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DETERMINATION OF VERTICAL DEFLECTIONS USING THE GLOBAL POSITIONING SYSTEM AND GEODETIC LEVELING

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Abstract. The capabilities of the Global Positioning System (GPS) for accurately determining geodetic quantities are well established. Nevertheless, no comparison between deflections of the vertical as determined through GPS with leveling and values conventionally computed by astrogeodetic methods has yet been published. This investigation demonstrates that accurate deflections of the vertical components (η, ξ) can be obtained independently of classical astronomic observations by combining geodetic leveling with satellite GPS positioning. The approach uses a radial configuration of baselines to determine the "best" (in a least squares sense) values of (η, ξ) at the central station.

Introduction

In recent years, several investigators have used GPS techniques and methods to determine geometric quantities, primarily baselines and ellipsoidal (geodetic) height differences [Engelis et al., 1984; Bock et al., 1985]. In conjunction with geodetic leveling, some computations of physical parameters such as geoid-ellipsoid separation (denoted N_g and also referred to as undulation or geoid height) have been published [Engelis et al., 1985; Denker and Wenzel, 1987]. The major drawback with analyses of this type is that a set of independent "true" geoid heights are needed as a source of comparison. Unfortunately, undulations cannot be directly observed, and the gravimetrically computed values are not as accurate as the leveled orthometric heights or the GPS determined ellipsoidal heights. Relative gravimetric geoid undulations obtained using standard computational methods (geopotential models and Stokes' integration, Fast Fourier Transforms (FFT) and/or least squares collocation) and terrestrial gravity are at best accurate to a few centimeters. Estimates are 2.5 to 5 ppm (parts-per-million) of the baseline distance depending on the number of refinements incorporated [Kearsley, 1988]. However, recent advancements in GPS instrumentation and software provide repeatability of relative ellipsoidal heights of about 1 part in 10^7 [Dong and Bock, 1989]. Orthometric height differences are measured even more precisely with high order geodetic leveling. This one order of magnitude improvement of observed over gravimetrically predicted quantities becomes more critical when small angles such as vertical deflections are analyzed.

The deflection of the vertical is the angle at the point of observation between the normal to the adopted reference surface (generally an ellipsoid of revolution) and the direction of the plumb line. Like the undulation, it can be predicted using gravimetric methods (e.g., Vening Meinesz's equations complemented by earth gravitational models). Nevertheless, astrogeodetic methods are considered more accurate. When deflections of the vertical are predicted using gravity and the most modern and sophisticated algorithms, the reported agreement with the observed astrogeodetic deflections is not better than 1" [Kearsley et al., 1985]. Furthermore, to attain these results extensive gravity coverage and regional detailed digitized topographic height information are imperative.

The full relationship between the GPS determined local geodetic frames and the "natural", directly observed, local astronomic frames, is an important problem (e.g., it establishes unequivocally the direction of the gravity vector). The three differential rotations involved in this transformation, respectively around the first (pointing east), second (north), and third (zenith) axes, are the two components of the deflection of the vertical along the geodetic meridian and prime vertical, denoted respectively ξ and η, and the so called Laplace's correction (i.e., the difference between the astronomic and geodetic azimuths). Soler et al. [1988] compared GPS and astronomically determined azimuths and concluded that GPS determined geodetic azimuths can be accurately converted to astronomic azimuths once the deflections of the vertical are known. See also Evans et al., [1989].

This paper compares, for the first time, vertical deflections determined by GPS and spirit leveling with classical astrogeodetic deflections. The investigation demonstrates the feasibility of determining deflections of the vertical at least as accurate as astrogeodetic deflections although completely independent of stellar observations. A major advantage of this alternative procedure will be the reduction of time spent in the field, thus, its cost effectiveness. GPS is an all weather operation and its practicality can be dramatically increased once kinematic surveys are perfected. Also, relative leveling requirements may already exist or can be measured in a matter of hours if an appropriate short radial network is established.

Accurate and uncomplicated determination of vertical deflections, when done periodically, may become a significant tool in tectonic and geo-

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dynamic studies. They complement the three-dimensional representation of regional crustal distortions. Repeated GPS baselines could define the area's strain field. Comparison of differences in ellipsoidal and/or orthometric heights will provide the uplift or subsidence of the region. Finally, changes in the deflection components (η,ξ) are the only geodetic parameters able to disclose the direction of possible shallow underground mass displacements.

Test site and data collection

The test area selected for this feasibility study is located at the Naval Surface Warfare Center (NSWC) in Dahlgren, VA. The planning, observational schedule, and data collection were completed through the joint efforts of the NSWC, the National Geodetic Survey (NGS), and the Defense Mapping Agency (DMA). In addition to GPS and leveling observations, conventional terrestrial measurements of gravity, distances (using Electronic Distance Measurement Instruments, i.e., EDMI) and astronomic positions and azimuths (using first order theodolites) were also performed within the test area [Evans et al., 1989].

Eight days of GPS data were collected starting on August 17, 1987. Five Texas Instrument TI4100 geodetic receivers, with the GEodetic SAtellite Receiver (GESAR) operating software, were employed to obtain both pseudorange and dual carrier phase measurements on L1 and L2 frequencies. This equipment tracks up to four satellites simultaneously. In order to optimize the station-satellite geometric configuration an observation span of six hours, which allowed a total of six different satellites to be observed, was implemented. Specifically, satellites with Pseudo-Random-Noise (PRN) code numbers 3, 6, 8, 11, 12, and 13 were utilized. With the advent of a full GPS constellation and new instruments which can track all satellites in view, the observation time span per session can possibly be reduced considerably. Figure 1 depicts the four GPS baselines used in this investigation. They range in length from 1.2 to 4.8 km and radiate from the central reference station MBRE where the deflections were sought. Bench marks at individual baseline end points were connected by double-run first order spirit leveling. The leveling circuit was surveyed on Aug. 28-29 by NGS personnel and is shown with dotted lines in Figure 1. The total loop distance is 18.830 km and the misclosure 0.14 mm. Classical measurements of astronomic longitude (Λ) and latitude (Φ) at station MBRE required to compute the astrogeodetic vertical deflections were observed with a Wild T4 universal theodolite by DMA's Geodetic Survey Squadron (GSS) from Cheyenne, WY, in April 1988.

Methodology

The mathematical model selected herein is conceptually very simple. The fundamental aim is to establish a functional relationship between the unknown parameters (η,ξ) and the GPS and leveling observables.

The undulation total differential can be written

$$dN_g = \frac{\partial N_g}{\partial \lambda} d\lambda + \frac{\partial N_g}{\partial \phi} d\phi \tag{1}$$

where λ and ϕ are, respectively, the geodetic longitude and latitude. The components of the deflection of the vertical (η,ξ) when referred to an ellipsoid of revolution of semimajor axes a and flattening f are defined by the following:

$$\eta = -\frac{1}{(N+h)\cos\phi}\frac{\partial N_g}{\partial \lambda} \tag{2}$$

$$\xi = -\frac{1}{(M+h)}\frac{\partial N_g}{\partial \phi} \tag{3}$$

where

h = geodetic (or ellipsoidal) height;
N = a/W, principal radius of curvature in the

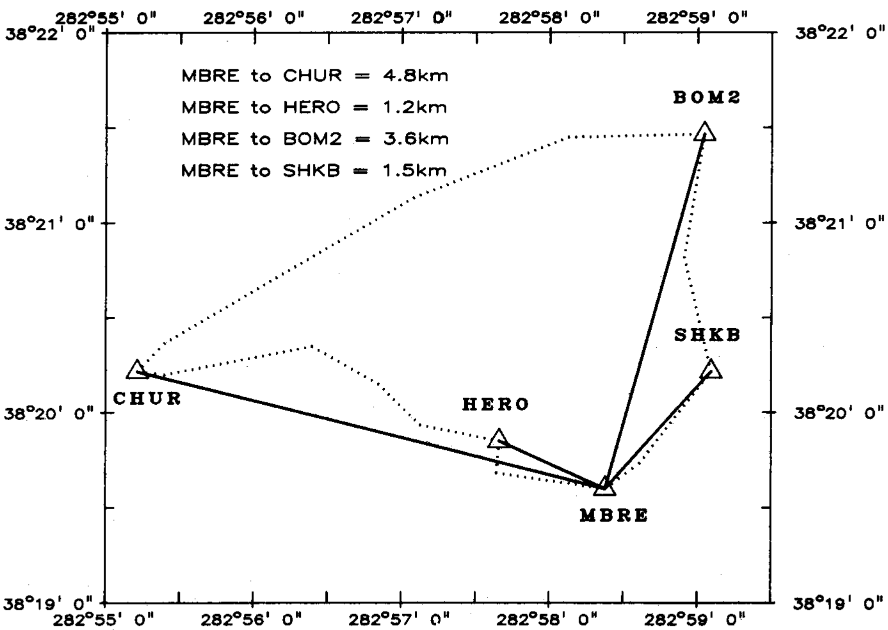

Fig. 1. GPS baselines and leveling loop.

plane of the prime vertical;

$M = a(1-e^2)/W^3$, principal radius of curvature in the plane of the meridian;

$W = (1-e^2\sin^2\phi)^{1/2}$;

$e^2 = 2f-f^2$.

Substituting the partial derivatives from (2) and (3) into (1), and assuming $N_g = h-H$ (this equality is not rigorously correct but the errors introduced are negligible), where H is the leveled orthometric height, we find the relationship

$$dh - dH = -(N+h)\cos\phi d\lambda\ \eta - (M+h)d\phi\ \xi . \quad (4)$$

Expression (4) can be written in its implicit form $F(X,L)=0$, and thus be adopted as a general least squares mathematical model, where dh, dH, $d\lambda$ and $d\phi$ are the observables (L), and η and ξ the unknowns (X). Several advantages are implicit in the above model. One is its linearity on the parameters; consequently, iterations are unnecessary. In addition, because normal heights (H^* as oppose to H) were used, N_g really represents the height anomaly ζ ($\zeta=h-H^*$), and the deflections are obtained at the earth's surface and not on the geoid. Hence, unreduced one to one comparisons with astrogeodetic deflections are feasible. Notice that $d\lambda$, $d\phi$, dh, denote the differences in geodetic coordinates between every end point in the radial network and the central station. These values are strictly geometric quantities and are determined through GPS observations. Finally, dH is the difference in observed leveled heights between the two ends of a radial baseline. This is a physical quantity defining approximately the separation between the geopotential surfaces of the two stations at the end point.

Because a least squares estimation is invoked, in actuality the best "average" values of (η,ξ) at the central station in terms of the height anomaly differences around it were estimated. Alternatively, the components of the deflection of the vertical along the E-W and N-S directions are calculated in terms of the deflections along arbitrary azimuths. Clearly, the computed deflections are closer to the astrogeodetic "point value" for shorter baselines. The main factors affecting the accuracy of the determined deflections as estimates of "point" rather than "average" deflections are the length of the baselines and the terrain roughness.

Results

All GPS solutions were obtained using precise post-fitted ephemerides derived at NSWC and referred to the World Geodetic System of 1984 (WGS84). Geocentric point positions of the central station MBRE were determined using pseudoranges, which avoid the integer cycle ambiguity problem inherent in phase observables, and the sequential filter algorithm developed at NSWC [Hermann, NSWC Internal Report, 1988]. Table 1 shows the "natural" ("true" observed physical quantities) and geodetic (strictly geometric) coordinates of the base station MBRE referred to the Geodetic Reference System 1980 (GRS80) ellipsoid.

TABLE 1. Curvilinear Coordinates of station MBRE

Natural	Coordinates[a]	Geodetic	coordinates[b]
Λ	282° 58′ 30".48±0".5	λ	282° 58′ 23".869±0".04
Φ	38° 19′ 33".71±0".5	ϕ	38° 19′ 36".293±0".03
H	(8.650±0.01)m	h	(-25.4±1.0)m

[a] Determined through astronomic (Λ,Φ) and leveling (H) observations.

[b] Determined through GPS observations. Refer to the GRS80 ellipsoid and WGS84 coordinate system.

Table 1 suggests that absolute determination of curvilinear "datum" coordinates can be currently determined with GPS, hence minimizing weather constraints, about ten times more accurately than with classical astronomic observations. From the tabulated values the two components of the astrogeodetic deflection of the vertical can be derived using the well known expressions

$$\eta = (\Lambda-\lambda)\cos\phi\ ; \qquad \xi = \Phi-\phi\ . \quad (5)$$

Several assumptions are implicit in the above equations. The Cartesian geodetic coordinate system (u,v,w) associated with the GRS80 ellipsoid and to which λ and ϕ refer, coincides or is parallel to the GPS realized WGS84 geocentric coordinate system (x,y,z). This (x,y,z) reference frame must be identical to the one implied in the reduction of astronomic observations, thus these must be accurately corrected for polar motion and UT1-UTC. The linear components of η and ξ refer to a local geodetic frame (e,n,u), being respectively positive toward east and north. Mathematical relationships between these three coordinate systems are reviewed by Soler and Hothem [1988].

The observable, dh, is a byproduct of relative position solutions of the four remote stations (BOM2, CHUR, HERO and SHKB) with respect to MBRE. They were determined using carrier phase measurements and the PHASER software developed at NGS which is based on a non-differenced phase model [Goad, 1985]. The geocentric position of the satellites was constrained to the ephemeris values, an assumption not very crucial considering the short length of the baselines. All session reductions were obtained from multi-baseline solutions using single frequency (L1) measurements sampled every 30 seconds and constraining the ambiguity biases to integers. A total of ten observation equations (i.e., eight degrees of freedom) corresponding to different combinations of repeated baselines and sessions were included in the adjustment. The standard errors for $d\lambda$, $d\phi$, and dh can be computed from the original GPS results (i.e., σ_x, σ_y, σ_z) after propagating errors. However, although no observations were rejected some dh observables were weighted differently according to their repeatability. The final standard errors applied to dh ranged from 10 mm (3 observations) to 2 mm (3 observations) with a mean of 4.5 mm for the 10 observations. The sigmas for the dH observables were computed according to the formula $1\text{mm}\sqrt{K}$ where K is the length of the leveled line in km. The adjustment "a posteriori" standard error of unit weight was 0.96, which is very close to the expected value of 1.0.

The final derived deflection components are shown in Table 2. Notice the remarkable agreement between the two sets of independently computed vertical deflections. Moreover, the standard errors for the astrogeodetic η and ξ are about four times the values obtained in the least squares adjustment for the combined GPS and leveling method. This may indicate that the standard errors estimated from

TABLE 2. Vertical deflection components at MBRE.

	Astrogeodetic	GPS & leveling	Diff.
η	5".19±0".5	5".28±0".10	-0".09
ξ	-2".58±0".5	-2".76±0".14	0".18

first-order astronomic observations (see Table 1) are generally much smaller than the often quoted value of 0".5. Due to the orientation of the baselines (none is located south of MBRE) the determined value of ξ is less reliable than that of η. Even with this limitation, the results corroborated the potential of GPS and leveling for this type of computation. To simplify future operational procedures as much as possible, only four or five baselines oriented quasi-symmetrically along the four quadrants will suffice to provide a strong geometry. One important factor, though, is to achieve repeatability of ellipsoidal height differences. Thus, several independent sessions (a minimum of three) should be scheduled for each baseline arrangement in order to detect possible outliers (e.g., errors in measured instrument height) or discern the proper weighting scheme for the observables of each individual baseline. The only observational errors which directly affect the solutions are errors in the measured height differences dh and dH. Errors as large as 10 m in horizontal position (λ,ϕ) caused insignificant changes in the parameters (η,ξ) or their variances. It is extremely difficult to simulate astrogeodetic deflections consistent with a set of given orthometric heights and GPS observables, thus an exhaustive error analysis is precluded. Based strictly on the geometry and observations of this particular survey, it was estimated that if the standard errors of all dh observations, as originally used in the adjustment, were systematically increased by 1 cm, it would still be possible to recover the deflections at the 0".4 level, which is well inside the predicted accuracy of presently applied methods.

Summary

Excellent agreement between astrogeodetic and GPS and leveling determined deflections of the vertical is reported for this feasibility test. Thus, the conclusion is reached that we can expand still further the geodetic applications of the GPS. What to date was a complicated and tedious astronomic operation, requiring inherent skills and limiting considerably the number of deflection-control points, can now be replaced by simple procedures and concepts based on new GPS techniques and standard spirit leveling methods.

The area where this test was performed is not mountainous. Thus, it may be argued that the reported agreement is a consequence of the local smoothness of the geoid and may not be a valid indication of the accuracy to be expected under more typical conditions. However, astrogeodetic deflections were also computed at CHUR (4.8 km from MBRE and 27 m higher) and differences of 1".18 and -0".48 respectively in η and ξ with respect to the values at MBRE were noted. These changes in vertical deflections from point to point are significant and much higher than the differences presented in Table 2 between the two observational methods. Encouraged by the results of this study, more tests in different geographic regions with varied terrain characteristics are clearly needed to rigorously compare the two methods at the 0".2 level.

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References

Bock, Y., R.I. Abbot, C.C. Counselman, S.A. Gourevitch, and R.W. King, Establishment of a three dimensional geodetic control network via interferometry with the Global Positioning System, J. Geophys. Res., 90, 7689-7703, 1985.

Denker, H., and H.G. Wenzel, Local geoid determination and comparison with GPS results, Bull. Géod. 61, 349-366, 1987.

Dong, D., and Y. Bock, Global Positioning System network analysis with phase ambiguity resolution applied to crustal deformation studies in California, J. Geophys. Res., 94, 3949-3966, 1989.

Engelis, T., R.H. Rapp, and C.C. Tscherning, The precise computation of geoid undulation differences with comparisons to results obtained from the Global Positioning System, Geophys. Res. Lett., 11, 821-824, 1984.

Engelis, T., R.H. Rapp, and Y. Bock, Measuring orthometric height differences with GPS and gravity data, Manuscr. Geod., 10, 187-194, 1985.

Evans, A.G., T. Soler, L.D. Hothem, W.L. Stein, B.R. Hermann, and A.E. Carlson, Jr., Vertical deflections and astronomic azimuth derived from GPS and leveling: Joint test description and results, Proc. Fifth Intl. Geodetic Symp. on Satellite Positioning. Physical Science Lab., New Mexico State Univ., Las Cruces, NM, 1989.

Goad, C.C., Precise relative position determination using Global Positioning System carrier phase measurements in a nondifference mode, Proc. First Intl. Symp. on Precise Positioning with the Global Positioning System, National Geodetic Information Center, NOAA, Rockville, MD, 1985.

Kearsley, A.H.W., Tests on the recovery of precise geoid height differences from gravimetry, J. Geophys. Res., 93, 6559-6570, 1988.

Kearsley, A.H.W., M.G. Sideris, J. Krynski, R. Fosberg, and K.P. Schwarz, White Sands revisited. A comparison of techniques to predict deflections of the vertical, UCSE Report 30007, 166 pp., Div. of Surv. Engineering, The Univ. of Calgary, Edmonton, Canada, 1985.

Soler, T., L.D. Hothem, A.G. Evans, and B.R. Hermann, GPS methods vs. conventional geodetic techniques. Preliminary results from the Dahlgren test (abstract), EOS Trans. AGU, 69, 325, 1988.

Soler, T., and L.D. Hothem, Coordinate systems used in geodesy: Basic definitions and concepts, J. Surv. Engrg., 114, 84-97, 1988.

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