Rigorous Transformation of GPS-determined Vector Components

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BIOGRAPHY

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ABSTRACT

When GPS-determined vector components are compared or integrated into simultaneous network adjustments, one must be certain that the components of the vectors are rigorously referred to the same epoch and station reference point. This paper gives rigorous matrix transformations to achieve this aim, including changes in coordinate systems, displacements due to plate tectonics, and possible centering and height measuring errors. These transformations are critical to infer accurate geodetic coordinates or when crustal motions or deformation studies are investigated. A practical case involving the shifting of Global Positioning System (GPS) observables from the L1 phase center to the antenna reference point (ARP) was investigated.

INTRODUCTION

Generally, the primary post-processed output of Global Positioning System (GPS) reduction software is the linearly independent vector components between base stations and one or more remote stations. These components, grouped by common observing periods called sessions, include their corresponding variance-covariance matrix. Also known are the date when the observations were taken and the starting and ending time of the observation window. It is important to realize that the components of these spatial vectors are referred to a particular 3D terrestrial geocentric reference frame (e.g., ITRF96, WGS84), specifically the one implicit in the precise ephemeris selected by the processor at the reduction stage. The epoch of this frame, and actually of the GPS vectors themselves, could be assumed to be the mean epoch of the session observation span, which is always designated by a year and its fraction (e.g., 0^h UTC, September 15, 1999 = 1999.707).

Then, if we want to combine vectors observed at different epochs into a simultaneous least-squares adjustment, we need to be clear about the characteristics of the coordinate frames to which the available vector components refer. For example, are all these frames the same? If not, rigorous transformation of vector components to a common frame and epoch should be performed before combining the vectors into a simultaneous adjustment. Furthermore, the geocentric location of the two points defining each vector surely has moved due to the rotation of the plate on which they are located. This important geophysical phenomena, neglected years ago when GPS observations were not so accurate, should not be ignored now and requires appropriate consideration. In Soler (1998) equations were presented to transform vector components referred to two arbitrary reference frames assumed known at epochs t_0 and t $(t \geq t_0)$ by taking into account their differences in orientation and scale as well as the motion of the plates where the points are located. Since the vector components are always given with respect to local terrestrial frames with origin at the base station A, the possible shifts between the origins of the conventional geocentric frames do not even enter into the formulation. However, small displacements in x, y, and z, caused by possible antenna centering and height measuring errors, may have occurred when the station was reoccupied, making the position of the antenna at time *t* not exactly the same as at time t_0. An extreme case of this problem could be the misidentification of a mark at a site during surveys done at two arbitrary epochs t_0 and *t*.

This situation is more common than it appears at first glance, because until recently there was not total agreement among the geodetic community about what should be recognized as the "antenna reference point (ARP)" when reducing GPS carrier-phase observables. It was general practice not long ago to assume that the L1 phase center was the best reference point to tie the observations to the reference mark or monument, usually a brass disk at ground level. This has changed in recent years and now the ARP, which is located at the center of the antenna at the base of the preamplifier ("preamp"), is considered the logical physical reference point on the antenna. The main argument favoring this preference alludes to the fact that the spatial position of the L1 phase center is not a well-defined electronic point since it changes position as a function of the satellite signal elevation angle. This was empirically corroborated as a result of several investigations that modeled antenna-phase-center patterns (Mader and MacKay, 1996; Meertens et al., 1996; Rothacher and Schär, 1996; Mader, 1999).

Another term frequently quoted in GPS literature is that of "antenna parameters." This applies to the various constants peculiar to each individual antenna, establishing the relationship between the fundamental hardware elements, e.g., nominal phase centers L1 and L2, ARP, ground plane, etc. These quantities are given by the receiver manufacturer or otherwise should be precisely calibrated by the user. NGS has calibrated most GPS geodetic antennas. Diagrams of GPS antennas and their calibrated parameters can be accessed at the following web address [➲http://www.grdl.noaa.gov/GRDL/GPS/Projects/ANTCAL/]

One clarification is in order, the ARP is not necessarily the "station reference point (SRP)." The SRP in more often than not the center of the physical disk attached to a steel pipe buried in concrete in the ground and used to permanently mark the location of the station. In classical geodesy and/or surveying practice, the SRP is traditionally the so-called monument. This is a logical choice considering that it is the only remaining permanent marker once the observations are completed and the antenna removed from the site. Thus, when a permanent mark is available, all GPS observations should be reduced to this mark. However, many GPS "fiducial" stations do not have proper ground marks *per se* and, consequently, the ARP is assumed to be coincident with the SRP. The term "fiducial" is loosely applied to name continuously operating GPS sites whose RINEX2 data are made available electronically to the GPS community. Examples include the National Geodetic Survey (NGS) Continuously Operating Reference Stations (National CORS) network [➲http:/www.ngs.noaa.gov/CORS/] or the International GPS Service (IGS) global network of GPS permanent trackers [➲http://igscb.jpl.nasa.gov]. All fiducial stations have station logs where information about site history is given. This includes the different type of antennas used during the years, the adopted antenna constants, ARP height over the mark if any, ties to nearby points, etc.

THEORY

The rigorous transformation of GPS-determined vector components from a geocentric coordinate system, e.g., ITRFyy, epoch t_0, to ITRFzz, epoch *t*, designated symbolically by the mapping ITRFyy(t_0) → ITRFzz(*t*), could be implemented according to the matrix equations given in Soler (1998) and rewritten here in its explicit form for clarity:

$$\begin{Bmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{Bmatrix}_t = (1+s) \begin{bmatrix} 1 & \epsilon_z & -\epsilon_y \\ -\epsilon_z & 1 & \epsilon_x \\ \epsilon_y & -\epsilon_x & 1 \end{bmatrix} \times \left\{ \begin{Bmatrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{Bmatrix}_{t_0} + (t - t_0) \begin{Bmatrix} v_{B_x} - v_{A_x} \\ v_{B_y} - v_{A_y} \\ v_{B_z} - v_{A_z} \end{Bmatrix} \right\} \qquad (1)$$

The variables ϵ_x, ϵ_y, and ϵ_z (expressed in radians) are the differential rotations about the axes of the ITRFyy frame required to make it parallel to ITRFzz. Counterclockwise (anticlockwise) rotations are assumed positive. The parameter *s* (unitless in ppm×10^{-6}) is the differential scale factor required to change the unit of scale of the ITRFyy frame to make it consistent with the ITRFzz frame. ν_A and ν_B (generally given in meters per year) are velocities, respectively, of points A and B caused by plate motion or any other known secular tectonic displacement. Predicted velocities at any location in the United States can be obtained interactively from the web site [➲http://www.ngs.noaa.gov] by clicking on «Products and Services» and then «HTDP-- Horizontal Time-Dependent Position». To know more about HTDP, the reader may consult Snay (1999). Finally, the geocentric Cartesian coordinates of points A and B are given according to standard notations. Some authors use $\Delta x = x_A - x_B$, etc., as an alternative nomenclature to denote vector components.

Not included in Eq. (1) are the corrections to the vector components due to centering and/or errors caused by incorrect measurement of the ARP height above the mark. Let's start by assuming that one wants to correct the vector components for possible centering and height measurement errors detected at epoch t_0. The most general case must consider displacements at both ends of the vector $\overrightarrow{AB}$, i.e., its origin at the base station A, and the tip of the vector's arrow at the remote station B. Furthermore, in practice these displacements are known on a local (geodetic horizon) right-handed frame defined by the east, north, and up (vertical) directions. In normal situations, if the antenna set up was done with rigor, the antenna ground plane would be leveled and approximately oriented towards astronomic north. The centering errors along the east and north directions, although included in this discussion for generality sake, are practically negligible. The same assumption does not apply to the vertical component. A typical example was previously mentioned, change of station reference point from the L1 phase center to the ARP or vice versa. Small changes δe, δn, and δu along the east, north, and up directions, respectively, that could have been directly measured in the field at time t_0 or may be known due to new calibration of the antenna parameters, correspond to the following changes δx, δy, and δz along the local Cartesian frame for points A and B:

$$\left\{\begin{matrix} \delta x_B - \delta x_A \\ \delta y_B - \delta y_A \\ \delta z_B - \delta z_A \end{matrix}\right\}_{t_0} = [R]_{B_{t_0}} \left\{\begin{matrix} \delta e_B \\ \delta n_B \\ \delta u_B \end{matrix}\right\}_{t_0} - [R]_{A_{t_0}} \left\{\begin{matrix} \delta e_A \\ \delta n_A \\ \delta u_A \end{matrix}\right\}_{t_0} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(2)$$

where the symbol [R] denotes the orthogonal matrix that rotates the local geodetic frame (e, n, u) into the local terrestrial frame (e.g., ITRF96) and is given explicitly by:

$$[R] = \begin{bmatrix} -\sin\lambda & -\cos\lambda\sin\varphi & \cos\lambda\cos\varphi \\ \cos\lambda & -\sin\lambda\sin\varphi & \sin\lambda\cos\varphi \\ 0 & \cos\varphi & \sin\varphi \end{bmatrix} \qquad (3)$$

The above rotation matrix should always be computed at some specific point and epoch.

The resulting vector in Eq. (2) is added to the vector components at t_0 in Eq. (1). Thus, the GPS vector components at time t can be computed from its initial values at t_0, the changes in orientation and scale of the frame from t_0 to t, the antenna displacements at t_0, and the rotation of points A and B from t_0 to t due to plate motions:

$$\left\{\begin{matrix} x_{B'} - x_{A'} \\ y_{B'} - y_{A'} \\ z_{B'} - z_{A'} \end{matrix}\right\}_t = (1+s) \begin{bmatrix} 1 & \epsilon_z & -\epsilon_y \\ -\epsilon_z & 1 & \epsilon_x \\ \epsilon_y & -\epsilon_x & 1 \end{bmatrix} \times$$

$$\left\{ \left\{\begin{matrix} x_B - x_A \\ y_B - y_A \\ z_B - z_A \end{matrix}\right\}_{t_0} + \left\{\begin{matrix} \delta x_B - \delta x_A \\ \delta y_B - \delta y_A \\ \delta z_B - \delta z_A \end{matrix}\right\}_{t_0} + (t-t_0) \left\{\begin{matrix} v_{B_x} - v_{A_x} \\ v_{B_y} - v_{A_y} \\ v_{B_z} - v_{A_z} \end{matrix}\right\} \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(4)$$

Notice that at time t, and due to the antenna displacements, vector $\overrightarrow{AB}$ is replaced by another one termed $\overrightarrow{A'B'}$. If after all vector components are reduced to time t using Eqs. (1) and (2), one wants to implement additional antenna height changes in the position of the antennas introduced at time t, then the final values of the vector components should be computed according to:

$$\left\{\begin{matrix} x_{B''} - x_{A''} \\ y_{B''} - y_{A''} \\ z_{B''} - z_{A''} \end{matrix}\right\}_t = \left\{\begin{matrix} x_{B'} - x_{A'} \\ y_{B'} - y_{A'} \\ z_{B'} - z_{A'} \end{matrix}\right\}_t + \left\{\begin{matrix} \delta x_B - \delta x_A \\ \delta y_B - \delta y_A \\ \delta z_B - \delta z_A \end{matrix}\right\}_t \qquad (5)$$

where the vector of δ displacements on the right-hand side of Eq. (5) is computed using Eq. (2), but replacing the epoch t_0 by t.

DATA COLLECTION AND PROCESSING

The test area selected for this feasibility study was the Federal Base Network (FBN) in Wisconsin (see Fig. 1). This GPS network was observed from Sept. 18, 1997 (doy = 261) to Dec. 1, 1997 (doy = 335). The planning, observation schedule, and data collection were completed through the joint efforts of NGS and the Wisconsin Department of Transportation (WisDOT). The field data used in this study contain a total of 49 sessions, involving 7 receivers observing simultaneously (in 8 sessions); 6 (16); 5 (18); 4 (6); and 2 receivers (in 1 session). A total of 12 Trimble 4000SE and 10 Ashtech Z-XII receivers were used in the field. The project occupied 98 stations. In addition to

Figure 1. Wisconsin FBN Test Network

the receivers used in the field, five CORS stations (KEW1, MIL1, STB1, STP1, and WIS1) were also used at the processing stage. It should be emphasized that the U.S. Coast Guard has installed Ashtech Z-XII3 receivers with Geodetic III antennas (model # 700829.A1) at the five CORS sites.

In order to achieve the best possible set of coordinates, every session included data from the accurately-known CORS points. In 10 sessions one of the CORS points was the reference station, in all other cases they were used as remote stations. A total of 268 independent GPS vectors were determined in the data reduction (Fig. 1). All data were processed using NGS software PAGE4 (Schenewerk, 1993; [➲http://www.grdl.noaa.gov/GRD/GPS/DOC/toc.html]) adapted for Windows NT. This program applies antenna/elevation-dependent phase corrections, which is a must when an observing session involves stations that have different antenna types. GPS data were collected during a 6-hour observing window (approximately: 17^h UTC to 23^h UTC). All 25 satellites in the GPS constellation at the time of processing were used in the reductions of each individual day-session. At any given time, a minimum of four and a maximum of seven satellites were simultaneously visible above the horizon. Although raw data were collected at 15-second intervals, the selected sampling rate to reduce the observations of this test was set at 30 seconds, a restriction imposed by the data collection interval at the fiducial (CORS) stations. A minimum elevation angle of 15^o was chosen as the cut-off angle for all carrier-phase observables during the processing stage.

At the time GPS observations were collected (September-December, 1997) IGS orbits were expressed in the frame

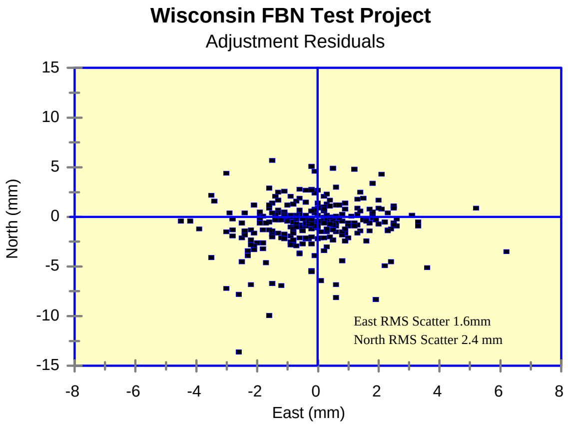

Figure 2. Adjustment Residuals Plotted on the Geodetic Horizon Plane

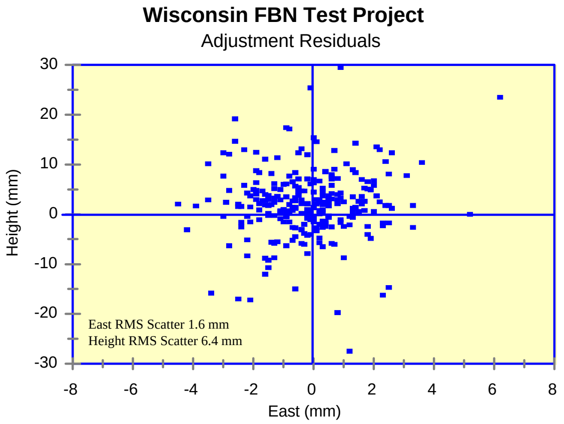

Figure 3. Adjustment Residuals Plotted on the Prime Vertical Plane

ITRF94. The adopted starting coordinates for the fiducial stations used in the reductions described herein are consistent with the frame of the precise satellite ephemeris employed, namely ITRF94, epoch 1996.0. However, because observations were collected in 1997, before the processing of GPS observations began, the known velocities of the CORS stations were used to update to the proper observation epoch the coordinates of the fiducial stations originally known at epoch 1996.0. As a result of this precaution, the components of all determined vectors are, in a sense, "instantaneous" and refer to ITRF94 (the ephemeris frame) and a variable epoch which is determined by the time at which the observations were actually taken. This ITRF frame and epoch identification tag becomes very critical in case the processed vectors are used for future scientific applications.

Only static, multi-station, relative GPS solutions between selected "base" and "remote" stations were utilized. All non-automatically corrected data outliers and cycle slips, if any, on frequencies L1 and L2 were manually accounted for (by relying on post-fitted residual plots for quality control). Final solutions were determined using double-difference carrier phase measurements and the ionosphere-free linear combination of the L1 and L2 model [Leick 1995, p. 306]. A zenith tropospheric scale factor was estimated for every 3 hours. Ambiguity biases were fixed whenever possible.

RESULTS

Following NGS standard procedures, a GFILE was created for each GPS session. This file [➲ http:// www.ngs.noaa/ FGCS/tech_pub.html] contains the components of the vectors along the local terrestrial x, y, and z axes defined by the frame and epoch of the precise GPS ephemeris used in the reduction process (e.g., ITRF94, epoch 1997.822 for year 1997 at 0^h UTC, doy = 300). Each vector component is followed by its corresponding standard error and the correlation matrix of all vector components include in the session. This particular file must be modified when the reference frames at epochs t_0 and t are not the same, when correcting the vector components for plate motions, or when antenna centering and height errors are considered. The case discussed here is exclusively restricted to a switch from L1 phase center to ARP at epoch t while enforcing the assumption that the frames were identical at both epochs. Consequently, $t = t_0$; $\epsilon_x = \epsilon_y = \epsilon_z = s = 0$; $\delta e_A = \delta e_B = \delta n_A = \delta n_B = 0$. The values of t for two arbitrary points A and B defining vector $\overrightarrow{AB}$ will contain the correction from the L1 phase center to ARP at both ends of the vector if appropriate. Every point in the Wisconsin network –except the CORS stations– were referred, as usual, to the ground mark and, therefore, no height corrections to the GPS vectors at these points were required. However, the station reference point for the CORS stations was originally assumed to be the L1 phase center, but was later changed to the ARP. Consequently, according to the type of antenna used δu_{CORS} = - 9.2 cm. Thus, instead of reprocessing the GPS vectors, the GFILE was modified to account for the shift from L1 phase center to ARP.

With the assumptions mentioned above enforced, each individual session in the original GFILE was modified and finally combined into a GFILE$_{final}$ containing new transformed vector components for each session of the project. Afterwards, two minimally constrained least-squares adjustments were performed using the original GFILE with the CORS stations referred to the L1 phase center and the new GFILE$_{final}$ referred to ARP. In both adjustments the same CORS station STP1 was held fixed using its published L1 phase center and ARP coordinates in each adjustment referred to the ITRF94, epoch 1997.0. Then, the coordinates of all the points in the network from the two solutions were compared. As expected, the results were identical for the points where the reference point was the ground mark. However, the coordinates of the CORS stations showed a discrepancy which is exactly the shift between the L1 phase center and the ARP. This proves that the analytical procedure using matrices and the consequently generated program worked. Figures 2 and 3 depict all adjustment residuals from the solution projected on the planes of the geodetic horizon (east versus north) and prime vertical plane (east versus ellipsoid height = up).

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