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Full Length Research Paper

Utilizing non-iterative linear transformations between non-uniformly dilated 3D frames

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Linear transformations have been routinely applied in various field applications since they permit a straightforward geometric relationship between different coordinate systems. The most general form of linear transformations which is also referred to as the affine model allows coordinate transformations between non-uniformly dilated systems. This particular model is intrinsically useful for datum applications in deforming areas. In this study, a twelve-parameter 3D affine transformation is proposed to model the physical properties between the positional coordinates from different sets of reference systems. Its transformation parameters can be estimated using a novel Non-Iterative Solution for Linear Transformations (NISLT) algorithm. In order to illustrate the capability of the proposed approach, numerical analyses were performed for the coordinate transformations in two real cases: 1) a mild non-uniform dilatation case transforming coordinates between two continental networks, an International Terrestrial Reference Frame 2000 (ITRF2000) solution and its corresponding International GNSS Service (IGS) solution, and 2) a strong non-uniform dilatation case transforming coordinates between two regional networks around Taiwan based on IGS solutions at two different epochs. Results reveal that a significant improvement on the transformation quality can be achieved when the proposed procedure is implemented in modern-day datum applications, especially for those in a deforming area.

Key words: Coordinate transformation, affine transformation, deformation analysis, non-uniform dilatation.

INTRODUCTION

Transformation of coordinate systems plays an important role in many scientific applications since it makes possible a unified analysis of multi-system spatial information. Among many types of transformations, the similarity transformation model is one of the most commonly used in view of its simple mathematical configuration and the easy interpretation of its parameters. Many applications of this model can be found in geodesy for relating a geocentric frame to a geodetic datum-defined frame, or for relating two terrestrial reference frames obtained from different realizations (Molodenskii et al., 1962; Badekas, 1969; Leick and van Gelder, 1975; Soler and van Gelder, 1987; Costa et al., 2008; Pino and Firkowski, 2009; Başçiftçi et al., 2010). In recent years, this similarity transformation model has been further extended to a time-variant version to accommodate time-dependent variations of modern terrestrial reference frames (Soler, 1998; Altamimi et al., 2002; Han and van Gelder, 2006). In addition to datum applications, a similarity transformation model is also used for relating the R^2 image space and the R^3 object space in photogrammetry (Mikhail et al., 2001), and for combining multiple network solutions from various techniques and/or at different epochs (Altamimi et al., 2002; Han et al., 2008; Aktuğ, 2009). Despite its wide applicability, a similarity transformation model however does not provide a satisfactory solution for non-uniformly deformed systems, due to the fact that it allows only a uniform scale factor. Alternatively, an affine model characterizes non-uniform dilatations with different scale

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factors in three principal directions, and thus works well in many cases where a similarity model is not applicable. Applications of an affine model can be found, for instance, in photo coordinate registrations with fiducial marks (Mikhail et al., 2001).

In a traditional approach, the parameters of a transformation model are estimated by applying a least-squares adjustment technique. It usually requires a good a-priori estimation of the parameters to initiate sound iterative computations. This is not often a problem in cases when the two systems are almost aligned. However, in circumstances where the coordinate systems are poorly aligned (for example, the transformation from a local datum frame to a global frame or between arbitrarily defined frames), it usually requires additional efforts to find appropriate initial values for the parameters. Furthermore, the iterative computations could become very inefficient when a large number of reference points is involved in a transformation.

In order to improve the computational performance, recent efforts have been made to find explicit solutions of linear transformations as alternatives to the traditional least-squares solutions. In Awange and Grafarend (2003), it is illustrated that a closed-form solution for a seven-parameter similarity transformation is possible by means of the Gauss-Jacobi combinatorial algorithm. Awange and Grafarend (2005) further developed a Procrustes algorithm to compute the similarity transformation parameters without iterations. Later, Awange et al. (2008) proposed an ABC-Procrustes algorithm for a nine-parameter transformation problem which works well in the cases of mild anisotropy. More recently, Han (2010) proposed a non-iterative solution for linear transformations (NISLT) algorithm for the transformations using a seven-parameter similarity model or a twelve-parameter affine model. This alternative provides a reliable solution at the same quality level as the least-squares approach does, but with improved computational performance. All aforementioned works make possible a rigorous and efficient analysis for coordinate transformations.

In this study, the twelve-parameter affine transformation is proposed for modeling the deformation behavior of coordinate systems. By incorporating the NISLT algorithm, the proposed affine model can be readily applied in coordinate transformations with a reliable and proficient performance. As will be demonstrated, the proposed procedure is feasible for modern-day datum applications and makes substantial contributions particularly to the cases with non-uniformly dilated coordinate systems.

MATERIALS AND METHODS

A linear 3D transformation model with uniform dilatations

It is common practice to relate coordinate vectors referred to two coordinate frames with a similarity transformation model, which can be written as follows:

$$\mathbf{x}' = s\mathbf{R}\mathbf{x} + \mathbf{t} \tag{1}$$

Where $\mathbf{x}$ and $\mathbf{x}'$ are the coordinate vectors in the original and transformed systems, s is a scale factor, $\mathbf{R}$ is a rotation matrix and $\mathbf{t}$ is a translation vector. It is well-known that in the 3D case, the rotation matrix and the translation vector can be expressed as:

$$\mathbf{R} = \begin{bmatrix} \cos(r_z) & \sin(r_z) & 0 \\ -\sin(r_z) & \cos(r_z) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(r_y) & 0 & -\sin(r_y) \\ 0 & 1 & 0 \\ \sin(r_y) & 0 & \cos(r_y) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(r_x) & \sin(r_x) \\ 0 & -\sin(r_x) & \cos(r_x) \end{bmatrix} \tag{2}$$

and

$$\mathbf{t} = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix} \tag{3}$$

Where r_x, r_y, and r_z are the counter-clockwise rotation angles about the three axes, and t_x, t_y, and t_z are the translations with respect to the x, y, and z axes, respectively. A total of 7 parameters are required for transforming coordinates between two frames using a similarity transformation model.

As illustrated in Equation 1, a similarity transformation model postulates a unique scale between the two systems. In other words, only a uniform dilatation (that is, the same scale in all directions) is allowed when two coordinate systems are related by this model.

A linear 3D transformation model with non-uniform dilatations

In order to model the non-uniform dilatations between two coordinate systems, a second-rank symmetric tensor (Billington and Tate, 1981) is employed:

$$\mathbf{E} = \begin{bmatrix} e_{11} & e_{12} & e_{13} \\ & e_{22} & e_{23} \\ sym. & & e_{33} \end{bmatrix} \tag{4}$$

The symmetric tensor $\mathbf{E}$, which is typically referred to as a deformation tensor can be decomposed into the product of an orthogonal matrix $\mathbf{S}$ and a diagonal matrix $\mathbf{\Lambda}$:

$$\mathbf{E} = \mathbf{S}\mathbf{\Lambda}\mathbf{S}^T = \mathbf{S} \begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix} \mathbf{S}^T \tag{5}$$

The diagonal elements λ_i ($i = 1,2,3$) in the matrix $\mathbf{\Lambda}$ represent the principal dilatations in three orthogonal directions which are defined by the column vectors in the matrix $\mathbf{S}$. In the special case that $\mathbf{S} = \mathbf{I}_3$ (the 3×3 identity matrix), the principal dilatations are into the directions along the three coordinate axes. With the addition of this symmetric tensor, the coordinate system is allowed to have a homogeneous deformation with non-uniform dilatations. Figure 1 depicts the deformation behavior on a plane of a uniformly dilated system and a non-uniformly dilated system.

By substituting the deformation tensor for the uniform scale parameter, Equation 1 now becomes:

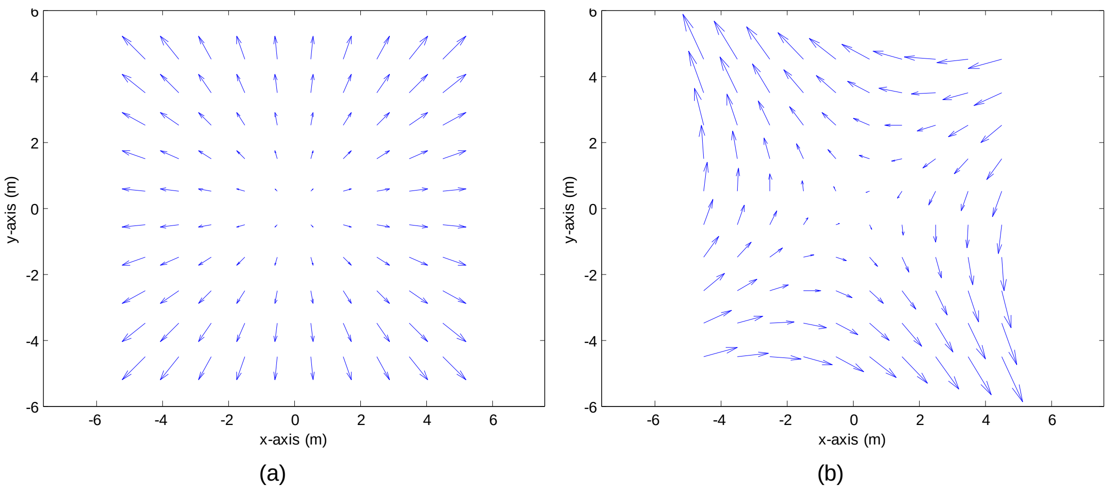

Figure 1. Visualization on a plane of two types of deformations; a uniformly dilated system (a) and a non-uniformly dilated system (b).

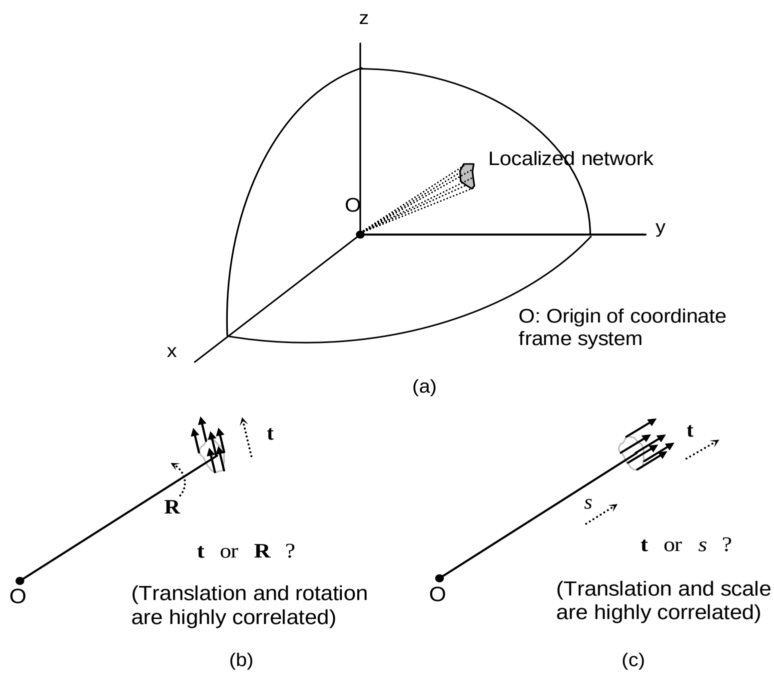

Figure 2. A localized reference network with respect to (a) a global frame and (b and c) parameter correlations for the transformation of such a reference network.

$$\mathbf{x}' = \mathbf{E}\mathbf{R}\mathbf{x} + \mathbf{t} = \mathbf{S}\Lambda\mathbf{S}^T\mathbf{R}\mathbf{x} + \mathbf{t} \tag{6}$$

Where $\mathbf{R}$ and $\mathbf{t}$ still represent a rotation and a translation, and $\mathbf{E}$ indicates that a non-uniform dilatation between two coordinate systems is present. The transformation expressed in Equation 6 is mostly referred to as an affine model. In a general 3D case, a total of 12 parameters (3 in $\mathbf{t}$, 3 in $\mathbf{R}$ and 6 in $\mathbf{E}$) are present in this model.

Molodenskii-Badekas treatment

When a transformation is performed between a frame defined by a regional reference network and a global coordinate frame, the estimated transformation parameters become highly-correlated due to a localized geocentric distribution of reference sites. Consequently, the parameter estimation is of a lower quality and thus less interpretable. This concept is intuitively illustrated in Figure 2. It can be seen in the figure that, for the transformation of a small network, scale and rotations are on a high correlation with the translations. As a result, observational errors in scale or rotations could produce large uncertainties in translations. To amend this problem, the so-called Molodenskii-Badekas transformation model with seven parameters could be invoked (Molodensky et al., 1962; Badekas, 1969). This model removes the high correlations between the parameters by relating them to the center of the reference network, using a similarity transformation of the type:

$$\mathbf{x}' = s\mathbf{R}\left(\mathbf{x}-\bar{\mathbf{x}}\right)+\mathbf{t}_M + \bar{\mathbf{x}}' \quad (7)$$

Where $\bar{\mathbf{x}}$ and $\bar{\mathbf{x}}'$ represent the centroids of the reference network in the original and transformed systems defined as:

$$\bar{\mathbf{x}} = \frac{\sum_{i=1}^{n}\mathbf{x}_i}{n} \text{ and } \bar{\mathbf{x}}' = \frac{\sum_{i=1}^{n}\mathbf{x}'_i}{n} \quad (8)$$

The physical interpretation of the Molodenskii-Badekas model is identical to the seven-parameter similarity model but with the coordinate origin shifted to the center of the reference network (Turgut, 2010). After this modification, the reference network becomes evenly distributed and high correlations between parameters are thus removed.

For the affine model, the same treatment can be applied and Equation 6 is then rewritten as:

$$\mathbf{x}' = \mathbf{S}\mathbf{\Lambda}\mathbf{S}^T\mathbf{R}\left(\mathbf{x}-\bar{\mathbf{x}}\right)+\mathbf{t}_M + \bar{\mathbf{x}}' \quad (9)$$

Since only a coordinate shift is performed in the Molodenskii-Badekas treatment, the scale and rotation parameters in Equations 7 and 9 should remain the same as those in the original similarity or affine model (that is, Equations 1 and 6), but with a modified translation vector $\mathbf{t}_M$ now representing the offset between the centroids of two reference networks.

NISLT algorithm

When a transformation model is used to relate two coordinate frames, the primary task is to obtain the transformation parameters. In the traditional approach, the least-squares adjustment technique is usually employed for finding the parameter estimates when the positional coordinates of reference points from both systems are known. Nevertheless, the success of a least-squares technique requires a good initial value of the parameters and an iterative process. This standard modus operandi could become inefficient and/or inaccurate when good initial values are not available or when a computation of a large network is involved. To improve the computational efficiency, the NISLT algorithm which was developed to compute the parameter estimates for general linear transformation models is explained in the following. In a linear transformation, its uniform scale can be estimated by:

$$\hat{s} = average\left(\frac{\left\|d\mathbf{x}'_{ij}\right\|}{\left\|d\mathbf{x}_{ij}\right\|}\right) \quad (10)$$

Where $d\mathbf{x}_{ij}$ is the coordinate difference between the original points i and j, $d\mathbf{x}'_{ij}$ is the coordinate difference of the transformed points i and j and $\hat{s}$ is the estimated scale. If the dilatations are non-uniform, one needs to compute an intermediate matrix $\mathbf{A}$ by:

$$\mathbf{A} = \Delta\mathbf{X}'^T\,\Delta\mathbf{X}\left(\Delta\mathbf{X}^T\Delta\mathbf{X}\right)^{-1} \quad (11)$$

Where $\Delta\mathbf{X}$ is a matrix formed by stacking the transposed coordinate differences of the original points:

$$\Delta\mathbf{X} = \begin{bmatrix} d\mathbf{x}_{12}^T \\ d\mathbf{x}_{13}^T \\ \vdots \\ d\mathbf{x}_{ij}^T \end{bmatrix}_{i\neq j} \quad (12)$$

and $\Delta\mathbf{X}'$ is a matrix formed by stacking the transposed coordinate differences of the transformed points:

$$\Delta\mathbf{X}' = \begin{bmatrix} d\mathbf{x}'^T_{12} \\ d\mathbf{x}'^T_{13} \\ \vdots \\ d\mathbf{x}'^T_{ij} \end{bmatrix}_{i\neq j} \quad (13)$$

In a 3D case, the dimensions of $\Delta\mathbf{X}$ and $\Delta\mathbf{X}'$ are both ($k\times 3$) where k is the total number of all possible coordinate differences between any two points in the network (that is, $k = C_2^n = n(n-1)/2$; n denotes the number of reference points). Furthermore, the matrix $\mathbf{A}$ in Equation 11 actually represents a minimum-norm least-squares solution for the matrix product $\mathbf{ER}$. By applying the singular value decomposition (SVD) theorem to the matrix $\mathbf{A}$, one immediately obtains:

$$\mathbf{A} = \mathbf{S}_a\mathbf{\Lambda}_a\mathbf{V}_a^T \quad (14)$$

Where $\mathbf{S}_a$ and $\mathbf{V}_a$ are two orthogonal matrices, and $\mathbf{\Lambda}_a$ is a diagonal matrix.

Consequently, the estimated symmetric tensor $\hat{\mathbf{E}}$ can be computed by:

$$\hat{\mathbf{E}} = \mathbf{S}_a\mathbf{\Lambda}_a\mathbf{S}_a^T \quad (15)$$

For both the uniform dilatation and non-uniform dilatation cases, the estimated rotation matrix $\hat{\mathbf{R}}$ can be obtained by:

$$\hat{\mathbf{R}} = \mathbf{S}_a\mathbf{V}_a^T \quad (16)$$

Finally, the translation vectors are estimated according to the following two cases:

$$\hat{\mathbf{t}} = \begin{cases} average\left(\mathbf{x}'-\hat{s}\hat{\mathbf{R}}\mathbf{x}\right) & \text{(similarity model)} \\ average\left(\mathbf{x}'-\hat{\mathbf{E}}\hat{\mathbf{R}}\mathbf{x}\right) & \text{(affine model)} \end{cases} \quad (17)$$

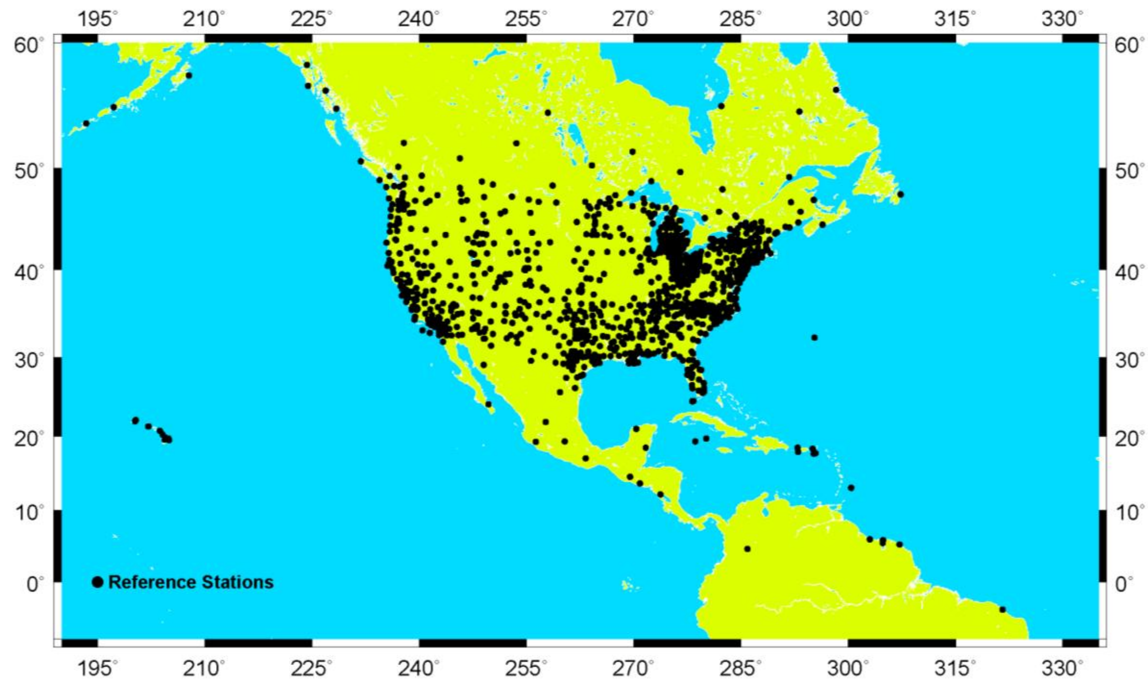

Figure 3. Site distribution of the CORS network investigated in case study 1.

To evaluate the quality of a transformation, the post-fit root-mean-square error of the transformed coordinates is computed by:

$$\mathrm{RMSE} = \sqrt{\frac{\boldsymbol{\varepsilon}_i^T \boldsymbol{\varepsilon}_i}{n}} \tag{18}$$

With $\boldsymbol{\varepsilon}_i$ representing the post-fit error vector for point i computed by:

$$\boldsymbol{\varepsilon}_i = \begin{cases} \mathbf{x}'_i - \hat{s}\hat{\mathbf{R}}\mathbf{x}_i - \hat{\mathbf{t}} & \text{(similarity model)} \\ \\ \mathbf{x}'_i - \hat{\mathbf{E}}\hat{\mathbf{R}}\mathbf{x}_i - \hat{\mathbf{t}} & \text{(affine model)} \end{cases} \tag{19}$$

RESULTS

Here, the coordinate transformations of two real cases are investigated. The first case concerns the transformation between coordinates referred to an ITRF2000 solution and to its corresponding International GNSS Service (IGS) solution of two continental networks which are expected to be almost aligned and similar (that is, no significant dilatation exists). The second case involves the transformation between two IGS solutions obtained at two different epochs of the same regional network around Taiwan. This network spans the intersection of two tectonic plates. Significant surface deformations are expected to produce strong non-uniform dilatations in this region. For both cases, the similarity and the affine transformation parameters are estimated by the NISLT algorithm using the post-fit root-mean-square errors of solutions to evaluate the fit of the model to the data.

Case study 1: A coordinate transformation between two quasi-aligned frames

The data used in this case is obtained from the two coordinate solutions at the same epoch of the continuously operating reference stations (CORS), which constitutes a GPS network principally covering North America (Figure 3) and that is operated by the National Geodetic Survey, NOS, NOAA in the United States (Snay and Soler, 2008). Continuous GPS measurements are processed to compute weekly solutions used in the IERS network combination (Ferland et al., 2000). Of the two solutions involved here, one was produced by tying itself to the ITRF2000 reference stations; the other solution refers to the IGS05 frame and was obtained using inner constraints (constrain-free). There were 1215 common stations extracted from these two weekly solutions at the same epoch (GPS week 1525). The estimated transformation parameters are listed in Table 1. Table 1 illustrates that the transformation parameters for both the similarity model and the affine model are small and almost identical, indicating a good alignment between the two solutions. Furthermore, the post-fit RMSE's for the two

Table 1. NISLT parameter estimates between two quasi-aligned and similar systems.

Variable	Similarity model		Affine model	
			$\lambda_1 - 1$	0.000
Scale factor (part-per-million)	$s-1$	-0.003	$\lambda_2 - 1$	-0.004
			$\lambda_3 - 1$	-0.006
	r_x	0.0003	r_x	-0.0001
Rotations (arc seconds)	r_y	0.0003	r_y	0.0004
	r_z	0.0002	r_z	0.0002
	t_x	0.026	t_x	0.017
Translations (m)	t_y	-0.008	t_y	-0.025
	t_z	-0.000	t_z	-0.022
Post-fit error (m)	RMSE	$\pm$0.006	RMSE	$\pm$0.006

models are of the same magnitude level (both are $\pm$6 mm), meaning that the two models work equally well for the coordinate transformation between these two particular solutions.

Case study 2: A coordinate transformation between non-uniformly dilated systems

The data used in this second case is from a GPS tracking network bordering the strait of Taiwan. This network contains 7 continuously operated stations which are evenly distributed in this region (Figure 4). One solution was obtained at epoch 2003.2 by fixing IGS reference stations while the other solution was obtained by using the same IGS constraints at epoch 2007.1. These two solutions are thus expressed in the IGS reference frame at each corresponding epoch. Furthermore, since the entire network covers only a limited area of the earth surface, its points have a localized geocentric distribution with respect to the global reference frame. In order to avoid high correlations between parameters, the Molodenskii-Badekas treatment has been applied before the transformation parameters for the similarity and affine models were estimated as listed in Table 2. In Table 2, large transformation parameter values can be found for both the similarity model and the affine model. It clearly represents a significant reference frame variation between the two coordinate solutions under investigation. Furthermore, the post-fit RMS error for the transformed coordinates is about $\pm$4.1 cm in the similarity model, but significantly reduces to $\pm$0.9 cm when the affine model is applied. This statistical fact indicates a strong non-uniform dilatation between these two sets of solutions. For a physical interpretation, we further projected the three principal scales obtained in the affine model into a local geodetic E-N frame (Figure 5).

The diagram indicates a 0.848 ppm shortening roughly in the E-W direction (azimuth = 98.1868°) and a 0.041 ppm shortening in the N-S direction (azimuth = 188.1868°) during the time interval of the investigation (~3.9 years). The corresponding averaged strain rates are 0.218 and 0.011 ppm/year, respectively.

DISCUSSION

The numerical results obtained from the two case studies illustrated the feasibility of the proposed approach. In Case Study 1, the network covers a large region in North America where significant continental-wide dilatations should not exist at the same epoch. Consequently, the proposed approach produced a transformation solution of the same level of quality as the classical similarity transformation did. In case study 2, when the proposed approach was applied to the transformation between non-uniformly dilated systems, it successfully captured the deformation signals. These deformation patterns and their magnitudes obtained from the affine transformation model are consistent with the surface velocity field caused by tectonic activities in this region as explicitly reported in previous studies (Yu et al., 1997; Hsu et al., 2003). Since the Taiwan Island appears to be located at the intersection of the Eurasia Plate and the Philippine Sea Plate, tectonic

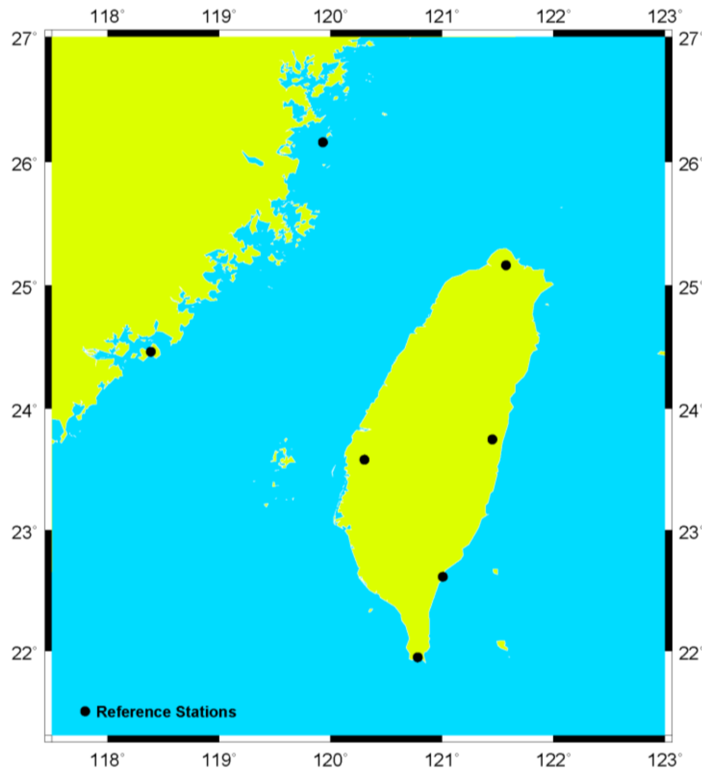

Figure 4. Site distribution of the regional network investigated in case study 2.

Table 2. NISLT parameter estimates between two non-uniformly dilated systems (with Molodenskii-Badekas treatment).

Variable	Similarity model		Affine model	
			$\lambda_1 - 1$	8.903
Scale factor (part-per-million)	$s-1$	-0.080	$\lambda_2 - 1$	-0.067
			$\lambda_3 - 1$	-34.075
	r_x	-0.0228	r_x	-0.9672
Rotations (arc seconds)	r_y	0.0300	r_y	1.1527
	r_z	0.1389	r_z	-3.1599
	t_x	-0.009	t_x	-0.009
Translations (m)	t_y	-0.059	t_y	-0.059
	t_z	-0.097	t_z	-0.097
Post-fit error (m)	RMSE	$\pm$0.041	RMSE	$\pm$0.009

activity produces significant surface deformations particularly along the west-northern and east-southern direction. A traditional similarity model is not sufficient to model the behavior of the geocentric frames defined by

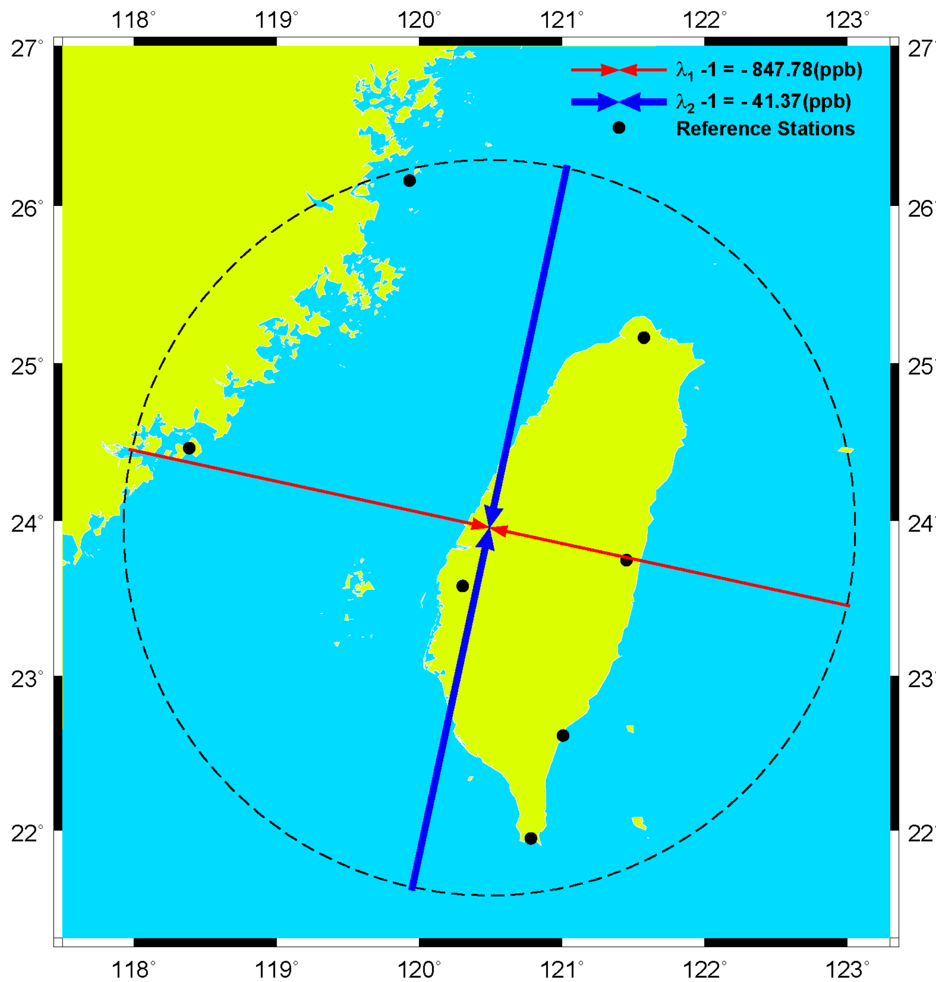

Figure 5. Projected principal scales obtained from the affine model.

the positions of terrestrial reference stations in this region. On the other hand, the affine model, which allows non-uniform dilatations in multiple directions provides a better fit to the actual crustal deforming behavior associated with station coordinates. Consequently, a significant improvement on the transformation quality and its interpretation can be achieved when this model is applied in tectonic active areas.

Conclusions

Recent advances in spatial information techniques enable high quality spatial measurements throughout every region of the earth. As a result, the deforming signal of a reference network can be precisely determined and interpreted. In the traditional approach, the similarity model is routinely applied to obtain coordinate transformations. However, this model fails to give satisfactory solutions in deforming areas where non-uniform dilatation is present. In this study, a twelve-parameter affine transformation model is proposed for transforming coordinates between non-uniformly dilated systems. Its parameters can be estimated by the NISLT algorithm which has been proven very efficient and reliable. In the case of uniform dilatations, the affine model works equally well as the traditional similarity model does. On the other hand, the affine model produces a better fit to the behaviors of a deforming coordinate system with the added advantage of attaining a more realistic physical interpretation. A significant quality improvement can be achieved when this twelve-parameter model is implemented in the transformation between non-uniformly dilated systems.

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