Subseasonal GNSS positioning errors

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[1] Global Navigation Satellite System (GNSS) station coordinate errors over seasonal and longer time scales are known to be spatially and temporally correlated with flicker noise spectra. Overlaying this are strong annual and semiannual variations that cannot be explained by any single phenomenon. Next most prominent are harmonics of the GPS draconitic year with periods of (351.4/N) days. One explanation is that errors in the standard model for Earth orientation parameter (EOP) tidal variations near 12 and 24 h periods are absorbed into the resonant GPS orbit and daily EOP estimates, resulting mainly in draconitic and fortnightly alias signatures for 24 h product sampling. With the change in International GNSS Service (IGS) station coordinates from weekly to daily resolution in August 2012, it is now possible to study subseasonal performance. All IGS Analysis Centers (ACs) show fortnightly signals, but the resolution will not be sufficient to distinguish direct from aliased subdaily tidal error sources till two more years of data are available. Nevertheless, aliased errors from the subdaily EOP tide model are expected. All but one of the ACs that includes GLONASS data have signals at ~8 day periods, the ground repeat period for GLONASS orbits. This most likely arises from larger geographically correlated orbit errors for GLONASS. Two ACs possess unique short-period features that appear to be caused by peculiarities of their analysis strategies. **Citation:** Ray, J., J. Griffiths, X. Collilieux, and P. Rebischung (2013), Subseasonal GNSS positioning errors, *Geophys. Res. Lett.*, *40*, 5854–5860, doi:10.1002/2013GL058160.

1. Introduction

[2] GPS geodetic results first found widespread scientific application in measuring linear surface velocities for tectonic studies. Accuracy considerations required an understanding of technique errors at seasonal and longer time scales where it was learned that coordinate errors are spatially and temporally correlated [*Zhang et al*., 1997; *Mao et al*., 1999; *Williams et al*., 2004]. The background spectra of nonlinear position residuals closely follow a flicker noise power law process, with modest white noise at the highest frequencies. Since then, reliable velocity uncertainties have usually been based on empirical measures of power law noise levels rather than on formal errors that neglect time correlations [*Mao et al*., 1999; *Williams*, 2008; *Bos et al*., 2008].

[3] In addition, strong annual and semiannual GPS variations are routinely observed, which cannot be explained by any single phenomenon [*Dong et al*., 2002]. Crustal displacements due to pressure loading variations from the atmosphere, ocean, and continental water account for only about half of the nonlinear vertical motions [*Dong et al*., 2002], namely, ~2.4 mm global median annual amplitude [*Ray et al*., 2011]. Peak annual vertical load amplitudes reach ~1 cm in central Asia due to atmospheric pressure and in the Amazon basin due to soil moisture. Much smaller portions of the GPS horizontal annual variations (10 to 20%) are caused by surface fluids, assuming the load models are reliable [*Ray et al*., 2011]. These seasonal effects must be taken into account for reliable GPS velocity estimates, for instance, by requiring observing spans at least 2.5 years long [*Blewitt and Lavallée*, 2002].

[4] Next most prominent and more recently discovered are harmonics of the GPS draconitic year (the interval between repeats of the Sun-GPS constellation inertial orientation), with periods of (351.4/N) days for N = 1, …, 6 or higher [*Ray*, 2006; *Ray et al*., 2006, 2008; *Amiri-Simkooei et al*., 2007]. These features are pervasive in nearly all products of the International GNSS Service (IGS). One explanation is that demonstrated errors in the International Earth Rotation Service (IERS) model for subdaily EOP tidal variations near 12 and 24 h periods [*Petit and Luzum*, 2010] are absorbed into the resonant GPS orbit and daily Earth orientation parameter (EOP) estimates, resulting mainly in draconitic and fortnightly alias signatures for standard 24 h product sampling [*Griffiths and Ray*, 2013]. Such an orbit-linked mechanism predicts that the draconitic signals should be spatially correlated, as found by *Collilieux et al*. [2007] and *Amiri-Simkooei* [2013], whereas station linked causes like local multipath do not lead to large-scale ground patterns.

[5] *Griffiths and Ray* [2013] showed that errors in the subdaily EOP tide model are efficiently transmitted into the orbits, or even amplified on the prograde side, for lines within about 0.1 cycles per day (cpd) of the GPS orbital period (near K2) or its multiples. Some longer-period aliases in GPS orbit error spectra coincide with simple predictions based on 24 h sampling [*Penna and Stewart*, 2003], but some are shifted in frequency. Broad alias bands form mainly near fortnightly periods with smaller signals around 9, 7, and 29 days. A simulation assuming errors of about 20% in the IERS model for subdaily EOP tides [*Petit and Luzum*, 2010] closely matches

Additional supporting information may be found in the online version of this article.

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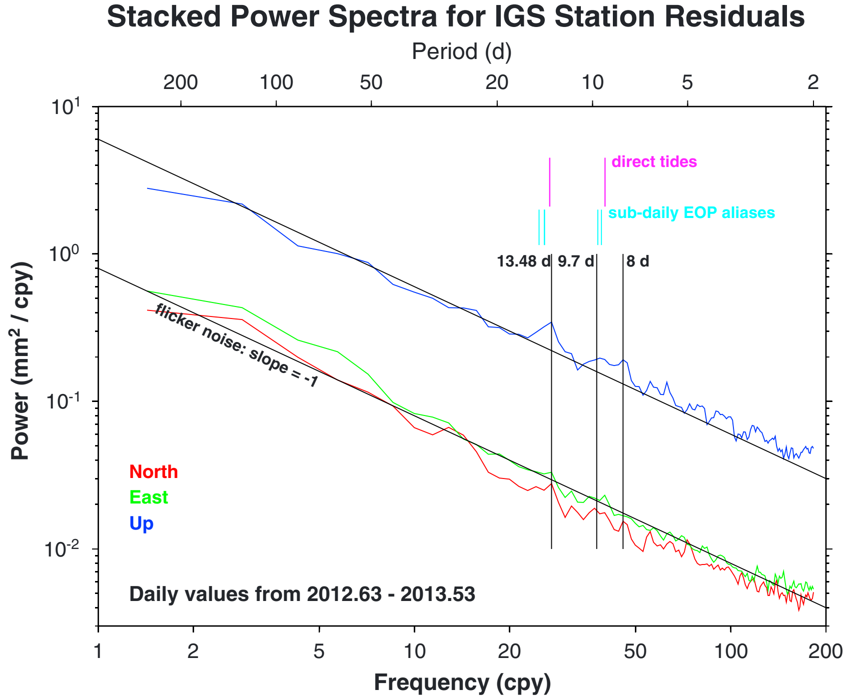

Figure 1. PSDs by local North, East, Up component for the 306 IGS combined station residuals having at least 256 days of data during 20 August 2012 to 13 July 2013. Fast Fourier transform power spectra were computed for each time series then stacked and averaged. Three spectral peaks are identified by their periods, and some major tidal lines are indicated ("direct tides") together with nearby aliases of subdaily EOP tidal lines.

these submonthly features in the spectra of actual IGS orbit errors, as well as explaining large near-annual power and some odd draconitics.

[6] It has not been possible to search for confirmation of the submonthly lines in IGS station coordinates as these have historically been weekly integrations. However, earlier studies reported fortnightly signals in GPS daily network solutions [*Nikolaidis*, 2002, Figure III.18; *Williams et al.*, 2004, Figure 5] and daily precise point positioning solutions [*Amiri-Simkooei et al.*, 2007, Figure 7]. Starting 19 August 2012, the IGS switched to daily terrestrial frame products, which enables subseasonal error analysis for series from a number of different analysis software and strategies. All ACs of the IGS now claim to use strictly 24 h data arcs, except that JPL (NASA's Jet Propulsion Laboratory) and GRG (Groupe de Recherche de Géodésie Spatiale) use 30 h overlapping arcs, and some ACs include some secondary multiday constraint conditions. The purpose of this contribution is to inspect the first year of IGS daily coordinate time series for spectral features that could confirm the role of subdaily EOP tidal errors, clarify the origin of GPS draconitic harmonics, and reveal other indications of tide mismodeling or analysis errors. Not only is this relevant for the fullest geophysical interpretation of Global Navigation Satellite System (GNSS) results, but understanding short-period technique errors is a precondition for the development of sensitive GNSS-based systems to detect sudden natural hazards, such as earthquakes and tsunamis [e.g., *Blewitt et al.*, 2009]. While the IGS products have previously relied solely or primarily on GPS data, by early 2011, four ACs began to include significant volumes of GLONASS data also. The combined data usage is referred to as GNSS.

2. Data and Analysis

[7] The main source of station positioning results used here comes from the official IGS coordinate frame combinations except that the daily time series residuals for JPL come from their website sideshow.jpl.nasa.gov/post/series.html. Details on the data files and the spectra computations are given in the supporting information.

3. Results

[8] The most striking feature of the power spectral densities (PSDs) for the IGS combined time series in Figure 1 is how closely each component follows the slope -1 power law of flicker phase noise in the subannual frequency domain. White noise flattening is only apparent for periods shorter than about 2.6 days. No draconitic harmonics can be resolved with the current span of IGS daily coordinates. However, a fairly clear spectral peak can be seen in all three components near 13.5 days. The resolution at that period corresponds to about ±0.8 day. Smaller peaks in the dU (Up) power are possible near 9.7 days and especially near 8.0 days (also in dN (North)), but neither is very well resolved at this stage.

[9] The leading subseasonal tidal lines one might expect to affect daily GNSS positions if the tide models have significant errors are at periods 9.12/9.13, 13.63/13.66, 14.77, 27.56, and 31.81 days. We refer to these as "direct" tidal signatures. The 13.63/13.66 days pair has the largest tidal potential among these. For standard 24 h (S1 period) data processing, errors in the tides near 12 and 24 h can also affect geodetic estimates via longer-period aliases [*Penna and*

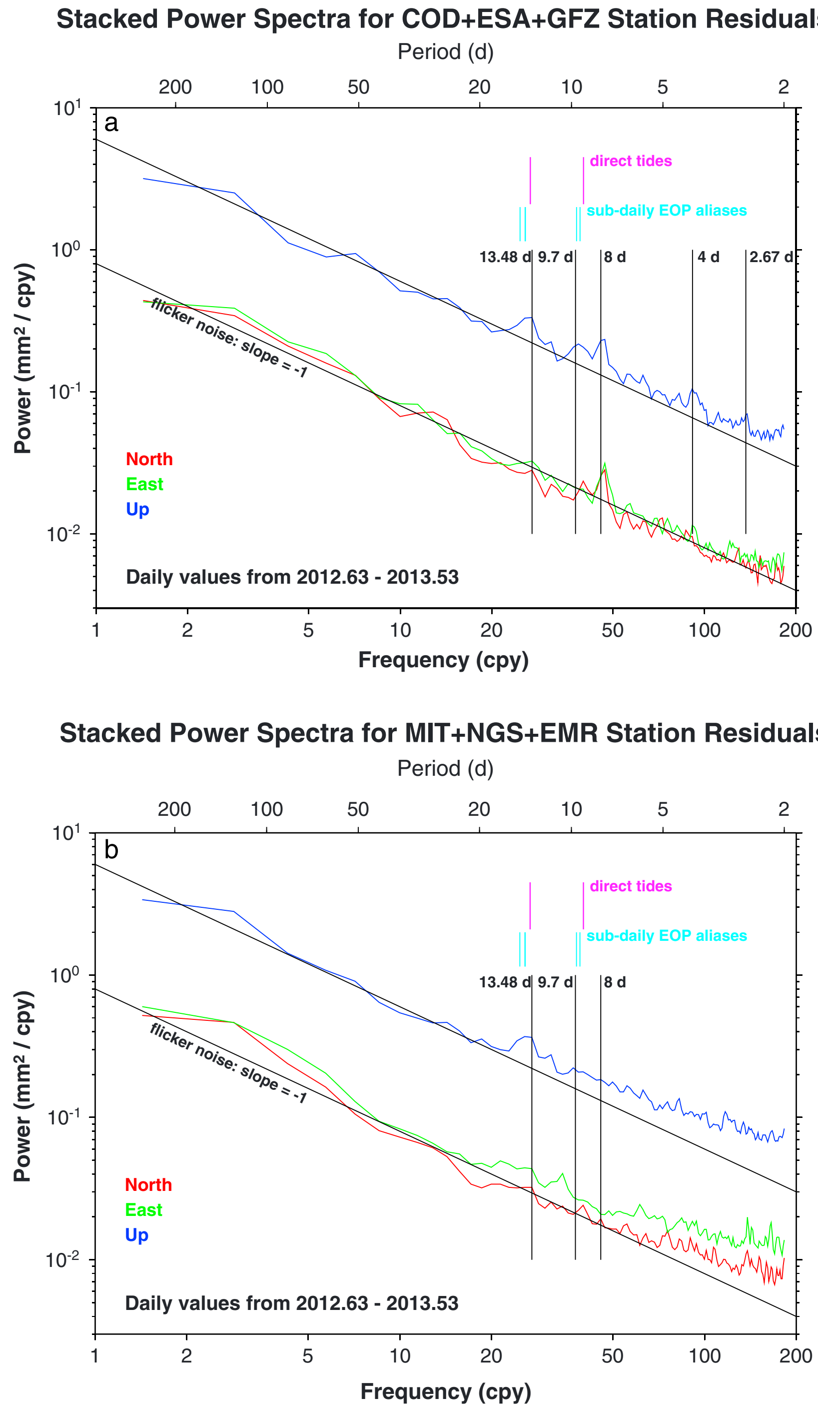

Figure 2. Same as Figure 1 except using only those stations contributed by (a) ACs COD (Center for Orbit Determination in Europe), ESA (European Space Agency), and GFZ (GeoForschungsZentrum) that use GLONASS as well as GPS data; and (b) ACs MIT (Massachusetts Institute of Technology), NGS (National Geodetic Survey), and EMR (Energy, Mines and Resources) that use GPS data only. COD + ESA + GFZ include a total of 474 stations, while MIT + NGS + EMR comprise 419 stations.

Stewart, 2003]. The main aliases are at periods 7.13, 7.38, 9.37, 9.61, 14.16/14.19, 14.73/14.76, 29.80, and 31.81 days [*Griffiths and Ray*, 2013]. The tides at M2 (alias at 14.76 days) and O1 (alias at 14.19 days) are largest in this range, but their alias magnitudes depend on the transfer mechanism and its efficiency. We distinguish these "aliased" tidal signatures from the "direct" ones and rely on them as tracers of the underlying error sources, provided that the frequency resolution is sufficient: primarily, the main lines 9.37/9.61 days versus 9.12/9.13 days and 14.19/14.76 days versus 13.63/13.66 days, respectively. These diagnostic lines are indicated in Figure 1. It is clear that the IGS daily time series are not yet long enough to resolve these differences. When the IGS accumulates at least 1024 days of daily results, the direct and

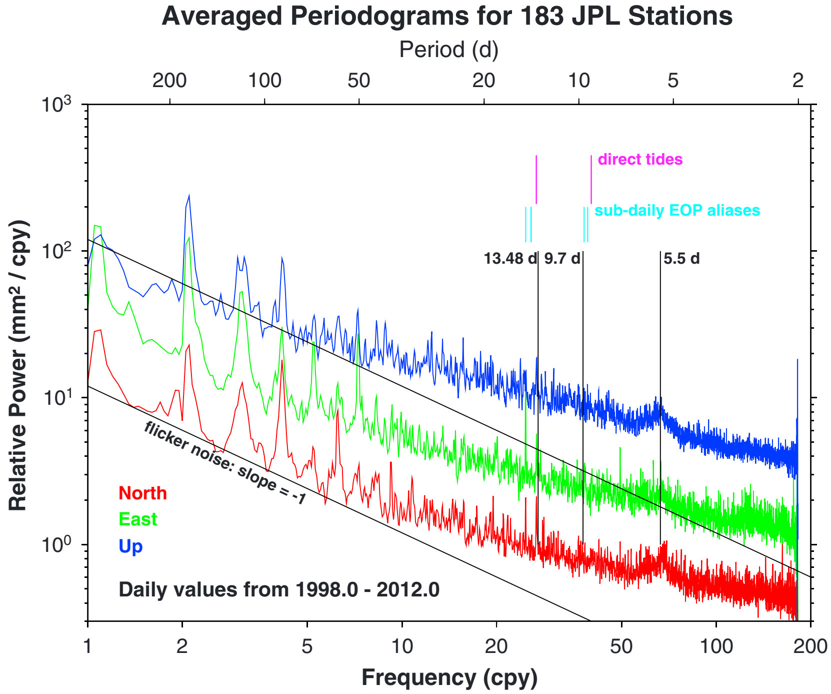

Figure 3. Normalized periodograms for the 183 daily JPL stations that are at least 80% complete for the period 1998.0 through 2012.0 have been computed after annual and semiannual fits have been removed for each. The periodograms were then stacked and averaged, and the results shown were multiplied by 3 for dE and by 9 for dU.

aliased fortnightly lines should be minimally distinguishable with a resolution of about 0.36 cpy, about one third of the interval separating them.

[10] The 8.0 days peak in Figure 1, if confirmed, is in a range where tidal lines are absent or relatively minor. Given that the ground repeat period for GLONASS satellites is also 8.0 days (nominal orbital period of 11h 15m), we investigate that possible source by comparing separate stacked spectra for the group of GNSS ACs (COD + ESA + GFZ) in Figure 2a to GPS-only ACs (MIT + NGS + EMR) in Figure 2b. Levels of small peaks in the 13–14 days and ~9 days bands are similar for the two AC groups, but the 8 days peak is only evident for the GNSS ACs. In fact, 8.0 days peaks are quite clear for all three components whereas no trace is seen in the GPS-only spectra.

[11] If the 8.0 days GNSS peak results from systematic errors in the GLONASS orbits that recur as the satellite ground tracks repeat, then one might expect to see associated harmonics, especially keeping in mind that GLONASS has three orbit planes. The second harmonic should be at 4.0 days and the third at 2.67 days. There are faint indications of both harmonics in the GNSS AC dU spectra. The underlying error source could be any effect that gives rise to geometrically repeating orbit errors, such as due to a strongly nonuniform distribution of GLONASS tracking stations. Indeed, major gaps in the network coverage occur over most of the Pacific Ocean, North Africa and the Mideast, most of the U.S., and western South America; concentrations of GLONASS stations occur in Europe, Canada, and Australia.

[12] Two IGS ACs, one GNSS, and the other GPS only, were not included in the Figure 2 groupings. GRG (GPS + GLONASS) spectra are shown in Figure S1 of the supporting information. They do not show evidence of the 8.0 days peak, which is probably because of the fact that they weight GLONASS data much more weakly than GPS, but there is a very pronounced peak at about 3.66 days in all three components. There is no matching feature in their orbit results compared to the IGS combination, so it is evidently not satellite linked. However, a connection to tide model errors seems remote. It might not be coincidental that this period is very close to half of the GRG processing week (3.625 days), which consists of seven overlapping arcs of 30 h each or 174 h. That could happen if GRG has a subtle station constraint or coding bug related to their weekly processing batches. Curiously, there is no matching peak at 7.25 days. In any case, this strong peak is unique to some aspect of the GRG data analysis and not a general GNSS feature.

[13] JPL (GPS only) was also not included in the Figure 2 groups for reasons explained in section 2 (see supporting information). Instead, stacked normalized periodograms using 14 year long daily residuals for 183 stations from JPL are shown in Figure 3; annual and semiannual fits have been preremoved. Draconitic harmonics are clear in all three components up to the fourth, but their amplitudes vary by component at higher frequencies (fifth is strong in dE (East), sixth in dN, seventh in dE and dU). Another general aspect of the JPL power distributions is the much higher level of white noise flattening than the other ACs, which becomes apparent already at monthly periods. White noise levels are also higher than average in the AC spectra of GRGS, MIT, and particularly NGS, but their deviations from flicker noise are at much shorter periods, around 10 days. In the submonthly range, JPL is distinctive in several other respects too. A broad excess of power centered roughly at 5.5 days stands out, most prominently for dN and dU. This feature was previously noticed by *Amiri-Simkooei* [2013] who claimed to see "many small peaks indicating no clear, sharp, and unique peak for

these signals" and concluded them likely to be quasiperiodic and perhaps station dependent. We suggest an alternative explanation. Recalling that JPL processes global network GPS data in 30 h overlapping arcs, their tidal aliases will differ from most other IGS ACs that use 24 h data sets. Tides near 12 and 24 h will cluster into alias bands near 2.5 to 3 days and around 4 to 7 days, respectively. While some very diffuse excess power might exist in the predicted 2.5 to 3 days band (120 to 150 cpy), it is too noisy to confirm aliasing of tidal errors as the explanation for the 5.5 days band.

[14] Also singular to JPL are well-resolved fortnightly lines at 14.76 days (strong 24 h alias of M2 or the weak direct 14.77 days tide) and 13.63 days (direct tide), thanks to the long span of daily samples. These are relatively large in dE and dN but hardly visible in dU (13.63 only maybe), unlike the fortnightly features of the other ACs. *Amiri-Simkooei* [2013] reported lines in the same JPL data (though for a different selection of stations) at 13.63, 14.2, 14.6, and 14.8 days, which he attributed to aliasing effects of subdaily unmodeled periodic signals even though neither 13.63 nor 14.6 days is a common tidal alias. Although able to resolve spectral features very effectively, his multivariate method was applied to the 3-D coordinate series simultaneously and so aggregated all detected periods. We observe here that fortnightly peaks are much less distinct (or even absent) in the JPL vertical component than in the horizontal. While higher measurement noise in heights could reduce detection sensitivity compared with the horizontal components, most tidal effects are correspondingly larger for the vertical. So the most natural explanation of the JPL fortnightly lines points to a tidal rotational source rather than to station displacement errors (see below). There is also no indication of any discrete 9 days lines in any of the JPL spectra, though it is possible that some broadband horizontal power exists around 9.7 days and longer. On the other hand, a single, distinct line stands out at 7.38 days in dE, unmatched in the other two components. This falls at the 24 h alias period of the μ2 tide [*Griffiths and Ray*, 2013]. It is also close to the expected 30 h alias of the major M2 tide (7.72 days) or perhaps related to the JPL orbit processing week (7.25 days). But none of these possibilities explains why only the dE component is affected.

[15] Given the 30 h processing arcs of JPL global network solutions and the 24 h data files used for the bulk of their point positioning results, tidal aliases could be expected at the usual periods discussed by *Griffiths and Ray* [2013], including fortnightly, as well as the ~5.5 and ~3 days bands. However, the lack of dU signatures except for the broad ~5.5 days feature, unlike for the other ACs, points to a rotational source for the narrow lines rather than tidal displacement errors. Subdaily EOP tide errors normally couple into orbit radial motions and station heights, so these appear an unlikely source. The only hypothesis we can offer is that perhaps there are shortcomings in the JPL EOP modeling that directly impact the relative orbit-Earth orientation at these particular fortnightly and 7 days lines.

4. Discussion and Conclusions

[16] Several reports have claimed to detect errors in various tide models used a priori in GNSS data processing. The models [*Petit and Luzum*, 2010] include effects for tidal displacements of the solid (body) Earth (the largest single effect), ocean loading, atmospheric pressure loading (S1, S2, and higher harmonics of the solar day), as well as tidal variations in the geopotential due to the solid Earth, ocean, and atmosphere, and the corresponding oscillations in the Earth's rotations, mostly owing to the oceans. Of the tidal rotations, only those shorter than about 1 days rely on an external model (from the IERS Conventions 2010) applied a priori; longer-period tidal variations are included naturally in the standard global parameters consisting of daily polar motion offsets and rates plus length of day. Rotational tidal errors can therefore appear only as aliases of subdaily EOP model defects in 24 h (and longer arc) GNSS results. The IERS model for the body tide is thought to be accurate to about 1 mm [*Petit and Luzum*, 2010], but in their empirical study of tidal displacements using GPS, *Yuan et al*. [2013] inferred sensitivity to errors in the body tide down to the 0.24 mm level (vertical) that indicate lateral heterogeneities in the Earth's rheology. Ocean tidal loading magnitudes reach up to about 10 cm vertically, maximum, with the largest errors expected in areas adjoining mainly shallow seas where the global ocean tide models are least accurate or where tide heights are most extreme, such as around Brittany and coastal Antarctica. Elsewhere, typical model inaccuracies are expected to be in the few percent range [*Petit and Luzum*, 2010]. So models for tidal displacements should be generally reliable to about 1 to 2 mm globally. This is much smaller than the hypothetical 10 mm error level assumed in the *Penna and Stewart* [2003] simulation and probably explains why published examples of real GPS tidal aliases are sparse.

[17] By contrast, errors in the IERS model for subdaily EOP tides affect GNSS differently in two important respects. First, the existence of those errors can be detected very sensitively by computing differences of estimated GNSS polar motion (PM) values for adjacent days by extrapolating from noon estimation epochs to midnight day boundaries using the estimated PM rates. Errors in PM offsets for tidal lines close to 12.0 and 24.0 h average and alias to very small values (usually 10% or less), but such attenuation does not affect the PM rate extrapolation. So a spectrum of these PM rate differences will immediately reveal any tidal EOP aliases. *Kouba* [2003] used this approach to check that all IGS ACs implemented the most current IERS subdaily EOP tide model. Later, *Ray and Griffiths* [2009] used the same method and found aliased EOP tidal errors in GPS PM rates even after the latest IERS model was adopted by all ACs. The tidal signatures were traced to defects in the IERS model itself by comparing with differenced PM rate spectra generated using alternative EOP models provided by R. Ray (private communication, 2009) based on different global ocean tide models. Such spectra for EOP models derived from TPXO7.1 and GOT4.7, compared to the IERS model, closely match the periods of most peaks in the actual IGS PM rate results, namely, annual (K1, P1, T2 aliases), 14.2 d (O1), 9.4 d (Q1, N2), and 7.2 d (σ1, 2Q1, 2N2, μ2) [*Ray and Griffiths*, 2009]. (Real IGS peaks at odd draconitic harmonics are not matched by the tide model variants because those are generated via orbit fitting interactions; see *Griffiths and Ray* [2013].) The amplitudes of the IGS peaks do not always match any of the tested models, but those vary anyway among the three tide models. The general uncertainty in the IERS model is revealed by the variations among the tidal amplitudes of the models, at the 10 to 20% level but not the detailed frequency-dependent errors.

That same error level is consistent with the dispersion among EOP tide models considered by *Artz et al*. [2012]. Given that the total maximum subdaily EOP amplitudes reach about 1 milliseconds of arc (mas), the IERS model error is therefore very significant, around 0.1 to 0.2 mas. That corresponds to displacements of 3 to 6 mm at the Earth's surface or 13 to 26 mm at GPS altitude. In other words, the EOP tidal errors are considerably larger than any other type expected except for ocean loading within certain specific regions.

[18] The second important aspect of the subdaily EOP tidal errors for GNSS processing is that they couple directly, without dilution or averaging, into the estimated GPS orbital parameters [*Griffiths and Ray*, 2013] for resonant terms within about 0.1 cpd of K1 and its multiples. This ensures that they must thereafter influence station position estimates. We note in passing that the "repeat orbit" aliasing simulated by *Penna and Stewart* [2003] is not valid because they assumed a GPS orbital period of exactly K2, which is not quite correct. The mean GPS repeat period is about 7.5 s longer and varies significantly among individual satellites [*Agnew and Larson*, 2007], which leads to a frequency shift and broadening of the aliases they predict. Furthermore, the complex interaction of the tidal variations of the rotating Earth with the orbital parameters [*Griffiths and Ray*, 2013] was totally neglected.

[19] Contrasting these considerations against the results of section 3, the IGS daily spectra are perhaps consistent with subdaily EOP tidal aliasing, but they presently lack the resolution needed to confirm alias signatures over direct tidal errors. At least two more years of data will be needed to really improve the situation. The highly resolved JPL results are anomalous in this respect by showing direct and possibly alias lines but prominent only in the horizontal components. Because the JPL orbit modeling is quite different from all other ACs except EMR (both ACs use the same software system, but EMR data arcs are 24 h), it is possible that the propagation of tidal errors is also different and does not affect station heights much. On the other hand, the much lower resolution EMR daily spectra (not shown) match other IGS ACs, not JPL, in having distinct dU fortnightly power but no 5.5 days band. In any case, JPL and GRG both show signs of unique spurious effects probably derived from aspects of their data processing.

[20] The indications of 8 days errors for those ACs (COD, ESA, GFZ) that include GLONASS data is interesting. While aliasing of subdaily EOP tide errors as the underlying source is a possibility, that should be less effective than for GPS because the shorter orbital period means less efficient coupling via orbital parameters. A quantitative simulation is needed similar to that by *Griffiths and Ray* [2013]. Meanwhile, GLONASS orbit errors are definitely larger than for GPS in IGS products by about a factor of 3, based on internal AC agreement, and geographical correlation is likely. So this is our preferred hypothesis.

[21] In the context of GLONASS-related errors, we observed large PM rate discrepancies in early 2013 for the same GNSS ACs with the 8 days coordinate residual peak. In a private intra-IGS email exchange (17 February 2013), G. Gendt (a member of the GFZ AC team) showed results with and without GLONASS that demonstrated a GLONASS data connection. T. Springer (a member of the ESA AC team) confirmed (email, 18 February 2013) the same result in their independent with and without GLONASS solutions. While the mechanism for the GLONASS degradation of the EOP estimates remains unknown, a relationship with the coordinate errors should be kept in mind.

[22] In conclusion, the arrival of IGS daily frame products opens the door to deeper understandings of errors in standard tidal models. That will only be fully realized after longer data spans become available, but we can already see prominent analysis dependencies that introduce spurious effects. Those need to be addressed. In particular, a better weighting needs to be achieved when including GLONASS data to avoid adding artificial short-period signals into station positions.

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