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DETERMINATION OF NORTH AMERICAN DATUM 1983 COORDINATES OF MAP CORNERS

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ABSTRACT. This publication describes the use of Doppler data in predicting approximate changes of coordinates of map corners from the North American Datum 1927 (NAD 27) to the North American Datum 1983 (NAD 83) system. A brief description of the computer program and pertinent mathematical formulas are included.

GENERAL CONSIDERATIONS

The redefinition of the North American Datum (NAD), scheduled for 1983, envisions the use of the world reference ellipsoid and a simultaneous adjustment in a geocentric system of all available observations, including geocentric positions derived from Doppler data. It now becomes necessary to predict the impact of this readjustment on cartographic materials published to date or to be published. Specifically, cartographers need to know by how much and in which direction the coordinates of map corners are expected to change. The accuracy of the prediction should be compatible with a plotting accuracy of 0.2 millimeter (5 meters at 1:24,000 scale).

Doppler geocentric x,y,z coordinates are computed in a system known as NWL 9D. Before they can be used in an adjustment of a geodetic network, they must be corrected for a small scale error and rotated in the xy plane to reduce them to the proper longitude origin. At the present time two such best fitting correction constants have been adopted for transforming NWL 9D coordinates to a system known as WGS 72 (NWL 10F).

The principle of the prediction is simple. Doppler station positions are known to be accurate to about 1 meter (1 sigma) in each component and when adjusted simultaneously with terrestrial observations are not expected to change by much more than a meter. Therefore, for the present purpose they can be considered errorless and can be used as control coordinates to which map corners will be fitted.

The validity of the prediction presupposes the knowledge of certain parameters on which the readjustment will be based. These are now considered.

1. Ellipsoid parameters. The ellipsoid to be used for the NAD 83 datum will have very nearly the same defining parameters as those of the WGS 72 (NWL 10F) ellipsoid. A change of 10 meters in equatorial radius or of one unit in the second decimal of the reciprocal of flattening will not change horizontal positions by more than 5 meters.

2. Scale of Doppler data. The correction value now adopted for application to Doppler (NWL 9D) scale is -0.8263 ppm. This is equivalent to lengthening the equatorial radius by 5.27 meters in transforming rectangular geocentric to geodetic coordinates.

3. Longitude origin of Doppler data. This correction is being re-evaluated by the National Geodetic Survey (NGS) and is expected to change in the first decimal. If the prediction is to be accurate to ±5 meters, this parameter must be known to $\pm 0''.18$.

It should be noted that the new datum will have no datum origin associated with any station (such as MEADES RANCH in NAD 27). No stations will be held fixed in the adjustment; absolute positioning will be accomplished by Doppler data.

COMPUTER PROGRAM FOR DETERMINATION OF SHIFTS

A computer program has been developed for the purpose of determining shifts of coordinates of map corners and related data. The shifts are expressed in seconds and in millimeters at a specified scale. Graticule interval, map scale, and all parameters discussed above are a part of the input.

The program accepts x, y, and z coordinates of Doppler control stations in the NAD 27 and the NWL 9D systems. The latter set of coordinates is corrected for scale and rotation about the z-axis. This yields for each control station Δx, Δy, Δz values (Doppler minus NAD 27). If Doppler data were errorless and if NAD 27 contained no distortions, these values would be the same at all stations. Changes in latitude, longitude, and geodetic height (height above the ellipsoid), however, would still exist because of a change in ellipsoid parameters and orientation. The departures of Δx, Δy, Δz from the means or from values at any arbitrary station reflect distortions of NAD 27 in all three components, assuming that Doppler coordinates contain no errors.

The model for predicting shifts of map corners is unsophisticated but quite satisfactory for the intended purpose. Given latitude and longitude of a map corner and assuming geodetic height as zero, the corresponding x,y,z values are computed in the NAD 27 system. The Δx, Δy, Δz shifts for the corner are computed from shifts at all control (Doppler) stations which are weighted inversely as the square of the straight line distance from the map corner to the control station. In this way the closest control station has the largest weight, while a very distant station contributes practically nothing to this determination.

The weighted means of Δx, Δy, Δz of each map corner are then converted to changes in map coordinates.

The program also computes related data which may be of interest in analyses performed for other purposes. These include maximum, minimum, and mean Δx, Δy, Δz shifts; changes in latitude, longitude, and geodetic height at control stations; distortions of NAD 27 in three components based on departures from the mean shifts; and NAD 27 errors in straight line distances, azimuths, and vertical angles over lines between widely separated points.

Mathematical formulas used in the program are given in appendix 1. Contours of changes in latitude, longitude, and geodetic height from NAD 27 to the best present estimate of NAD 83 are shown in figures 1, 2, and 3.

CONCLUSIONS

Comparisons of x, y, and z coordinates at control stations within the conterminous States give the mean shift values (Doppler NWL 10F minus NAD 27)

$$\Delta x = -22 \text{ m} \qquad \Delta y = 157 \text{ m} \qquad \Delta z = 176 \text{ m}$$

and the spreads in Δx, Δy, and Δz of 24 m, 13 m, and 16 m respectively.

In Alaska the mean shifts are:

$$\Delta x = -13 \text{ m} \qquad \Delta y = 143 \text{ m} \qquad \Delta z = 175 \text{ m}$$

with spreads of 47 m, 46 m, and 22 m respectively. At a scale of 1:100,000 a plotting error of 0.2 mm corresponds to 20 meters on the ground. Therefore, for medium scale maps the mean Δx, Δy, Δz shifts may be adopted and the changes in latitude and longitude (in meters) computed by the following approximations:

$$\Delta\phi = \sin\phi(-\cos\lambda\ \Delta x + \sin\lambda\ \Delta y) + \cos\phi\ dz + 2(a\ \Delta f + f\Delta a)\sin\phi\cos\phi$$

$$\Delta\lambda = -(\sin\lambda\ \Delta x + \cos\lambda\ \Delta y)$$

Using $\Delta f = -0.0000373$ and $\Delta a = -71.4$ m, $2(a\ \Delta f + f\Delta a) = -476$.

At 1:500,000 scale the shifts are within plotting accuracy and need not be applied.

APPENDIX 1. CONSTANTS AND FORMULAS

Longitude positive west.

1. Ellipsoid parameters.

	NAD 27	NWL 9D	NWL 10F (WGS 72)
Equatorial radius	6,378,206.4 m	6,378,145 m	6,378,135 m
Flattening	1/294.978698	1/298.25	1/298.26

2. Conversion of ϕ, λ, h to x, y, z.

$$e^2 = 2f - f^2 \tag{2.1}$$

$$N = a(1-e^2\sin^2\phi)^{-1/2} \tag{2.2}$$

$$x = (N + h) \cos\phi \cos\lambda \tag{2.3}$$

$$y = -(N + h) \cos\phi \sin\lambda \tag{2.4}$$

$$z = [N(1-e^2)+h] \sin\phi \tag{2.5}$$

3. Conversion of x, y, z to ϕ, λ, h. Formula by B. R. Bowring.*

$$p = (x^2 + y^2)^{1/2} \tag{3.1}$$

$$\tan u = (z/p)(a/b) \tag{3.2}$$

$$\tan\phi = \frac{z + e'^2 b \sin^3 u}{p - e^2 a \cos^3 u} \tag{3.3}$$

$$\tan u = (1-f) \tan\phi \tag{3.4}$$

$$h = \pm [(p-a\cos u)^2 + (z-b\sin u)^2]^{1/2} \tag{3.5}$$

$$\tan\lambda = -y/x \tag{3.6}$$

The sign of h is the same as the sign of (p-a cos u).

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*Bowring, B. R. (Directorate of Overseas Surveys, Tolworth, Surrey, England), 1975 (personal communication); and Bowring, B. R., 1976: Transformation from spatial to geographical coordinates. Survey Review (Tolworth, England), 23, No. 181, 323-327.

4. Conversion of rectangular NWL 9D to NWL 10F coordinates.

Scale correction: -0.8263 ppm

Rotation in xy plane:

$$\delta x = y\ \delta\lambda \qquad \delta y = -x\ \delta\lambda \tag{4.1}$$

where $\delta\lambda$ = -0.00000 12605 (= -0.26").

5. Inverse solution in space.

S is straight line distance, A is astronomic azimuth (from north), V is vertical angle positive upwards from astronomic horizon. Δx, Δy, Δz are $x_2 - x_1$ etc. ϕ' and λ' are astronomic values at P_1.

$$S = (\Delta x^2 + \Delta y^2 + \Delta z^2)^{1/2} \tag{5.1}$$

$$\tan A = \frac{\Delta x \sin\lambda' + \Delta y \cos\lambda'}{-\sin\phi'\ (\Delta x \cos\lambda' - \Delta y \sin\lambda') + \Delta z \cos\phi'} \tag{5.2}$$

$$S \sin V = \cos\phi'(\Delta x \cos\lambda' - \Delta y \sin\lambda') + \Delta z \sin\phi' \tag{5.3}$$

6. Errors in x, y, and z expressed as errors in latitude, longitude, and geodetic height. Results in linear units.

R is mean radius of the Earth.

$$p = (x^2 + y^2)^{1/2} \tag{6.1}$$

$$\delta\phi = (-z\ x\ \delta x - z\ y\ \delta y + p^2 \delta z)/(p\ R) \tag{6.2}$$

$$\delta\lambda = (y\ \delta x - x\ \delta y)/p \tag{6.3}$$

$$\delta h = (x\ \delta x + y\ \delta y + z\ \delta z)/R \tag{6.4}$$

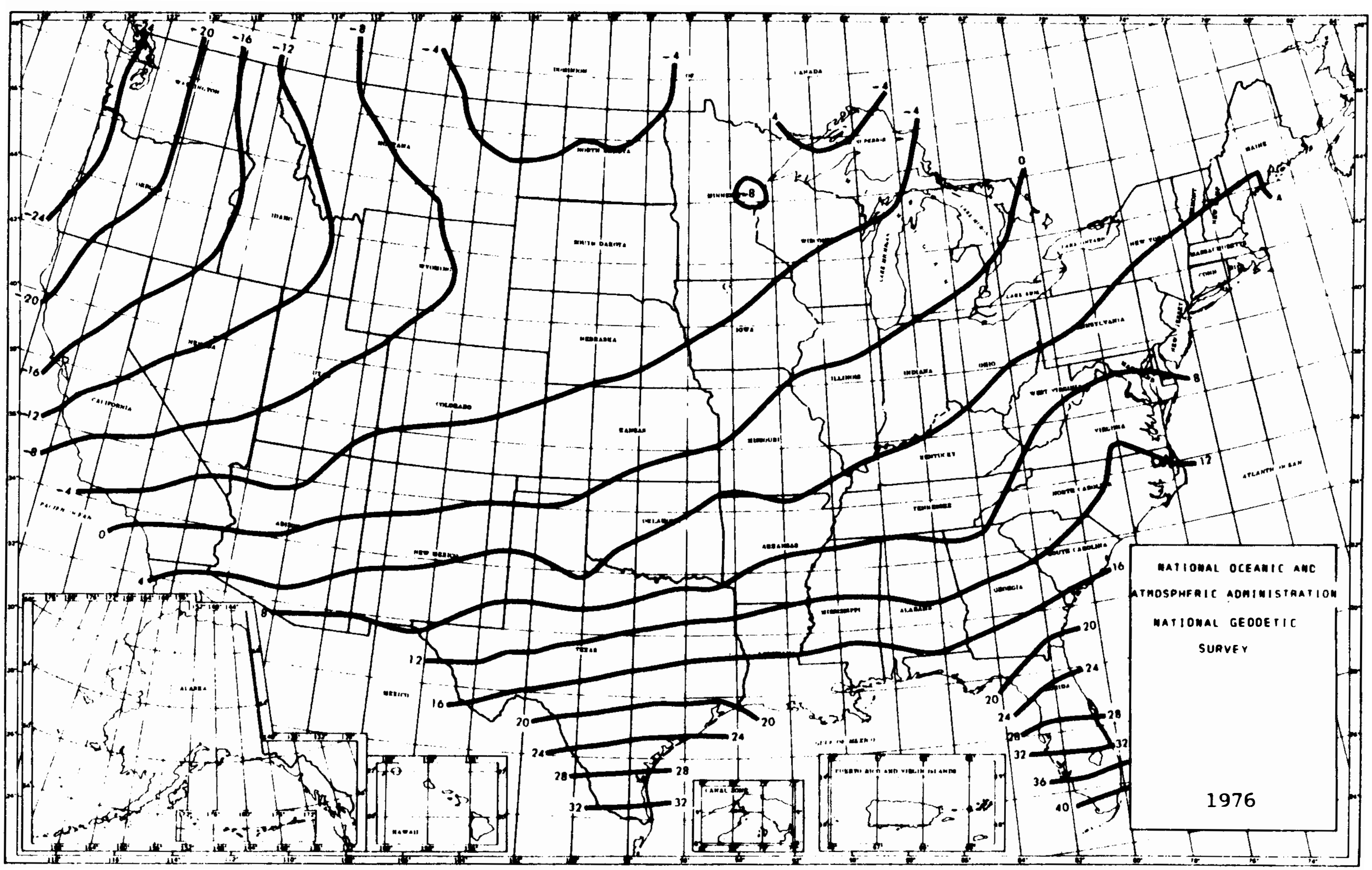

Figure 1.--Expected latitude change from NAD 27 to NAD 83 (in meters).

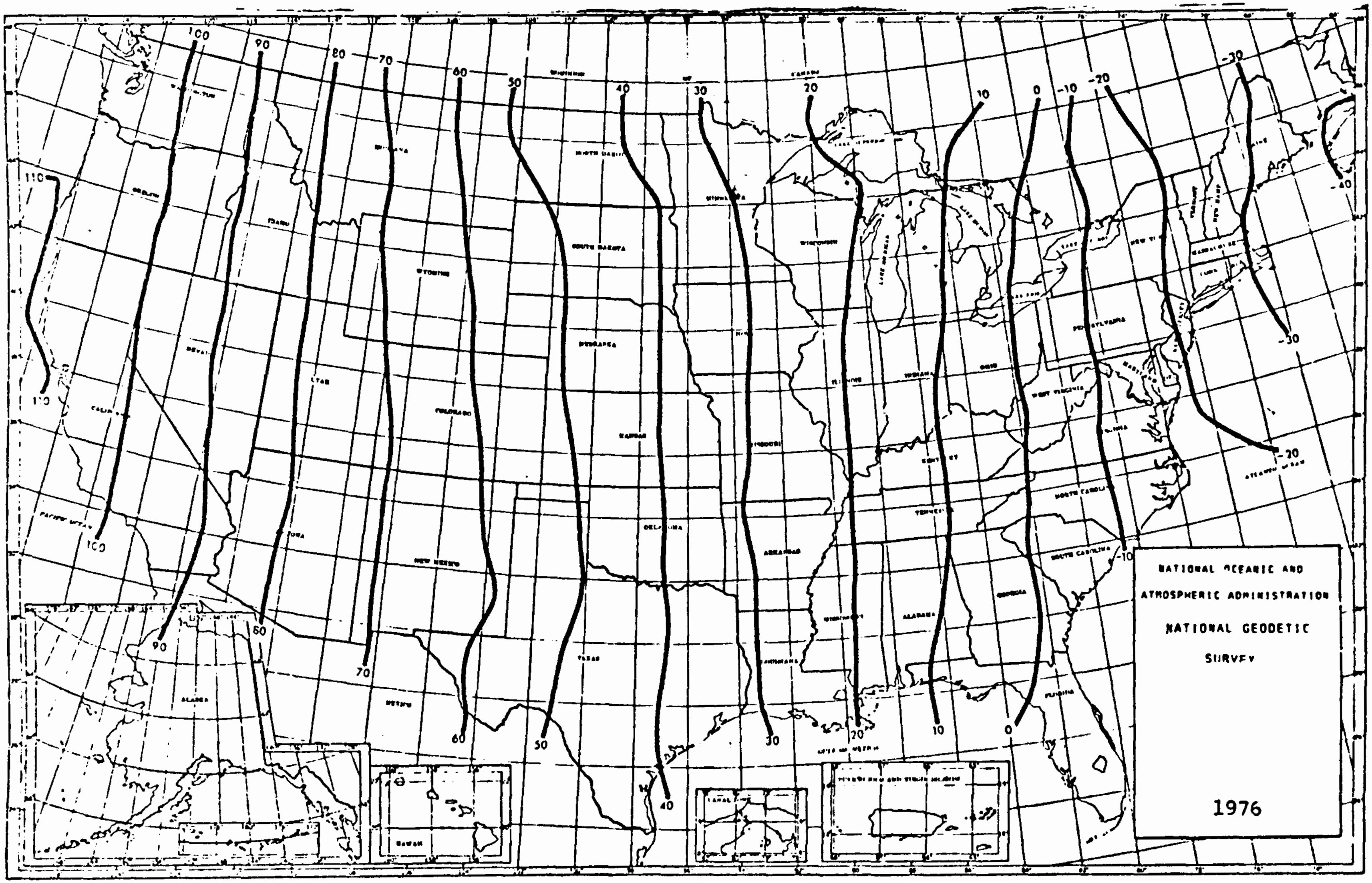

Figure 2.--Expected longitude change from NAD 27 to NAD 83 (in meters).

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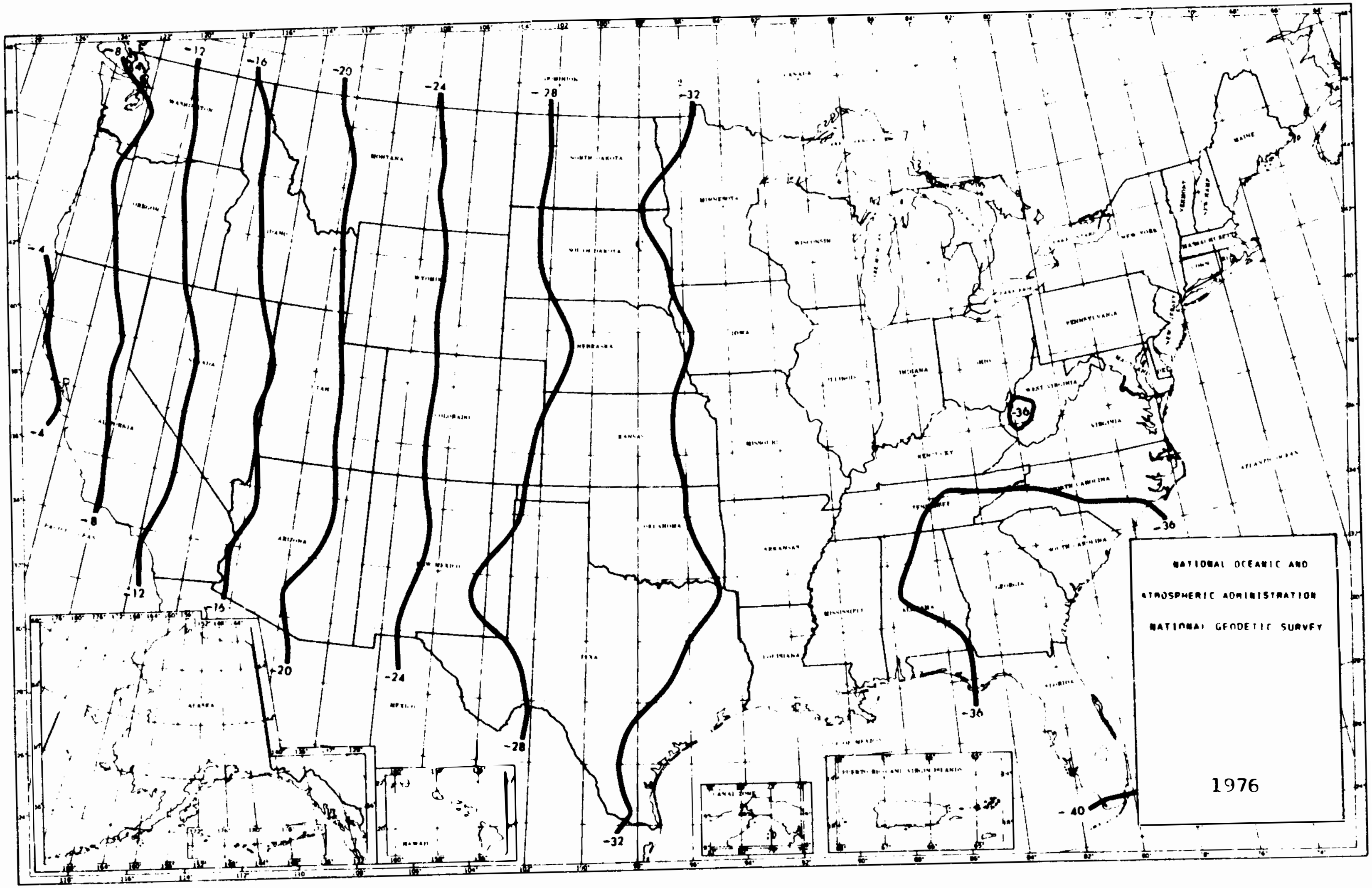

Figure 3.--Expected geoid height change from NAD 27 to NAD 83 (in meters).