Transformations Between NAD83 and WGS84

One of the questions most frequently asked by GPS users is: How do I transform coordinates between the frames defined by NAD 83 and WGS84?

In effect, the GPS-user is trying to convert the positional coordinates that the receiver outputs or the manufacture proprietary software gives from one particular frame into the other. First of all, one should understand that the 3-D Cartesian frames to which the coordinates of the NAD83 and WGS84 refer are not identical. Their origin, axes orientation in space, and the unit of scale differ. Why? Simply, because the definitions of these two frames are based on different sets of observations, processing algorithms, and perhaps, geodetic assumptions.

In practice, any terrestrial frame is defined by a set of points with assigned coordinates and velocities. These parameters materialize the reference frame in question. The velocities are important to change positional coordinates from one epoch to another. Thus, it is critical to understand that if we refine the coordinates and/or the velocities of the defining points by introducing new and more accurate observations, then, the origin, orientation and scale of the frame may change, but to a small degree. That is why sequence of reference frames has been developed. The latest version of NAD83 and WGS84 frames are specifically named NAD83 (CORS96) and WGS84 (G1150).

Thus, when the receiver tell us that the computed coordinates using GPS observations are referred to the WGS84, they actually mean WGS84(G1150). On the other hand, if we use CORS coordinates data, or the OPUS service, the NAD83 reference frame to which these coordinates refer is actually NAD83 (CORS96).

Consequently, rigorously speaking, a transformation between NAD83 and WGS84 should be interpreted as a transformation between NAD83 (CORS96) and WGS84 (G1150). Transformations between older realizations of NAD83 and WGS84 are now outdated and will not be discussed here.

The question is then reduced to: How do we transform between NAD83 (CORS96) and WGS84 (G1150)?

In absolute positioning, the frame WGS84 (G1150) is materialized not by the coordinates of marks on the surface of the Earth - as the NAD83 (CORS96) is - but by the coordinates of points in space, namely, the 3-D location of the GPS satellites (given by their precise or broadcast satellite ephemerides). Recent investigations [True, 2004] have shown that for all practical purpose, the WGS84 (G1150) frame is identical to the International Terrestrial Reference Frame of year 2000 (ITRF2000). Thus, except for very accurate geophysical calculations (crustal dynamics; tectonic strain determination, etc.), the WGS84 (G1150) is supposed to be aligned with the ITRF2000. In another words, both have the same origin, orientation, and scale.

Once the WGS84 (G1150) and ITRF2000 are assumed equivalent [≡ is the mathematical symbol for equivalent], the question can be taken one step further and finally be formulated as: How do we transform between NAD83 (CORS96) and ITRF2000?

In order to do this transformation the epoch at which the coordinates defining the frames were determined must be known. The National Geodetic Survey (NGS) provides coordinates and velocities of all CORS stations on these two frames: ITRF2000, epoch 1997.0, and NAD83 (CORS96), epoch 2002.0, for the conterminous states. The epoch 2003.0 was selected for the state of Alaska due to the displacements originated by the Denali earthquake on November 2, 2002. The value of these coordinates and velocities as well as a description of previous ITRF frames can be consulted in the following CORS Web page: http://www.ngs.noaa.gov/CORS/metadata1/.

Let's assume now that we want to transform coordinates between the WGS84 (G1150) ≡ ITRF2000 and NAD83 (CORS96). The first thing that we need to keep in mind is that, in theory, the WGS84 (G1150) or ITRF2000 coordinates are the first one computed by the receiver software. However, they are originally obtained at the time of the observation, that is, the epoch corresponding to the mid-point of the observation window during the GPS data were collected. Consequently, we cannot compare coordinates from a set of observations taken today to results we determined one year ago. These two sets of coordinates must be reduced to a common epoch before the comparison is made. Lets assume, that we want to compare the coordinates we obtained today referred to the ITRF2000 to the ones at epoch 1997.0. Some receivers may be already doing this transformation from epoch to epoch and the final result may be already expressed in the frame WGS84 (G1150) ≡ ITRF2000 at epoch 1997.0. This will simplify the work involved, otherwise we should know the velocity of the GPS station due to the rotation of the tectonic plate on which is located. These velocities are not rigorously known until we have constantly monitored the point for a number of years. This is not generally the case and we must rely on geophysical models. The most common model used these days for correcting for plate rotations is called NNR-NUVEL-1A. This model provides the parameters required to correct for plate motion for all major plates (macroplates) forming the crust of the earth.

Assume for an instant, that the velocities of the three components vx, vy, and vz, are known in cm/year for a particular point, then, the transformation between epochs could be simply written:

x (epoch 1997.0) = x (epoch today) + vx * (1997.0 - epoch today)

with similar equations for y and z.

Once we have all observed points referred to a common frame and epoch, e.g. ITRF2000, epoch 1997.0, we will be able to compare apples to apples. Remember that this change from epoch to epoch may be already included in the software that you are using. In any case, one should know that all coordinates must refer to a common epoch.

Finally, we are ready to transform between WGS84 (G1150) ≡ ITRF2000 at epoch 1997.0 and NAD83 (CORS96) at epoch 2002.0.

In order to do this final step, the 14 transformation parameters (three shifts, three rotations, one scale and their variations with time) between the two frames are needed. The rigorous transformation is somewhat involved [Soler and Snay, 2004], thus, the easiest approach is to use the software developed at NGS which is interactively available through the Internet. Its name is Horizontal Time Dependant Positioning (HTDP) and can be located at the following URL: http://www.ngs.noaa.gov/TOOLS/Htdp/Htdp.shtml.

In fact, HTDP [Snay, 1999] has also the capability of transforming coordinates from epoch to epoch using in the process model NNR-NUVEL-1A mentioned above. This software also could provide transformations from the WGS84 (G1150), assumed identical to ITRF2000, to NAD83 (CORS96). The text included on the HTDP Web page is self explanatory and sufficiently detailed for doing all types of transformations necessary when performing GPS work.

Final comment. Although on November 5, 2006, the GPS satellite orbits provided by the International GNSS Service switch to frame ITRF2005, everything mentioned above is still valid. Nothing practically will change considering that the differences between ITRF2000 and ITRF2005 are smaller than the differences between WGS84 (G1150) and ITRF2000. Except for accurate geodetic investigations these differences could be assumed negligible.

References

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Further Reading

Below is a list of references that expand into the theory and practical applications of a topic as important as frame transformations for GPS users. We suggest that the persons desiring more information should start by reading and comprehend the series of articles about coordinate frames and datums written by Snay and Soler. For those interested on expanding into the theoretical intricacies of the problem a more technical set of citations are included. The reason that we are mainly concentrating on NGS publications is because all these references can be downloaded for free from the CORS Web page.

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