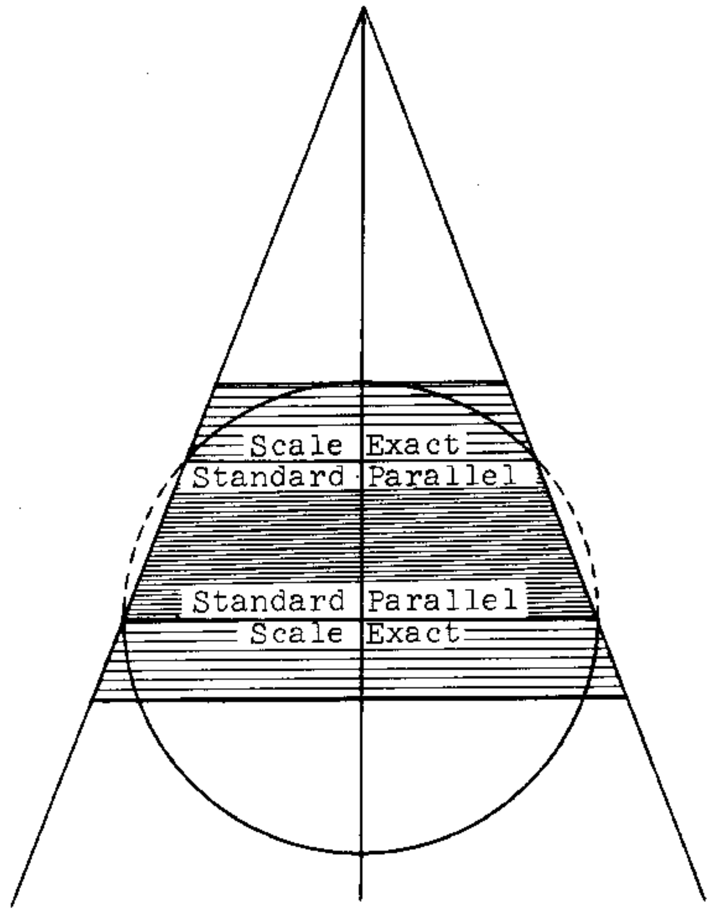


Figure 16

Lambert Projection - Cone Secant to Sphere

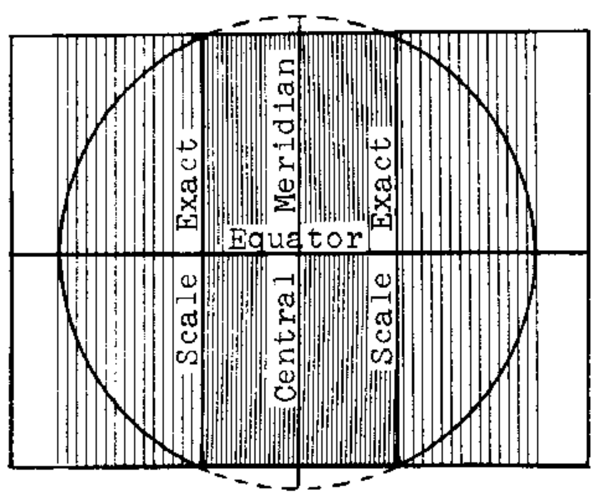
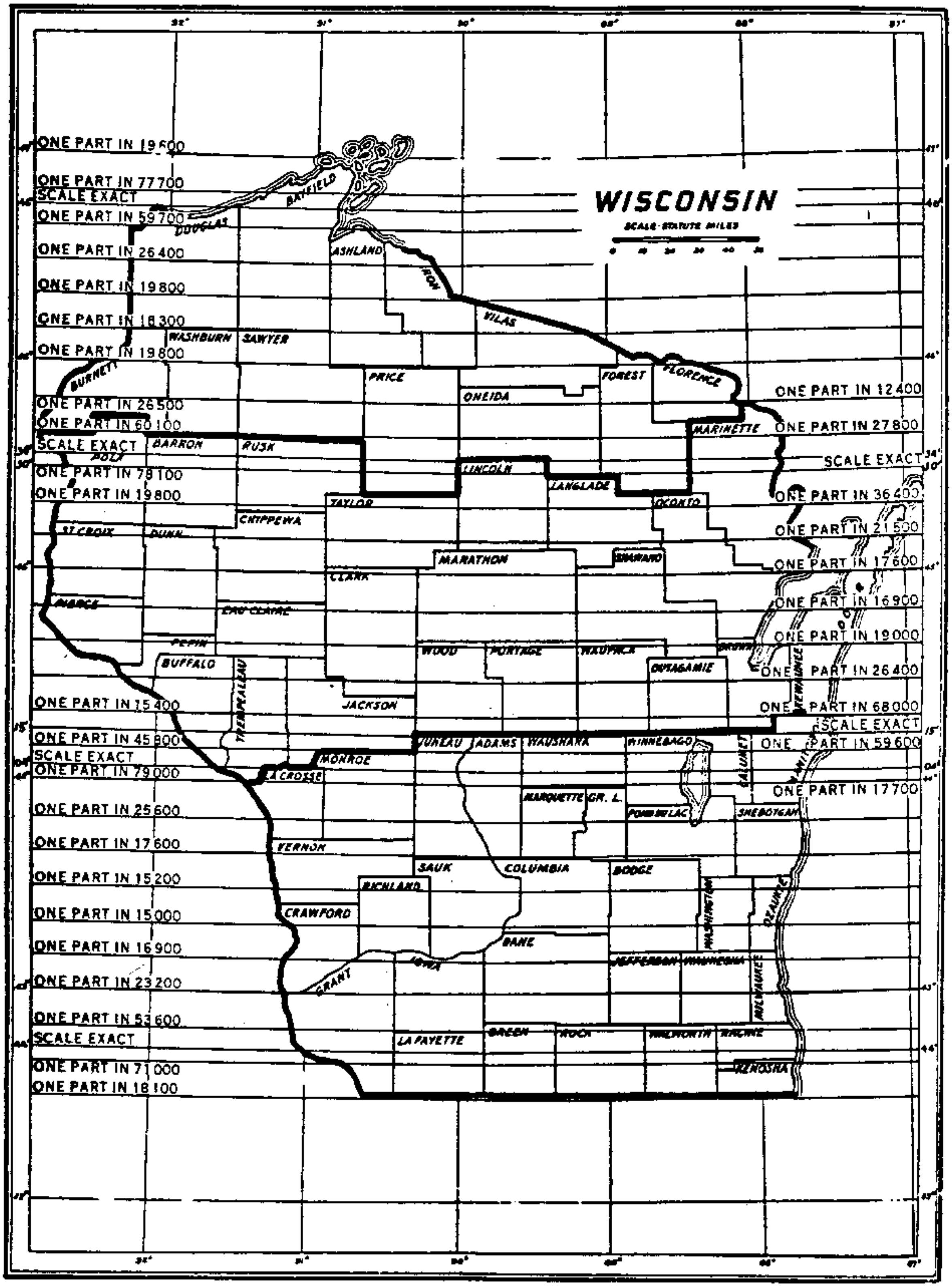


Figure 17

Transverse Mercator Projection - Cylinder Secant to Sphere



STATE PLANE-COORDINATE ZONES AND SCALE FACTORS

Figure 18

LAMBERT PROJECTION FOR WISCONSIN (CENTRAL)

Table I

Lat.	R (feet)	y' y value on central meridian (feet)	Tabular difference for 1 sec. of lat. (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
44° 26'	21,212,180.19	218,733.71	101.26517	-128.6	0.9999704
27	21,206,104.28	224,809.62	101.26517	-138.1	0.9999682
28	21,200,028.37	230,885.53	101.26533	-147.2	0.9999661
29	21,193,952.45	236,961.45	101.26533	-156.0	0.9999641
30	21,187,876.53	243,037.37	101.26550	-164.4	0.9999621
44° 31'	21,181,800.60	249,113.30	101.26567	-172.5	0.9999603
32	21,175,724.66	255,189.24	101.26583	-180.2	0.9999585
33	21,169,648.71	261,265.19	101.26600	-187.5	0.9999568
34	21,163,572.75	267,341.15	101.26600	-194.5	0.9999552
35	21,157,496.79	273,417.11	101.26617	-201.1	0.9999537
44° 36'	21,151,420.82	279,493.08	101.26633	-207.3	0.9999523
37	21,145,344.84	285,569.06	101.26650	-213.2	0.9999509
38	21,139,268.85	291,645.05	101.26667	-218.7	0.9999496
39	21,133,192.85	297,721.05	101.26700	-223.8	0.9999485
40	21,127,116.83	303,797.07	101.26700	-228.6	0.9999474
44° 41'	21,121,040.81	309,873.09	101.26733	-233.0	0.9999463
42	21,114,964.77	315,949.13	101.26750	-237.1	0.9999454
43	21,108,888.72	322,025.18	101.26767	-240.8	0.9999446
44	21,102,812.66	328,101.24	101.26800	-244.1	0.9999438
45	21,096,736.58	334,177.32	101.26817	-247.0	0.9999431
44° 46'	21,090,660.49	340,253.41	101.26833	-249.6	0.9999425
47	21,084,584.39	346,329.51	101.26867	-251.8	0.9999420
48	21,078,508.27	352,405.63	101.26900	-253.7	0.9999416
49	21,072,432.13	358,481.77	101.26917	-255.2	0.9999412
50	21,066,355.98	364,557.92	101.26950	-256.3	0.9999410
44° 51'	21,060,279.81	370,634.09	101.26967	-257.1	0.9999408
52	21,054,203.63	376,710.27	101.27000	-257.5	0.9999407
53	21,048,127.43	382,786.47	101.27050	-257.5	0.9999407
54	21,042,051.20	388,862.70	27067	-257.2	0.9999408
55	21,035,974.96	394,938.94	[illegible].27100	-256.4	0.9999410
56'	21,029,898.70	401,015.20	101.27133	-255.3	0.9999412
57	21,023,822.42	407,091.48	101.27167	-253.9	0.9999415
58	21,017,746.12	413,167.78	101.27200	-252.1	0.9999420
59	21,011,669.80	419,244.10	101.27233	-249.9	0.9999425
45° 00	21,005,593.46	425,320.44	101.27267	-247.4	0.9999430

Figure 19

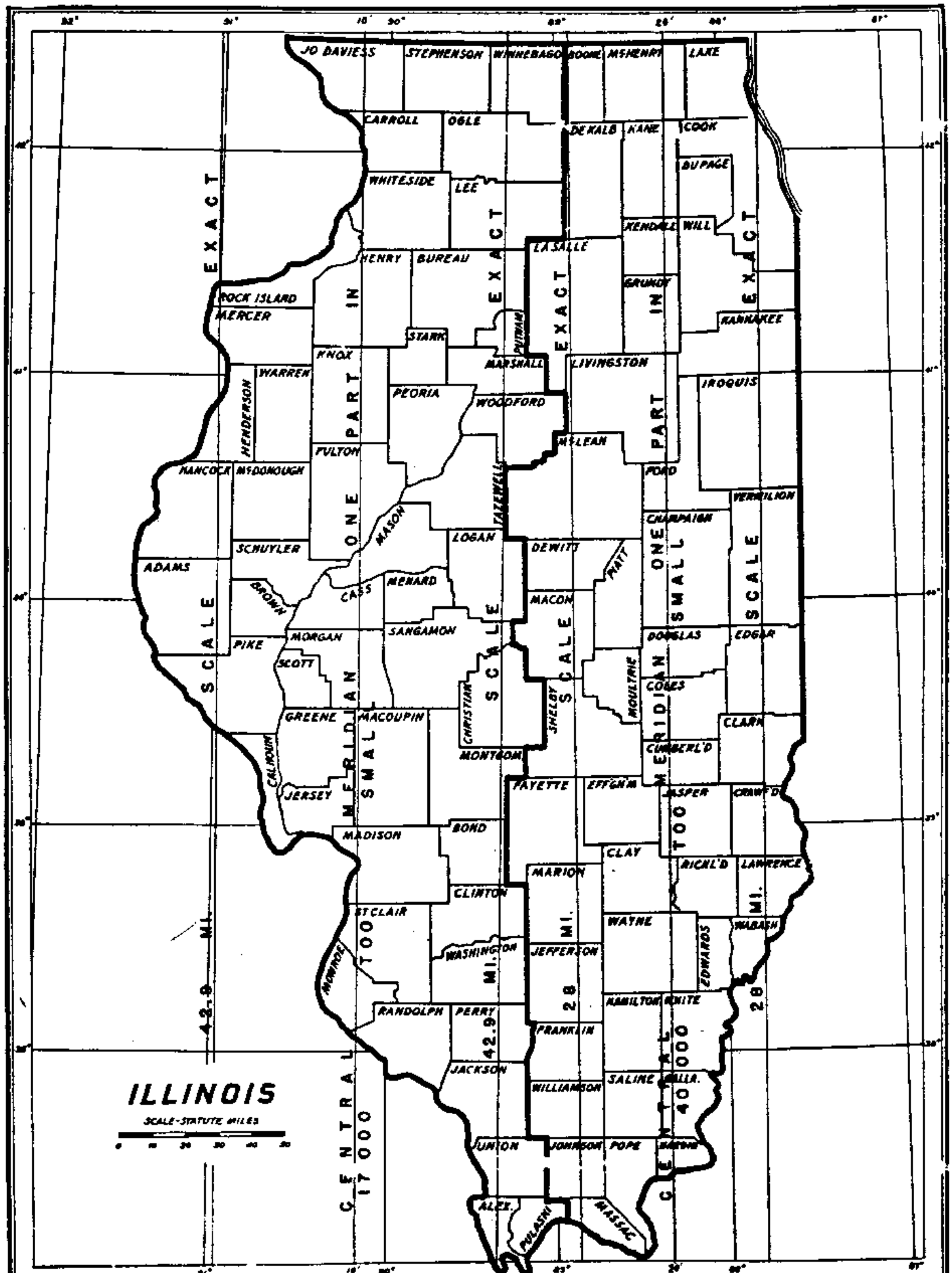
Scale factors for those states using the Lambert projection are given on Figure 19 as "Scale expressed as a ratio" on the extreme right side of the table. Since these factors are a function of the latitude, the scale factor for latitude (ϕ) 44° 41' = 0.9999463. When determining the average scale factor for a project, the mean latitude need not be too accurately determined. For example: if the scale factor for ϕ = 44° 36' rather than that for ϕ = 44° 41' (a difference of more than 5 miles), was used the grid lengths would only be in error by 6 parts per million or 1:166,667. A change of one in the seventh place of the scale factor is 1:10,000,000, similarly one in the sixth place is 1:1,000,000, one if the fifth place is 1:100,000 etc. Although the Lambert projection is being discussed here, the same changes in the scale factors for the transverse Mercator projection would produce identical results.

As indicated by Figure 17, the scale factors on the transverse Mercator projection extend east and west from the Central Meridian, and are functions of the X's or the distances in feet the points are from this meridian. The maximum scale factors less than one occur on the central meridian, and are exact on the grid meridians which are at the same distance in feet east and west of the central meridians. Beyond these grid meridians, the scale factors are greater than one.

Figure 20 is the sketch included with the projection table for Illinois, one of the states in which the SPCS coordinates are computed on the transverse Mercator projection. Note that the scale ratios are shown as being east and west of the central meridians.

For all states which employ the transverse Mercator projection the scale factors are given in tables as illustrated by Figure 21. The scale factors, as noted, are functions of the distances (X's) that the points extend east and west from the central meridians, and are given in the columns identified as "Scale expressed as a ratio." Figure 21 is for use with plane coordinates computed on the Illinois State system and applies only to coordinates computed in the East zone. There is a separate table for Illinois West zone. When one table is used for more than one zone, this indicates the scale reduction is identical on the central meridian for the zones. To use, simply determine a mean X' value for the project, enter the table and take out the scale factor. For example: X' = 272,000 feet, the interpolated value would be = 2000/5000 x (+0.0000032) + 1.0000583 = + 0.0000013 + 1.0000583 = 1.0000596. No great harm would be done if the scale factor for X' = 270,000 feet or X' = 275,000 feet or even X's of 255,000 or 285,000 feet had been used. The maximum error to the grid lengths at the extremes would only be about 1:100,000 in these cases.

Combined Factors: To combine the sea level and scale factors simply multiply them together or the same result may be obtained by adding the two quantities and then subtract one. This method of combining the factors applies for both projections. To



STATE PLANE-COORDINATE ZONES

Figure 20

TRANSVERSE MERCATOR PROJECTION

ILLINOIS

East Zone

x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio	x' (feet)	Scale in units of 7th place of logs	Scale expressed as a ratio
0	-108.6	0.9999750	175,000	+43.5	1.0000100
5,000	-108.5	0.9999750	180,000	+52.3	1.0000120
10,000	-108.1	0.9999751	185,000	+61.3	1.0000141
15,000	-107.5	0.9999752	190,000	+70.6	1.0000163
20,000	-106.6	0.9999755	195,000	+80.2	1.0000185
25,000	-105.5	0.9999757	200,000	+90.0	1.0000207
30,000	-104.1	0.9999760	205,000	+100.1	1.0000230
35,000	-102.5	0.9999764	210,000	+110.4	1.0000254
40,000	-100.7	0.9999768	215,000	+120.9	1.0000278
45,000	-98.5	0.9999773	220,000	+131.7	1.0000303
50,000	-96.2	0.9999778	225,000	+142.8	1.0000329
55,000	-93.6	0.9999784	230,000	+154.1	1.0000355
60,000	-90.7	0.9999791	235,000	+165.6	1.0000381
65,000	-87.6	0.9999798	240,000	+177.4	1.0000408
70,000	-84.3	0.9999806	245,000	+189.4	1.0000436
75,000	-80.7	0.9999814	250,000	+201.7	1.0000464
80,000	-76.8	0.9999823	255,000	+214.3	1.0000493
85,000	-72.7	0.9999833	260,000	+227.0	1.0000523
90,000	-68.4	0.9999843	265,000	+240.1	1.0000553
95,000	-63.8	0.9999853	270,000	+253.4	1.0000583
100,000	-58.9	0.9999864	275,000	+266.9	1.0000615
105,000	-53.9	0.9999876	280,000	+280.7	1.0000646
110,000	-48.5	0.9999888	285,000	+294.7	1.0000679
115,000	-42.9	0.9999901	290,000	+309.0	1.0000711
120,000	-37.1	0.9999915	295,000	+323.5	1.0000745
125,000	-31.0	0.9999929	300,000	+338.3	1.0000779
130,000	-24.7	0.9999943	305,000	+353.3	1.0000814
135,000	-18.1	0.9999958	310,000	+368.5	1.0000849
140,000	-11.3	0.9999974	315,000	+384.1	1.0000884
145,000	-4.2	0.9999990	320,000	+399.8	1.0000921
150,000	+3.1	1.0000007	325,000	+415.8	1.0000957
155,000	+10.7	1.0000025	330,000	+432.1	1.0000995
160,000	+18.5	1.0000043	335,000	+448.6	1.0001033
165,000	+26.6	1.0000061	340,000	+465.4	1.0001072
170,000	+34.9	1.0000080	345,000	+482.4	1.0001111
			350,000	+499.6	1.0001150

Figure 21

illustrate this point, let us assume that the maximum elevation in Wisconsin is located at the latitude of the Central zone where the scale factor is at the maximum less than one. A similar example is also given for Illinois.

(1) Wisconsin - maximum elevation = 1952 feet above sea level

Sea Level factor = 20,906,000/20,907,952 = 0.9999066

Central Zone scale factor (ϕ = 44° 52') = 0.9999407

Combined factor by addition minus one = 0.9998473

Combined factor by multiplication = 0.9998473

(2) Illinois - maximum elevation = 1235 feet above sea level

Sea Level factor = 20,906,000/20,907,235 = 0.9999409

East Zone Scale factor (X' = 5000 ft.) = 0.9999750

Combined factor by addition minus one = 0.9999159

Combined factor by multiplication = 0.9999159

Whem combining factors, it will often be found that the scale factors north and south of the standard parallels on the Lambert system and east and west of the grid meridians on the transverse Mercator projection being greater than one will cancel out the sea level factors.

Computation on the SPCS: The first step is to determine the sea level factor for the mean elevation of the project, then the scale factor for the mean latitude (Lambert system) or mean X' (transverse Mercator projection) and combine the two factors into a single multiplier.

For the example used here, the mean elevation was assumed to be 950 feet above sea level with an average latitude of 44°49' for the project.

Sea level factor = 20,906,000/20,906,950 = 0.9999546

Central Zone scale factor (ϕ = 44°49') from Fig. 19 = 0.9999412

Combined factor = 0.9998958

Each horizontal distance is then multiplied by this combined factor to obtain the grid distances which are then used in the computation of the SPCS coordinates (Figures 22 and 24).

From	To	Horiz. Distance	Grid Distance
MT TOM (CofEC)	A	1103.34	1103.23
A	B	1321.21	1321.07
A	D	1319.70	1319.56
B	C	1314.99	1314.85
C	D	1320.69	1320.55
C	K	1314.10	1313.96

The recommended procedures for computing SPCS coordinates for any survey connected to SPCS coordinated control are shown by Figures 22 and 24. Preliminary bearings for the computations given on Figure 22 are from Figure 11. As one can easily see, the only difference between these calculations and those made previously (Figures 13 and 14) and, in fact, those shown by Figures 5 and 6 is the application of a combined elevation and scale factor to the horizontal distances. Except for the Michigan Lambert grid, all coordinates computed in this manner are referenced to sea level. There is one other minor change, the azimuths used in the computation shown by Figures 23 and in determining the coordinates (Figure 24) are based on North as the origin. The final coordinates for Point D are identical to those that would have been obtained had azimuths from the south or bearings been employed. It is thought that by now the subject of bearings versus azimuths whether referenced with their origins to the North or South has been sufficiently emphasized and no further mention (except to indicate the origin) will be made in the computations which follow.

The azimuths A-B and C-D used to control the computation of the preliminary azimuths (Figure 23) were determined, as has been done in the previous instance from the adjusted Latitudes and Departures (Figure 22).

To obtain ground level adjusted lengths, simply divide the adjusted grid lengths by the combined factor (p. 40) or multiply them by the reciprocal of this factor. The ground level adjusted lengths are given in the following tabulation and will be used in the comparison with other computations. Rather than divide each distance, the reciprocal = 1/0.9998958 = 1.000104211 will be used to convert the adjusted grid distances to adjusted ground level values. This factor will also be used to determine ground level coordinates for the points. This listing follows the table of distances. As noted earlier, distances derived from such coordinates are at ground level. This subject will be covered in some detail in the section directed to Project Datum coordinates.

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	Plane Bearing	GRID DISTANCE	SIN Bearing / COS Bearing	Latitude	Departure	GRID COORDINATES Northing (Y)	GRID COORDINATES Easting(X)
	° ' "	Feet		Feet	Feet	Feet	Feet
MT TOM (CofEC)					**	362,611.25	1,615,233.86
			+0.7063762	+ 780.91			
	N 44 56 27 E	1103.23			*	363,392.16	1,616,013.16
			+0.7078366		+ 779.30	- 0.09	- 0.04
A					**	363,392.07	1,616,013.12
			+0.0200166	+1320.81			
	N 1 08 49 E	1321.07			*	364,712.97	1,616,039.60
			+0.9997996		+ 26.44	- 0.19	- 0.09
B					**	364,712.78	1,616,039.51
			+0.9998306	- 24.20			
	S 88 56 43 E	1314.85			*	364,688.77	1,617,354.23
			-0.0184073		+1314.63	- 0.30	- 0.13
C					**	364,688.47	1,617,354.10
			+0.9998282	- 24.36			
	S 86 56 16 E	1313.96			*	364,664.41	1,618,667.96
			-0.0185382		+1313.73	- 0.40	- 0.18
K	Sum of Lengths	5053.11			**	364,664.01	1,618,667.78

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Closure = $\Delta N = \Delta Y = -0.40$ ft. $\Delta E = \Delta X = -0.18$ ft. $\sqrt{(0.40)^2 + (0.18)^2} = 0.44$ ft.
0.44/5053.11 = 1:11,484

N or Y Factor = -0.40/5053.11 = -0.07916 per 1000 feet

E or X Factor = -0.18/5053.11 = -0.03562 per 1000 feet

* Preliminary Coordinates

** Adjusted Coordinates

Figure 22

Adjusted Latitudes, Departures, Distances and Bearings (Azimuths)

From	To	Latitude	Departure	Distance	Bearing Azimuth
MT TOM (CofEC)	A	+ 780.82	+ 779.26	1103.14	N 44°56'34"E 224 56 34
A	B	+1320.71	+ 26.39	1320.97	N 1 08 41E 181 08 41
B	C	- 24.31	+1314.59	1314.81	S 88 56 26E 271 03 34
C	K	- 24.46	+1313.68	1313.91	S 88 56 00E 271 04 00

Figure 22 (Cont.)

STATION FROM	STATION TO	PRELIMINARY AZIMUTH ° ' "	CORRECTION FOR CLOSURE ' "	CORRECTED AZIMUTH ° ' "
A	B	1 08 41		1 08 41
∠		89 54 00*		
A	D	91 02 41	+ 4	91 02 45
D	A	271 02 41		
∠		89 54 35*		
D	C	0 57 16	+ 9	0 57 25
C	D	180 57 16		
∠		89 53 55L		
C	B	91 03 21	+13	91 03 34
∠ Azimuths from north			13/3= +4.33	per angle
* Azimuth Angle				

Figure 23

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	Plane Azimuth from North	GRID DISTANCE	SIN Azimuth / COS Azimuth	Latitude	Departure	GRID COORDINATES Northing (Y)	Easting (X)
	° ' "	Feet		Feet	Feet **	Feet 363,392.07	Feet 1,616,013.12
A							
			+0.9998334	- 24.08			
					*	363,367.99	1,617,332.46
	91 02 45	1319.56					
			-0.0182522		+1319.34	+ 0.05	- 0.20
					**	363,368.04	1,617,332.26
D							
			+0.0167011	+1320.37			
					*	364,688.36	1,617,354.51
	0 57 25	1320.55					
			+0.9998605		+ 22.05	+ 0.11	- 0.41
					**	364,688.47	1,617,354.10
C	Sum of Lengths	2640.11					

Closure $\Delta N = \Delta Y = +0.11$ ft. $\Delta E = \Delta X = -0.41$ ft. $\Delta S = \sqrt{(0.11)^2 + (0.41)^2} = 0.42$ ft.
0.42 ft./2640.11 = 1:6,286
N or Y Factor = +0.11/2640.11 = +0.04166 per 1000 feet.
E or X Factor = -0.41/2640.11 = -0.1553 per 1000 feet.

* = Preliminary Coordinates
** = Adjusted Coordinates

Adjusted Latitudes, Departures, Distances and Azimuths (Bearings)

From	To	Latitude	Departure	Distance	Azimuth (From North)/Bearing
A	D	- 24.03	+1319.14	1319.36	91° 02' 37" S 88 57 23 E
D	C	+1320.43	+ 21.84	1320.61	0 56 51 N 0 56 51 E

Figure 24

From	To	Adjusted Ground Level Distances	
MT TOM (CofEC)	A	1103.25	= 1103.14 (Fig. 22) x 1.000104211
A	B	1321.11	= 1320.97 (Fig. 22) x 1.000104211
A	D	1319.50	All other distances obtained in the same manner
B	C	1314.95	
C	D	1320.75	
C	K	1314.05	

Station	Adjusted Ground Level Coordinates n = y	e = x
MT TOM (CofEC)	362,649.04	1,615,402.18
A	363,429.94	1,616,181.53
B	364,750.79	1,616,207.92
C	364,726.47	1,617,522.65
D	363,405.91	1,617,500.80
K	364,702.01	1,618,836.46

As an example of the computation: the n = y and e = x for Point B were determined as shown below:

n = y = 364,712.78 (Fig. 22) x 1.000104211= 364,750.79

e = y = 1,616,039.51 (Fig. 22) x 1.000104211 = 1,616,207.92

To conclude this discussion, a few remarks are thought necessary.

When SPCS coordinates are raised to the average elevation of the site whether or not the combined factor is used to accomplish this, it is strongly suggested that the values be made unique in some fashion such as dropping off one or more numbers or by adding constants. Further, all pertinent documents should contain sufficient information such that no one would mistake these coordinates for SPCS values. Additional remarks will be made in this regard in the section concerned with Project Datum coordinates.

In the most accurately determined surveys, there are additional corrections to the angles known as the (t-T) or second-term corrections which should be applied. Furthermore, such surveys should be adjusted by the method of Least Squares. Both of these subjects are covered in great detail in ACSM Control Surveys Division publication "Surveying Instrumentation and Coordinate Computation Workshop Lecture Notes." The following comments are made purely as matters that may be of interest to some surveyors. In geodetic surveys made at the higher elevations the angles (directions) are corrected for the reduction to sea level and to account for the deflection of the vertical or the so-called "deviation of the plumb line." These quantities are quite small in all, but a very few cases and generally would be exceeded by the errors resulting from not having the bubble perfectly centered on slope lines. The bubble correction can be very significant (10"-15"), is not a function of the distance, and should be considered when vertical angles exceed 10°. No implication is made that the corrections should be applied except perhaps that related to the mislevelment of the transit or theodolite.

Project Datum Coordinates: Long before the advent of SPCS, there were many accurate local systems controlling areas as large as counties. Although some were referenced to sea level because the elevation variances were too large to neglect, many others were referenced to the average elevation of the locality. Thus, with few exceptions, all secondary or tie in surveys could use ground level lengths in the computations. With regard to elevation references, it is a matter of fact that in at least one such control network, the elevations used for all work were referenced to the water level of a lake. For practical purposes, this made a great deal of sense, especially in flood plain mapping projects.

With the introduction of the SPCS, larger areas could be placed on a single system, but two problems resulted. Since the systems were directly related to the National horizontal control network which is referenced to the sea level surface the plane coordinates were also referenced to this surface (the Michigan Lambert grid at 800 ft. above sea level came about much later). This was the first problem. The second resulted from the distortions inherent to map projections, the so-called "error in scale," which required that distances be corrected to fit the rectangular coordinates computed on these projections.

Although most, if not all, members of the Geodesy Division (now the NGS) of the Coast and Geodetic Survey (presently NOS) were well aware that surveyors and engineers deal primarily with ground level quantities little or no effort was made to explain how the end results could be expressed in this format yet still be directly related to the SPCS. Here and there in some of the publications, there are inferences and hints, but few direct explanations.

Finally, William T. Pryor, an engineer with the Bureau of Public Roads, wrote an article which suggested that the control data for a project be raised to the average elevation of the area and; furthermore, that the average scale factor for the same locality be also applied to these coordinates. For small areas, the resulting coordinates are ground level values and horizontal ground level distances could be employed. The results are identical to those obtained if the conventional computing practices had been employed (Figures 22 and 24) and the adjusted data then raised to the average elevation with the average scale reduction eliminated.

The question then arises - how small is a small area? There are two components to be considered, the variances in elevation and in scale as related to the desired precision of the results. In a sense, the desired accuracy of the survey also enters the picture, for if adjusted ground level lengths when compared with actual measurements need not be more accurate than 1:5000 then considerable leeway is permissible. For an example, let us take the worst case of the scale reduction at the center of a system for a zone within the contigious United States. This zone is Texas south central where a reduction of 1:7300 was applied along the central parallel. Should a survey be located near this central parallel then the average elevation should not be greater than 1500 feet above sea level. The maximum range of elevations would need to be between sea level and 3000 feet above sea level. If the survey progresses north or south of the central parallel, the relationship between the computed and measured distances would generally worsen. In this case, an area of about 130 miles in a north-south direction, unlimited east and west could be accommodated provided the elevations remain less than 1500 feet as the upper and lower limits are approached. The north-south range would be restricted somewhat if the higher elevations (those greater than 1500 feet above sea level) occurred at these locations.

The simplest approach is to examine the scale factors at the extremeties and the range of elevations and compare these values with those computed for the center of the system and at the mean elevation of the project. If the combined effect on the computed quantities is within acceptable tolerances of the anticipated accuracy of the ground measured data, then the single Project Datum concept is entirely satisfactory. Rarely would a project be of such a size and the topographic relief so severe to require more than one reference system.

Figures 25 and 27 are the computation of ground level coordinates. Azimuths (from the south) rather than bearings are employed (Figures 11 and 26). Again the azimuths shown on Figure 26 were controlled by the azimuths A-B and C-B computed from the adjusted data given on Figure 25. The distances for both computations are the horizontal ground distances. Computations follow the NGS format, that is - Departures, Latitudes, x and y. To control the survey at ground level and to eliminate the scale factor, the published X and Y coordinates on the SPCS for MT TOM (CofEC) and Point K were multiplied by the reciprocal (1.000104211) of the combined factor to obtain the ground level coordinates x and y.

The closures in azimuth and position should be identical in the SPCS and Project Datum computations if a sufficient number of signifcant figures are carried in all the calculations. Also, the lengths and final computations when reduced to a common reference (either Project Datum to SPCS or vice versa) would also be the same under these cir-

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	Plane Azimuth	DISTANCE	SIN Azimuth / COS Azimuth	Departure	Latitude	Project Datum Coordinates x = (e)	y = (n)
	° ′ ″	Feet		Feet	Feet	Feet	Feet
MT TOM (CofEC)					**	1,615,402.18	362,649.04
			-0.7063762	+ 779.37			
	224 56 27	1103.34			*	1,616,181.55	363,430.02
			-0.7078366		+ 780.98	- 0.04	- 0.09
A					**	1,616,181.51	363,429.93
			-0.0200166	+ 26.45			
	181 08 49	1321.21			*	1,616,208.00	364,750.97
			-0.9997996		+1320.95	- 0.09	- 0.19
B					**	1,616,207.91	364,750.78
			-0.9998306	+1314.77			
	271 03 17	1314.99			*	1,617,522.77	364,726.76
			+0.0184073		- 24.21	- 0.13	- 0.28
C					**	1,617,522.64	364,726.48
			-0.9998282	+1313.87		1,618,836.64	364,702.40
	271 03 44	1314.10			- 24.36	- 0.18	- 0.39
K	Sum of Lengths	5053.64	+0.0185382		**	1,618,836.46	364,702.01

Closure = $\Delta x = \Delta e = -0.18$ ft. $\Delta y = \Delta n = -0.39$ ft. $\Delta s = \sqrt{(0.18)^2+(0.39)^2} = 0.43$ ft.
0.43/5053.64 = 1:11,753

x or e Factor = -0.18/5053.64 = -0.03562 per 1000 feet

y or n Factor = -0.39/5053.64 = -0.07717 per 1000 feet

* Preliminary Coordinates

** Adjusted Coordinates

Figure 25

cumstances. In any case, the adjusted azimuths or bearing would need no reduction and would also be identical when the significant figure condition is met.

Whenever Project Datum coordinates are determined, it is extremely important to note on all computations, maps, documents, etc. exactly the manner in which the coordinates and lengths were derived. This notation should also carry a full explanation of how the sea level and/or scale and/or combined factors were computed with all supporting evidence completely referenced.

There is nothing wrong with the idea of Project Datum coordinates when fully documented. However, numerous users have been disenchanted with the concept simply because some surveyors did not take the small effort necessary to positively identify the coordinates provided their clients.

Adjusted Departures, Latitudes, Distances and Azimuths (Bearings)

From	To	Departure	Latitude	Distance	Azimuth Bearing
MT TOM (CofEC)	A	+ 779.33	+ 780.89	1103.24	224°56'34" N 44 56 34 E
A	B	+ 26.40	+1320.85	1321.11	181 08 42 N 1 08 42 E
B	C	+1314.73	- 24.30	1314.95	271 03 32 S 88 56 28 E
C	K	+1313.82	- 24.47	1314.05	271 04 01 S 88 55 59 E

Figure 25 (Cont.)

STATION FROM	STATION TO	PRELIMINARY AZIMUTH ° ' "	CORRECTION FOR CLOSURE ' "	CORRECTED AZIMUTH ° ' "
A	B	181 08 42		181 08 42
∠		89 54 00*		
A	D	271 02 42	+ 3	271 02 45
D	A	91 02 42		
∠		89 54 35*		
D	C	180 57 17	+ 7	180 57 24
C	D	0 57 17		
∠		89 53 55L		
C	B	271 03 22	+10	91 03 32
Azimuths	from south			
∠ * Azimuth	Angle		10/3 = +3.33	per angle
		Figure 26		

TRAVERSE COMPUTATION BY LATITUDES AND DEPARTURES

STATION	PLANE AZIMUTH	Distance	SIN Azimuth / COS AZIMUTH	DEPARTURE	LATITUDE	Project Datum Coordinates x (e)	Project Datum Coordinates y (n)
	° ' "	Feet		Feet	Feet	Feet	Feet
A					**	1,616,181.51	363,429.93
			-0.9998334	+1319.48			
	271 02 45	1319.70			*	1,617,500.99	363,405.84
			+0.0182522		- 24.09	- 0.20	+ 0.06
D					**	1,617,500.79	363,405.90
			-0.0166962	+ 22.05			
	180 57 24	1320.69			*	1,617,523.04	364,726.35
			-0.9998606		+1320.51	- 0.40	+ 0.13
C	Sum of Lengths	2640.39			**	1,617,522.64	364,726.48

Closure = $\Delta x = \Delta e = -0.40$ ft. $\Delta y = \Delta n = +0.13$ ft. $\Delta S = \sqrt{(0.40)^2 + (0.13)^2} = 0.42$ ft.
0.42/2640.39 = 1:6,287

x or e Factor = -0.40/2640.39 = -0.1515 per 1000 feet.
y or n Factor = +0.13/2640.39 = +0.04924 per 1000 feet.

* Preliminary Coordinates
** Adjusted Coordinates

Adjusted Departures, Latitudes, Distances and Azimuths (Bearings)

From	To	Departure	Latitude	Distance	Azimuth/Bearing
A	D	+1319.28	- 24.03	1319.50	271° 02' 37" S 88 57 23 E
D	C	+ 21.85	+1320.58	1320.76	180 56 52 N 0 56 52

Figure 27

Areas: The simplest formula for computing the area of a figure using coordinates follows:

$$\text{Area} = 1/2 \quad \frac{E_1}{N_1} \times \frac{E_2}{N_2} \times \frac{E_3}{N_3} \times \frac{E_4}{N_4} \cdots \times \frac{E_1}{N_1}$$

Twice the area is equal to the sum of the products of the coordinates connected by the solid lines minus the sum of the products joined by the broken lines.

For the tract defined by the corners A-B-C-D the formula may be placed in the following form:

$$\text{Area} = \left[(N_A E_B - E_A N_B) + (N_B E_C - E_B N_C) + (N_C E_D - E_C N_D) + N_D E_A - E_D N_A\right] / 2$$

There were five computations of the coordinates for the corners of the tract and for comparison purposes, the area will be determined from the five sets of coordinate values. To simplify the computations, constants will be subtracted from the coordinates as noted in the description of each computation which follows. In some instances, all the multiplications will be shown to obtain twice the area; in others only the summation will be shown, followed by the area of the figure and the area divided by 43560 square feet to obtain the area in acres. One of the computations involves the coordinates on the SPCS which are at the sea level reference surface and the grid area must be divided by the combined factor squared to obtain the area in square feet at the average elevation of the tract.

The computations are identified by the numbers 1, 2, 3, 4, and 5 which also indicates the sequence that they appear in the paper.

Computation and Figure Number(s)	Comments
1 - 5	5000.00 feet was subtracted from both the Northings and Eastings.
2 - 6	5000.00 feet was subtracted from both the Northings and Eastings. The areas computed for computation 1 and 2 would be identical if sufficient figures were used throughout. Actually the areas differ by only 0.057 square feet.
3 - 13,14	363,368.04 feet was subtracted from the Northings and 1,616,013.11 feet from the Eastings.

Computation and Figure Number(s)	Comments
4 - 22,24	363,368.04 feet was subtracted from the Northings and 1,616,013.12 feet from the Eastings. Since the coordinates are sea level values, the grid area was divided by the combined factor squared $(0.9998958)^2 = 0.9997916$.
5 - 25,27	1,616,181.51 feet was subtracted from the X's and 363,405.90 feet from the Y's. As was the case with computations 1 and 2, the areas computed from the data for computations 4 and 5 should be identical if sufficient figures were used throughout. Actually the areas differ by only 13.360 square feet which is quite satisfactory.

The reduced coordinates (n and e and x and y) used in the computation of the areas follow:

Point		Comp. 1	Comp. 2	Comp. 3
A	n	0	0	+24.06
	e	0	0	0
B	n	+1321.13	+1321.13	+1344.85
	e	+ 2.44	+ 2.57	+ 26.31
C	n	+1320.79	+1320.66	+1320.49
	e	+1317.50	+1317.63	+1340.94
D	n	+ 0.03	- 0.10	0
	e	+1319.63	+1319.63	+1319.18

Point		Comp. 4	Comp. 5
A	n	+24.03	x = 0
	e	0	y = +24.03
B	n	+1344.74	x = +26.40
	e	+ 26.39	y=1344.88

Point		Comp.	Comp.
C	n	+1320.43	x = +1341.13
	e	+1340.98	y = +1320.58
D	n	0	x = +1319.28
	e	+1319.14	y = 0

The corresponding area computations are tabulated as follows:

(1)	0		0		0		
	1740588.775	-	3222.728	=	+ 1737366.047		
	1742954.108	-	39.525	=	+ 1742914.583		
	0		0		0		
			Twice the Area	=	3480280.630		
			Area	=	1740140.315	= 39.948 Acres	
(2)			Twice the Area	=	3480280.745		
			Area	=	1740140.372	= 39.948 Acres	
(3)	633.019	-	0	=	+ 633.019		
	1803363.159	-	34742.092	=	+ 1768621.067		
	1741963.998	-	0	=	+ 1741963.998		
	0	-	31739.471	=	- 31739.471		
			Twice the Area	=	3479478.613		
			Area	=	1739739.306	= 39.939 Acres	
(4)	634.152	-	0	=	+ 634.152		
	1803269.445	-	34846.148	=	+ 1768423.297		
	1741832.030	-	0	=	+ 1741832.030		
	0	-	31698.934	=	- 31698.934		
			Twice the Grid Area	=	3479190.545		
			Grid Area	=	1739595.272		
	Grid Area/0.9997916 = Area			=	1739957.879	= 39.944 Acres	
(5)			Twice the Area	=	3479942.478		
			Area	=	1739971.239	= 39.944 Acres	

Comparisons: The following differences in bearings (azimuths) distances and where applicable in coordinates or latitudes and departures, and areas will be based on Computation No. 5 (Project Datum computation) as the model. In other words, the corresponding ΔB or ΔA, ΔD, ΔL, ΔDE, ΔN, ΔE, and $\Delta S = \Delta$ Area will be determined by subtracting the quantities in Computations 1, 2, 3, and 4 from similar values derived in Computation 5. The bearings and azimuths for computations 1 and 2 have been corrected for the θ angle at Point A = + 1°02'39" before the differences with Computation 5 were obtained. The data for computation

4 were raised to the reference plane with the scale factor eliminated prior to the comparison with Computation 5.

From	To	Comp 5-1	Comp 5-2	Comp 5-3	Comp 5-4
A	B	- 18" ΔB -0.02 ft.ΔD	- 38" ΔA* -0.02ft. ΔD	+ 14 ΔB +0.06 ft.ΔD	+ 1 ΔB 0 ft. ΔD
B	C	0 ΔB -0.11 ft. ΔD	- 21 ΔA* -0.11 ft.ΔD	+ 10 ΔB +0.09 ft.ΔD	+ 2 ΔB 0 ft.ΔD
C	D	- 14 ΔB 0 ft. ΔD	- 35 ΔA* 0 ft.ΔD	+ 13 ΔA +0.09 ft.ΔD	+ 1 ΔA +0.01 ft.ΔD
D	A	- 3 ΔB -0.13 ft.ΔD	- 18 ΔA* -0.13 ft.ΔD	- 5 ΔA +0.10 ft.ΔD	0 ΔA 0 ft.ED

Point	Comp 5-1	Comp 5-2	Comp 5-3**	Comp 5-4
A	ΔN ΔE NA	ΔN ΔE NA	+0.06 ft.ΔL +0.09 ft.ΔDE	-0.01 ft.ΔN -0.02 ft.ΔE
B	ΔN ΔE NA	ΔN ΔE NA	+0.06 ft.ΔL +0.10 ft.ΔDE	-0.01 ft.ΔN -0.01 ft.ΔE
C	ΔN ΔE NA	ΔN ΔE NA	+0.09 ft.ΔL +0.09 ft.ΔDE	+0.01 ft.ΔN -0.01 ft.ΔE
D	ΔN NA ΔE	ΔN NA ΔE	+0.03 ft.ΔL +0.10 ft.ΔDE	-0.01 ft.ΔN -0.01 ft.ΔE
ΔS	- 169.076 sq. ft. - 0.004 Acre	- 169.133 sq. ft. - 0.004 Acre	+231.933 sq. ft. + 0.005 Acre	+13.360sq.ft. 0.000 Acre

* 20" was added to the initial azimuth A-B and hence to all following azimuths. A good agreement with the differences at Comp 5-1 is obtained if 20" is added to the ΔA for Comp 5-2.

** All ΔL's and ΔDE's in this column are between consecutive points.

Comparing the coordinates determined from horizontal ground level distances (Figures 13 and 14) and those computed using the distances corrected for the reductions to sea level and for scale shows (Figures 22 and 24) that the differences are rather small with a maximum of 0.11 ft. in the Northings and 0.09 feet in the Eastings. These differences could be considerably larger at the higher elevations and where scale factors smaller than that used in this example are involved.

Mapping Angles: There are occasionally requirements when employing SPCS coordinates to determine the true bearing or azimuth at a point. For this paper only, the simplest computations to determine the differences between grid and geodetic (true) quantities for the Lambert and transverse Mercator projections will be given. The ACSM Control Surveys Division publication "Horizontal Control as Applied to Local Surveying Needs" provides several methods for each grid and contains many step-by-step computations which have not been covered in this paper.

(1) Taking the Lambert projection first, the Tangent of the mapping angle $\theta = X'/R_b - Y = (E'/R_b - N)$. A table of six place tangent will suffice for this computation.

For the example, the coordinates for Point C from Figure 22 will be used: Point C N(Y) = 364,688.47 E'(X') = 1,617,354.10 - 2,000,000 = -382,645.90

Included with each of the projection tables for the states assigned the Lambert grid is a table of constants which included the value for R_b for each state or zone within a state. The constants for Wisconsin are shown on Figure 28. Note that for the Central zone R_b = 21,430,913.90 ft.

$$\text{Tan } \theta = -382645.90/(21{,}430{,}913.90 - 364{,}688.47) =$$

$$-382645.90/21{,}066{,}225.43 = 0.018164$$

$$\theta = -1°02'26''$$

This value for " θ " is applied to all grid bearings or azimuths radiating out from Point C. There are three such quantities in this example, all are shown.

C-B	= 91°03'34"*	C-D	=	0°56'51"*	C-K	=	271°04'00"*
θ	= -1 02 26			-1 02 26			-1 02 26
C-B	= 90 01 08**	C-D	=	359 54 25**	C-K	=	270 01 34**
	N88 58 52W			S 0 05 35E			S 89 58 26E

* Grid Azimuths **Geodetic Azimuths followed by corresponding bearings.

(2) There is a similar table of constants for all states which use the transverse Mercator projection. The table for Illinois is shown by Figure 29. This table will not be used in the computation of the mapping angle ($\Delta\alpha$) on the transverse Mercator grid in this paper and is given here for two reasons, one to provide a sample and two to furnish the information required to compute the $\Delta\alpha$ angle if other methods as described in the ACSM publication noted earlier are used.

Included among the tables contained in the publication which have been prepared for all states which use the transverse Mercator grid is one illustrated by Figure 30. As noted $\Delta\alpha'' = MX' - e$ where "M" is a function of "Y" or the Northing. Since "e" is small (less than one second) it generally may be neglected. The sign of $\Delta\alpha''$ takes that of X' where X' = X-500,000.00 or whatever constant is used. As an example: The Illinois East Zone coordinates for some point identified as Z = X (E) = 725,662.30 Y (N)=1,536,282.91

Interpolating for "M" as follows:

1,500,000.00 (from Figure 30 - East Zone) M = 0.0084917
Δ = 0.3628291 x 0.0000825 =+0.0000299
1,536,28291 M = 0.0085216

X' = 725,662.30 - 500,000.00 = +225,662.30

MX' = (0.0085216)(+225,662.30) = +1923".0 = $\Delta\alpha$ (approx.)
Interpolating for e = - 0.1
+1922.9 = $\Delta\alpha$ =+0°32'03"

This angle ($\Delta\alpha$) at Z would be applied to all grid azimuths or bearings from this point to obtain geodetic values. For example:

Z-V = 173°40'25" grid	Z-T = 346°13'46" grid
$\Delta\alpha$ + 0 32 03	$\Delta\alpha$ = + 0 32 03
Z-V = 174 12 28 geod.	Z-T = 346 45 49 geod.
N 5 47 32 W	S 13 14 11 E

<u>NOTE</u>: The geodetic values in both instances may not be exact since the (t-T) corrections have been neglected, but for the purposes intended within the area of coverage devoted to in this paper, they are quite satisfactory.

Constants for Wisconsin

Constant	North zone	Central zone	South zone
C	2,000,000.00 ft.	2,000,000.00 ft.	2,000,000.00 ft.
Central Meridian	90° 00' 00".000	90° 00' 00".000	90° 00' 00".000
R_b	20,489,179.67 ft.	21,430,913.90 ft.	22,672,134.66 ft.
y_o	365,046.62 ft.	380,166.91 ft.	510,702.41 ft.
ℓ	0.72137 07913	0.70557 66312	0.68710 32423
$\frac{1}{2\rho_o^2 \sin 1''}$	2.355×10^{-10}	2.355×10^{-10}	2.356×10^{-10}
$\log \frac{1}{2\rho_o^2 \sin 1''}$	0.371 9140 - 10	0.372 0471 - 10	0.372 1993 - 10
$\log \ell$	9.85815 85535 - 10	9.84854 41885 - 10	9.83702 19979 - 10
log K	7.58743 29354	7.59094 42620	7.59559 91991

Figure 28

Constants for Illinois

Constant	Zone	
	East	West
Central Meridian	88° 20' 00."000	90° 10' 00."000
log R	-108.6	-255.5
Scale reduction (Central Meridian)	1 : 40,000	1 : 17,000
$\log\left(\frac{1}{6\rho_0^2}\right)_g$	4.581 0473 -20	4.581 0767 -20
$\log\left(\frac{1}{6\rho_0^2 \sin 1''}\right)_g$	9.895 4724 -20	9.895 5018 -20
$\left(\frac{1}{6\rho_0^2 \sin 1''}\right)_g$	0.7861×10^{-10}	0.7861×10^{-10}

Figure 29

TRANSVERSE MERCATOR PROJECTION

Illinois

$$\Delta\alpha = Mx' - e$$

y	East zone M	East zone ΔM	West zone M	West zone ΔM
0	0.007 3294	735	0.007 3296	735
100,000	0.007 4029	740	0.007 4031	740
200,000	0.007 4769	746	0.007 4771	746
300,000	0.007 5515	751	0.007 5517	751
400,000	0.007 6266	757	0.007 6268	757
500,000	0.007 7023	762	0.007 7025	763
600,000	0.007 7785	768	0.007 7788	768
700,000	0.007 8553	774	0.007 8556	774
800,000	0.007 9327	780	0.007 9330	780
900,000	0.008 0107	786	0.008 0110	786
1,000,000	0.008 0893	792	0.008 0896	792
1,100,000	0.008 1685	798	0.008 1688	798
1,200,000	0.008 2483	805	0.008 2486	805
1,300,000	0.008 3288	811	0.008 3291	811
1,400,000	0.008 4099	818	0.008 4102	818
1,500,000	0.008 4917	825	0.008 4920	825
1,600,000	0.008 5742	831	0.008 5745	832
1,700,000	0.008 6573	838	0.008 6577	838
1,800,000	0.008 7411	846	0.008 7415	846
1,900,000	0.008 8257	853	0.008 8261	853
2,000,000	0.008 9110	860	0.008 9114	860
2,100,000	0.008 9970	868	0.008 9974	867
2,200,000	0.009 0838	875	0.009 0841	876
2,300,000	0.009 1713		0.009 1717	

e

y \ x'	100,000	200,000	300,000	400,000
0	0.0	0.0	0.2	0.5
1,000,000	0.0	0.1	0.2	0.6
2,000,000	0.0	0.1	0.3	0.8

Figure 30

Dracup, J. F., and Kelley, C. F., Horizontal control as applied to local surveying needs. American Congress on Surveying and Mapping, Control Surveys Division Publication.

Dracup, J. F., NOAA Technical Memorandum NOS NGS-5. National Geodetic Survey data: Availability, explanation, and application, June 1976. Available from the National Geodetic Survey, National Ocean Survey, Rockville, Md. 20852.

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<u>Special Publication No. 235. The State Coordinate Systems (A Manual for Surveyors)</u>. U.S. Coast and Geodetic Survey, Washington, D.C. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va. 22151. Price on request. (Order COM-71-50367)

<u>Special Publication No. 246. Sines, Cosines, and Tangents, 0° - 6°, for Use in Computing</u> Lambert Plane Coordinates. U.S. Coast and Geodetic Survey, Washington, D.C. Available from the National Technical Information Service, U.S. Department of Commerce, Springfield, Va. 22151. Price on request. (Order COM-72-50181)

<u>State Plane Coordinate Projection Tables (available for all States except Alaska.)</u> Computations for Alaska are made using the 2-1/2-minute intersection tables, <u>USC&GS Publication 65-1, Part 49 for zone 1, Part 50 for zones 2 - 9</u>, and Part 51 for zone 10. Tables are also available for most U.S. Possessions. (These tables are part of the <u>USC&GS Special Publications. When requesting a publication, please specify State.)</u> Available from the National Geodetic Survey, National Ocean Survey, Rockville, Md. 20852.

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