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ACCURATE DETERMINATION OF CARTESIAN COORDINATES AT GEODETIC STATIONS USING THE GLOBAL POSITIONING SYSTEM

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Abstract. Comparison of Cartesian coordinates determined at collocated sites using two independent space techniques, very long baseline interferometry (VLBI) and Global Positioning System (GPS), shows remarkable agreement even when the points in question span transcontinental distances. The results corroborate the capabilities of commercial dual-frequency GPS receivers to perform geodetic work at the highest available accuracy. Adjusted geocentric coordinates of a configuration of GPS stations well distributed along the eastern half of the United States were accurately determined (better than 10^{-8}) in the rigorously defined International Earth Rotation Service (IERS) terrestrial reference frame ITRF 89.

Introduction

In two significant campaigns, during 1987 and 1990, the National Geodetic Survey (NGS) initiated the formation of a regional A-order high accuracy reference network (HARN) covering the eastern half of the conterminous United States employing Global Positioning System (GPS) techniques. This network is in the process of being densified and extended to cover the entire country [Strange and Love, 1991]. Among the set of points comprising this GPS framework are 10 well-distributed stations previously occupied by mobile very long baseline interferometry (VLBI) instruments or tied by local surveys to other fixed VLBI antennas. Thus, in order to estimate the inherent accuracy of the GPS results, it was logical to compare them against the VLBI-inferred "ground truth." Although repeatability (i.e., precision) of GPS observables at regional scales is well documented [e.g., Lindqwister et al., 1991; Larson and Agnew, 1991], research to determine "true" accuracy of GPS three-dimensional results is practically non-existent. The obvious explanation is the scarcity, until recently, of results obtained from the comparison of GPS-derived data with other observational techniques collocated at the same geodetic monuments. To this date, the emphasis has been concentrated in comparing individual baseline length and orientation [Larson and Agnew, 1991].

The accuracy study presented here expands in complexity recent attempts [Larson et al., 1991] restricted to a single test involving three sites, a unique set of three fiducials and an observation interval of only three days. In this experiment we compared adjusted coordinates derived from a more intricate GPS network covering the eastern half of the U.S. against coordinates computed from an unrelated VLBI least-squares solution for collocated sites. These two solutions are independent and uncorrelated, each one influenced by its own intrinsic observational procedures and peculiar error sources.

Coordinate systems definition

It is well-known that the coordinate system to which the reduced GPS Cartesian coordinates refer is the one implied by the orbital ephemerides used to fix the satellite positions. Commonly, this is the World Geodetic System of 1984 (WGS 84). However, if an orbital adjustment is also invoked as part of the least-squares solution, the reference frame for all estimated Cartesian coordinates will be defined by the coordinates assigned to the selected fiducial stations. This orbit relaxation approach is necessary if one considers that the best satellite ephemerides available today are restricted to accuracies of a few parts in 10^7. Consequently, when accuracies better than 1 part per million (ppm) are intended, the only available recourse is to simultaneously solve for some of the orbital parameters (plus the typical clock terms, ambiguities, tropospheric biases, etc.) in conjunction with the receiver position.

Originally, the coordinates of the known VLBI points used in this investigation were given in a quasi-geocentric frame defined by a global unpublished VLBI solution completed at NGS on October 1, 1990, and hereafter abbreviated NGS VLBI 90 [M. Abell, personal communication, 1990].

For the purpose of this study, all NGS VLBI 90 coordinates were transformed to the more recognized and accessible International Earth Rotation Service (IERS) terrestrial reference frame ITRF 89. This is a combined [VLBI, satellite laser ranging (SLR), and lunar laser ranging (LLR)] solution based on adjustments finalized in 1988, thus the rigorous terminology ITRF 89, Epoch 1988 [Boucher and Altamimi, 1991]. Nevertheless, for simplicity, the epoch suffix will be dropped in the remaining discussion. The origin, orientation, and scale of ITRF 89 are implicitly defined by the coordinates adopted for the terrestrial sites. The origin of the ITRF 89 is located at the center of mass of the Earth (including oceans and atmosphere) with an uncertainty of 10 cm. The IERS terrestrial reference pole (TRP) and reference meridian are consistent within $0''.003$ (i.e., 3 mas), with the corresponding directions in the previously used BIH Terrestrial System (BTS).

To determine the relationship between the NGS VLBI 90 and ITRF 89, a seven parameter similarity transformation was performed using 65 stations common to both coordinate systems. The shifts, rotations, and differential scale change of the mapping NGS VLBI 90 → ITRF 89, as well as other pertinent information are given, using conventional notation [e.g., Soler and Hothem, 1989], in Table 1. These parameters were used to transform the coordinates of every NGS VLBI 90 station in the GPS network to the ITRF 89. Once coordinates for all (mobile and fixed) VLBI points were obtained in the ITRF 89 frame, priority was given to the fixed VLBI locations as candidates for fiducial points in the GPS reductions. This preference is supported by the empirical confirmation that larger antennas and *in situ* very stable atomic oscillators collect observations with better signal-to-noise ratios than the less sophisticated mobile VLBI equipment.

In 1987, the Cooperative International GPS Network (CIGNET) was at the incipient stages of development. Nonetheless, during that year's GPS campaign, several stations in the high accuracy network were already connected by local surveys to some fixed VLBI antennas anticipating their use as primary fiducial sites in future GPS reductions.

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TABLE 1.- Transformation parameters between NGS VLBI 90 and ITRF 89

From	To	No. of Sta.	Number of Observations Input	Number of Observations Used	RMS mm	Translations Δx mm	Translations Δy mm	Translations Δz mm	Rotations $\delta\epsilon$ mas	Rotations $\delta\psi$ mas	Rotations $\delta\omega$ mas	Scale δs $\times 10^{-8}$
NGS VLBI 10-01-90	ITRF 89	65	195	179	14	-23.9 ±3.3	-13.7 ±3.3	-3.6 ±3.0	-4.9 ±0.1	10.2 ±0.1	-1.0 ±0.1	0.38 ±0.05

Stations selected with this principal goal in mind included HAR3, MOJ1, TIME, OCP3, and WES1 (see Figure 1). In 1989, CIGNET placed continuous trackers at three U.S. locations (MOJA, RICH, and WEST). Since then, every GPS reduction at NGS requiring orbit relaxation methods uses, when possible, these three CIGNET fiducials. Unfortunately, receiver malfunctions and unexpected breaks in data collection, although unusual, do occur. During the reduction of the GPS 1990 project, and on occasions when there was not an adequate amount of data from some of the three available CIGNET stations, other marks connected to large VLBI observatories and occupied during the GPS campaign were held fixed as fiducials in the orbital adjustment.

Data collection and processing

Exclusively, dual-frequency receivers capable of recording carrier phase and pseudorange observables were required in all the surveys described herein. Only static multistation relative GPS procedures between a selected "base" station and several "remote" ones were implemented.

The 1987 campaign commenced on November 12 and was completed on December 10. The field data contain a total of 13 sessions, involving from a maximum of 16 (2 sessions) to a minimum of 4 (2 sessions) TI-4100 four-channel receivers co-observing simultaneously. They were equipped with GESAR (geodetic satellite receiver) data-acquisition software. This project occupied a total of 42 stations.

The 1990 venture started on March 9 and ended on April 21. The field data of this campaign contain a total of 30 sessions involving from 9 (2 sessions) to 5 (1 session) instruments. In this instance, all data were collected with eight-channel Trimble 4000SST receivers, except at the three CIGNET fiducial stations, where Mini-Mac 2816 receivers were permanently deployed. The total number of stations occupied was 54. Co-participants on this occasion were the Texas Highway Department (5 sessions observed, 4 stations occupied) and the New Mexico State Transportation Department (3 and 3). Included in the HARN presented here is a network (3 and 5) also observed in 1990 (June 29 to July 1) by Western Geophysical for the Wisconsin Department of Transportation using Mini-Mac 2816 receivers. Because of logistics problems, not every station visited in 1987 was reoccupied in 1990, and conversely, stations established in 1990 were not included in the 1987 survey.

Double-difference phase observables were processed in the multistation session mode solving for tropospheric biases at each remote station and cross correlations between vector components with the OMNI software package, version 2.0, developed at NGS [Mader et al., 1991]. In both projects a cut-off elevation angle of 20° and a time interval of 30 seconds were selected in the processing. In order to guarantee the best satellite geometry, observation spans of 5-6 hours were scheduled for each session. This time window assured at least six different satellites above the cut-off angle during the observing campaigns. Precise postfitted ephemeris generated at Naval Surface Warfare Center (NSWC) in 1987 and at Defense Mapping Agency Hydrographic Topographic

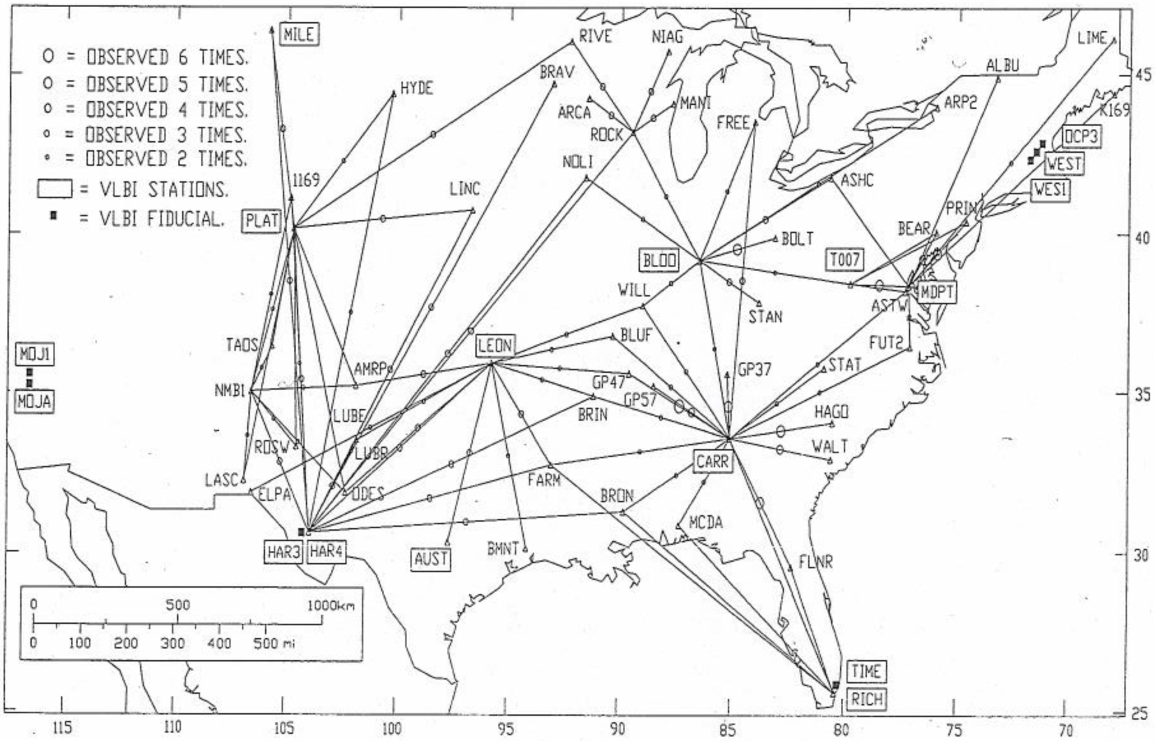

Fig. 1. GPS high accuracy reference network (HARN)

Center (DMAHTC) in 1990, both referred to WGS 84, were used as initial conditions for the positions of the satellites in the orbital adjustment. As described above, fiducial stations were fixed to the ITRF 89 values during an orbital relaxation solution where five Keplerian elements (a, e, i, Ω, t_0) were also estimated for each satellite orbital arc.

After all nonautomatically corrected data outliers and cycle slips (on frequencies L1 and L2) were manually cleaned (relying as reference on OMNI's postfitted residual plots), final solutions were determined using the L1, L2 combination option, which minimizes the effect of ionospheric refraction. Due to the length of the vectors involved, no attempt was made to fix ambiguity biases.

Results

A minimally constrained network adjustment using the NGS three-dimensional least-squares adjustment program (LAP) with GPS interstation vector components as observations was implemented. The coordinates in the ITRF 89-defined frame for the station at Richmond (RICH), Fl., which satisfied two important characteristics: rigorously known fixed VLBI station and reasonably connected by GPS to the rest of the network, were held fixed in this solution. A total of 55 stations were positioned using 215 nontrivial vectors, i.e., 645 observations, 27 (4.2%) of which were considered outliers and rejected, their individual residuals exceeding three times the RMS of all adjustment residuals.

Experimental evidence shows that *a priori* variance-covariance matrices of GPS-reduced vector components are too optimistic. They contain "formal errors," which do not reflect the complexities of the real physical world. This is primarily due to the difficulties in properly modeling systematic errors whose distribution is complicated by their spatial and/or temporal variations, e.g., satellite geometry, propagation delays, ephemerides inaccuracies, and multipath effects. Thus, the standard deviations commonly attached to the observables are not realistic. A number of numerical experiments confirm that a good representative value for the standard deviation of each GPS-reduced vector component is its residual as obtained by a preliminary minimally constrained network adjustment. These residuals are practically invariant with respect to the choice of fixed station, but they are significantly influenced by the unmodeled errors intrinsic to each particular observation. Thus, they reflect a better measure of dispersion for each observable than the original standard deviations generated by any GPS vector processing software. Large residuals are generally correlated with unmodeled conditions at the stations, e.g., high humidity, passing storm fronts, signal reflections from nearby surfaces, and ionospheric activity. Conceptually, they indicate how consistently each observation fits its average value which presumably represents the outcome of ideal standard observing conditions. This is particularly true if the repeated observations correspond to sessions with a wide range of epochs. This approach was noted to be superior to other often introduced alternatives, e.g., multiplying the original formal errors by a scale factor even when this factor may be distance dependant.

As expected, and corroborating the practicality of the weighting scheme described above, the adjustment *a posteriori* standard deviation of unit weight was reduced from 113.28, using the original formal errors generated by OMNI, to 1.72 when a residual-based weighting strategy was introduced. The former value reinforces the theory that statistics resulting from GPS reduction software are extremely optimistic or, alternatively, that they underestimate "true errors" when baselines predominately longer than 500 km are observed. The average vector length in this network is 791 km.

Another factor to consider when relative GPS networks are adjusted is the realization that reduced GPS vector components (the observables) have implicit definition of scale and orientation. Consequently, only three degrees of freedom specifying the origin (i.e., three coordinates = one site position) remain to be defined to account for the required minimal set of geometric constraints. Thus, similar to VLBI methods, no information about the origin of the network is known in advance. As VLBI relies on SLR and LLR (dynamic techniques) to refer its coordinates to the geocenter, in this study the coordinates of one GPS station (RICH) in the ITRF 89 frame are required to accomplish the same objective.

In contrast to standard VLBI analyses, no assumption about site velocities due to global plate rotations was introduced. This is consistent with the particular location of the stations involved in this comparison, all of them residing on the North American plate. However, the area in question may have detectable strain accumulation during the three year interval spanning the two sets of observations. In the near future, more specialized research will be directed to uncover this possibility, considering that very little is known about regional strain patterns in the eastern half of the United States.

Table 2 shows the results obtained after comparing the VLBI and GPS coordinates in a common frame of reference, the ITRF 89. Some of the differences, de, dn, and du, respectively, about the local east, north, and geodetic zenith (up) are remarkably small and were not anticipated before this investigation was completed. The largest tabulated value represents a positional absolute difference of 6.0×10^{-9} (6 parts per billion, ppb), a quantity difficult to imagine before current extraterrestrial observational methods became available. There is not an immediate explanation of why two of the differences in the table exceeded the 3 cm threshold. One plausible explanation is that the ties between main VLBI

TABLE 2. Coordinate differences in a local geodetic coordinate system (east, north, up) given in the sense GPS - VLBI

Location	Stat. ID	NGS No.	GPS Ant.	VLBI Type	de mm	dn mm	du mm	Total, ds mm	ds/R ppb	Stat. Tied	GPS Vectors	
											From	To
Austin (TX)	AUST	5966	TR	M	-11	33	2	35	5	1	0	3
Bloomington (IN)	BLOO	1192	TI&TR	M	8	-3	4	9	1	10	23	2
Carrolton (GA)	CARR	9449	TI&TR	M	-1	0	0	1	.3	18	52	1
Fort Davis (TX)	HAR4	8727	TI	F	-4	12	-3	13	2	14	40	0
Green Bank (WV)	T007	8440	TI&TR	F	13	6	-15	21	3	5	4	6
Leonard (OK)	LEON	9437	TI&TR	M	-15	4	17	23	4	12	26	3
Maryland Point (MD)	MDPT	5009	TI&TR	F	8	-8	-34	36	6	9	22	1
Miles City (MT)	MILE	9433	TI&TR	M	-1	14	5	15	2	2	0	6
Platteville (CO)	PLAT	6614	TI&TR	M	4	14	-3	15	2	12	18	4
Richmond (FL)	RICH	9721	MM	F	0	0	0	0	0	5	5	0

Notes: M = mobile; F = fixed; R = mean Earth's radius = 6371 km.

TABLE 3. Distance (adjusted) differences from station RICH. Values dS are given in the sense GPS - VLBI

Stat. ID	distance, S km	dS mm	Relative Accuracy
AUST	1773	-19	1.0×10^{-8}
BLOO	1605	5	0.3×10^{-8}
CARR	993	1	0.1×10^{-8}
HAR4	2363	2	0.1×10^{-8}
T007	1420	4	0.3×10^{-8}
LEON	1855	17	0.9×10^{-8}
MDPT	1443	-10	0.7×10^{-8}
MILE	3193	11	0.3×10^{-8}
PLAT	2755	3	0.1×10^{-8}

reference points and GPS monuments may be vulnerable to survey errors. In particular the connection at Maryland Point between the main VLBI antenna and MDPT could have a larger than desirable error along the height component. It is premature to confirm or deny this finding; but the concern is not new and has previously been expressed by other investigators, therefore resurveys of some of the most questionable connections should be contemplated.

At this time it is difficult to pinpoint the exact source of the detected discrepancies between the VLBI and GPS results. Probably both techniques are responsible in undetermined ways for the differences encountered in the present quest. A complete understanding of the specific error budget contribution will require additional comparisons. However, as a glance at Table 2 shows, the overall range of variation for the discrepancies between VLBI and GPS coordinates is not significant. Furthermore, the listed results improve on unrelated research previously published [Larson et al., 1991, Table 5].

Although the available sample of points is far from ideal, computed RMS values about the mean show, respectively, 9 mm, 12 mm, and 14 mm, in the east, north, and vertical components. As usual, the height component is the noisiest of the three, nevertheless, it is surprising that the magnitude of the disagreement between the VLBI and GPS vertical components is so small. Generally, there is a more accentuated degradation in the computed GPS heights, as previously noticed in more localized statewide networks [Soler et al., 1991]. Perhaps, simultaneous estimation of orbital parameters resolves the vertical component better than the standard procedure of fixing the coordinates of the satellite to the assumed perfect ephemeris. In this situation, the geometry of the mathematical model is not as rigid, and relative ellipsoidal heights between base (held fixed) and remote stations are better determined.

Finally, Table 3 depicts another measure of accuracy (external consistency) based on spatial distance comparisons. This avenue has the advantage of being invariant with respect to potential discrepancies in coordinate system definitions. The tabulated values are not an indication of repeatability; they do not represent differences between observed vector lengths, but between pairs of distances computed from the final adjusted coordinates of the VLBI and GPS solutions. Although the possibilities are endless, station RICH, a well-known CIGNET VLBI point, was selected as the common origin for all computed distances. The results show excellent agreement (largest relative accuracy is 10 ppb) between the two independent techniques substantiating the capabilities of GPS methods to establish geodetic positions at the highest achievable accuracy.

Conclusions

Results derived using advanced GPS technology and methods show good agreement with supposedly more precise and accurate VLBI techniques. The approach followed here involved comparison of coordinates and distances determined from two independently derived network adjustments of continental extent. It is concluded that measurements derived using simple GPS operational procedures can provide rigorous absolute geodetic positions at levels of accuracy comparable to mobile VLBI techniques whose instrumentation is more cumbersome and difficult to staff, hence, less cost-efficient. The only requirement is the availability of a few stations known in an accurate geocentric coordinate system such as ITRF 89. This should not be difficult to achieve as CIGNET improves its global coverage. Encouraged by these findings, NGS is continuing the consideration of a nationwide HARN in order to define an accurate reference framework to constrain lower level B-order statewide networks (relative accuracy = 1 ppm). The attainable degree of accuracy reported here can improve present surveying and mapping needs and are crucial for other related scientific studies.

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