COMPUTING NAD 83 COORDINATES USING ITRF-DERIVED VECTOR COMPONENTS

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ABSTRACT

Recently, a new set of 14 transformation parameters was jointly adopted by National Resources Canada (NRCan) and the U.S. National Geodetic Survey (NGS) for transforming coordinates between the International Terrestrial Reference Frame of 2000 (ITRF00) and the North American Datum of 1983 (NAD 83). Although a unique set of similarity transformation equations could be applied everywhere to transform positional coordinates between these two terrestrial frames, a more consistent set of NAD 83 coordinates can be obtained if, locally, ITRF00 intersite vectors are transformed and constrained in the NAD 83 frame. This is the procedure implemented by the OPUS (On-line Positioning User Service) software, available through the NGS Tool Kit web page, to derive positional coordinates on the NAD 83 datum. In this particular case, the final NAD 83 coordinates of the unknown point are computed from three GPS-derived intersite vectors radiating to nearby CORS (Continuously Operating Reference Station) sites.

INTRODUCTION

A significant breakthrough in the fields of geodesy and surveying was accomplished with the introduction of the technology and methods of the Global Positioning System (GPS). Credit should go to GPS for substantially increasing our knowledge of geocentric terrestrial coordinate systems. It represents a giant leap from outdated classical datums (North American Datum of 1927: NAD 27; South American Datum of 1956: SAD 56, etc.) and the TRANSIT predecesors. The absolute accuracies currently available in geocentricity, orientation, and scale of terrestrial coordinate frames will be difficult to improve significantly and should last well into the next decade. This achievement has been possible thanks to refinements in post-processed precise ephemeris which today provide the orbital positions of the GPS satellites at the 5 cm level.

As a result of this and other improvements, we have entered into a new specialized realm of science that is purely geometrical in concept and strictly coordinate-centered. A large amount of effort has been expended to exploit the advent of precise positioning. Several countries have readily deployed permanent GPS satellite tracking networks to provide accurate referencing for differential GPS positioning techniques in order to maintain geodetic control at sub-centimeter accuracies. GPS campaigns have proliferated in recent years and new superior continental datums have evolved around the globe (e.g., European Terrestrial Reference Frame of 1989: ETRF89; Geocentric Datum of Australia 1994: GDA94; NAD 83(CORS96); South American Geocentric Reference System: SIRGAS, etc.). Subsequently, local networks have been densified and rigorous control for mapping and surveying extended. Also, as a result of improved national geoid height models –so critical for GPS leveling—GPS now affords a more economical alternative to differential leveling for many heighting applications.

GPS users can now determine the three-dimensional coordinates of a point with centimeter accuracy relative to a control station located several hundred kilometers away (Eckl et al., 2001). As mentioned above, there is a proliferation of continuously operating reference stations (CORS) that can be operated for a myriad of scientific applications. Such is the case of the National Geodetic Survey (NGS) network named National CORS, at present –January, 2002--, a set of 253 permanently monumented GPS stations whose coordinates define the National Spatial Reference System (NSRS). The National CORS network is becoming not only a natural tool for accurate 3D geodetic positioning in the United States, but its GPS data holdings are used by a plethora of investigators interested in ionospheric research, crustal motions, water vapor studies, etc. (Snay and Weston, 1999). For more information about CORS, consult the web address http://www.ngs.noaa.gov/CORS/. The NSRS is based on the North American Datum of 1983, NAD 83 for short.

Coordinates of a CORS station are determined by NGS by exploiting the most advanced GPS methodologies and software. These points, initially, are rigorously known in a well-defined coordinate frame, such as the International Terrestrial Reference Frame of 2000 (ITRF00). Introductory information about ITRF frames can be found in Boucher and Altamimi (1996) and Snay and Soler (2000b). The final step is to transform ITRF00 (epoch 1997.0) coordinates into NAD 83 coordinates. This requires a set of transformation equations that will be explained in the following section. A brief description of the evolution of NAD 83 is given in Snay and Soler (2000a) where the concept of frame "realization" is explained. For example, the latest realization of NAD 83 based on GPS observations is denoted NAD 83 (CORS96), although to simplify writing, this nomenclature will be truncated in this article to NAD 83.

TRANSFORMATION OF COORDINATES

In order to know the relationship between geocentric ITRF00 (1997.0) coordinates and NAD 83 coordinates, a 14-parameter similarity transformation between the two frames was derived. Not too long ago, the Helmert (similarity) transformation between two independent sets of coordinates, referred to two cartesian 3D frames, was defined as a function of only seven parameters. However, present improvements in GPS accuracies are able to detect changes as a function of time in the customary seven parameters. In another words, today geodesists are not satisfied in knowing only the typical three translations, three rotations, and the scale differential factor connecting any two 3D frames, but, for highly accurate work, the rate of change of these parameters with time is also required. Consequently, the general formulation to transform coordinates between two frames, say ITRF00 to NAD 83, is given according to the following equations :

$$
\begin{aligned}
x(t)_{NAD83} &= T_x(t) + [1+s(t)]\cdot x(t)_{ITRF} + \varepsilon_z(t)\cdot y(t)_{ITRF} - \varepsilon_y(t)\cdot z(t)_{ITRF}\\
y(t)_{NAD83} &= T_y(t) - \varepsilon_z(t)\cdot x(t)_{ITRF} + [1+s(t)]\cdot y(t)_{ITRF} + \varepsilon_x(t)\cdot z(t)_{ITRF}\\
z(t)_{NAD83} &= T_z(t) + \varepsilon_y(t)\cdot x(t)_{ITRF} - \varepsilon_x(t)\cdot y(t)_{ITRF} + [1+s(t)]\cdot z(t)_{ITRF}
\end{aligned}
\tag{1}
$$

where for an adopted time of reference, denoted t_0:

$$
\begin{aligned}
T_x(t) &= T_x(t_0) + \dot{T}_x\cdot(t-t_0)\\
T_y(t) &= T_y(t_0) + \dot{T}_y\cdot(t-t_0)\\
T_z(t) &= T_z(t_0) + \dot{T}_z\cdot(t-t_0)
\end{aligned}
$$

$$
\begin{aligned}
\varepsilon_x(t) &= [\varepsilon_x(t_0) + \dot{\varepsilon}_x\cdot(t-t_0)]\cdot m_r\\
\varepsilon_y(t) &= [\varepsilon_y(t_0) + \dot{\varepsilon}_y\cdot(t-t_0)]\cdot m_r\\
\varepsilon_z(t) &= [\varepsilon_z(t_0) + \dot{\varepsilon}_z\cdot(t-t_0)]\cdot m_r
\end{aligned}
$$

$$s(t) = s(t_0) + \dot{s}\cdot(t-t_0)$$

$m_r = 4.84813681\times10^{-9} \equiv$ conversion factor from milli-arcseconds (mas) to radians

In the above expressions:

$T_x(t_0), T_y(t_0), T_z(t_0) \equiv$ Shifts of the origin of the NAD 83 frame on the geocentric ITRF00 frame at epoch t_0.

$\varepsilon_x(t_0), \varepsilon_y(t_0), \varepsilon_z(t_0) \equiv$ Differential anticlockwise (counterclockwise) rotations (expressed in radians) at epoch t_0, respectively, around the $x_{ITRF00}, y_{ITRF00}, z_{TRF00}$ axes to establish parallelism with the $x_{NAD83}, y_{NAD83}, z_{NAD83}$ axes.

$s(t_0) \equiv$ Differential scale change (expressed in ppb×10^{-9}; ppb ≡ parts per billion)

$\dot{T}_x, \dot{T}_y, \dot{T}_z \equiv$ Variation of the shifts with respect to time, e.g., $\dot{T}_x = dT_x / dt$

$\dot{\varepsilon}_x, \dot{\varepsilon}_y, \dot{\varepsilon}_z \equiv$ Variation of the rotations with respect to time, e.g., $\dot{\varepsilon}_x = d\varepsilon_x / dt$

$\dot{s} \equiv$ Variation of the scale factor with respect to time, $\dot{s} = ds / dt$

Originally, coordinates at 12 points in Canada and the United States, known on the ITRF96 and NAD 83, were used to determine the transformation parameters (Craymer et al., 2000). Later updates by the International Earth Rotation Service (IERS) provided us with the required information to complete a transformation according to the following mapping:

$$ITRF00 \to NAD83 = (ITRF00 \to ITRF97) + (ITRF97 \to ITRF96) + (ITRF96 \to NAD83)$$

The final values of the 14 transformation parameters adopted by National Resources Canada (NRCan) and NGS to transform coordinates given on the frame ITRF00 to the geodetic datum frame NAD 83 are:

t_0 = 1997.0

Shifts: $T_x(t_0) =$ 0.9956 m ; $T_y(t_0) =$ –1.9013 m ; $T_z(t_0) =$ –0.5215 m

Rotations: $\varepsilon_x(t_0) =$ 25.915 mas ; $\varepsilon_y(t_0) =$ 9.426 mas ; $\varepsilon_z(t_0) =$ 11.599 mas (mas ≡ milli-arcseconds)

Scale factor: $s(t_0) =$ 0.62 × 10^{-9} (unitless)

Shift rates: $\dot{T}_x =$ 0.0007 m· year $^{-1}$; $\dot{T}_y =$ –0.0007 m· year $^{-1}$; $\dot{T}_z =$ 0.0005 m· year $^{-1}$

Rotation rates: $\dot{\varepsilon}_x =$ 0.067 mas· year $^{-1}$; $\dot{\varepsilon}_y =$ –0.757 mas· year $^{-1}$; $\dot{\varepsilon}_z =$ –0.051 mas· year $^{-1}$

Scale factor rate: $\dot{s} =$ –0.18 × 10^{-9} year $^{-1}$

The above set of equations are the ones implemented by NGS interactive program HTDP (Horizontal Time-Dependent Position). This software is without cost, available at the NGS web site (http://www.ngs.noaa.gov) by clicking on «Geodetic Tool Kit», and then on «HTDP». Further technical information about HTDP can be found in (Snay, 1999).

NGS recognized the implications and promise of GPS in its early stages of development and embarked on the promotion and adaptation of GPS methods to improve the NSRS. However this is an iterative process marked by the constant demand of improving positional accuracies based on the latest advancements on technology and updated definitions of coordinate systems. A history of various sets of transformation parameters between several realizations of ITRF and NAD 83 is available on NGS web page (http://www.ngs.noaa.gov/CORS/ metadata1).

There is another consideration that should be kept in mind; the transformation parameters necessary to obtain NAD 83 coordinates from ITRF00 are the result of comparing coordinates at 12 points. However, when geodetic networks such as the Federal Base Network (FBN) are adjusted, some observation errors are propagated and small distortions appear when coordinates obtained using ITRF00 and the strict application of equation (1) are contrasted with the results of the FBN adjustment. These small differences (up to 2 cm in the horizontal components and 4 cm on the vertical) are also influenced by the fact that FBN network results are an ensemble of discrete state networks that were developed and adjusted independently although constrained to CORS. For this reason a continent-wide adjustment including all GPS observations stored in the NGS data base, is planned to by published by 2005. In the mean time, when no adjustments are performed, the procedure of using observed intersite vector components transformed to the NAD 83 was adopted as described in the next section.

TRANSFORMATION OF VECTOR COMPONENTS

Recall that, initially, GPS processing determines the coordinates of the unknown points at the epoch of observation *t* referred to the frame orientation implicitly defined by the ephemeris used during the processing stage. NGS usually relies on the precise ephemeris disseminated by the International GPS Service (IGS) which, presently, assures an orbit of highest accuracy. Generally, the final post-processed products available to the GPS user are vector components, that is, the difference between the determined coordinates of the unknown point P and the coordinates of the fixed (base; fiducial) point A at the time of observation *t*. The magnitude (modulus) of the vector $\overrightarrow{AP}$ is the baseline length *AP*. Mathematically, in matrix notation, we can write the vector of the three components in either of the following forms:

$$\begin{Bmatrix} \Delta x_{AP}(t) \\ \Delta y_{AP}(t) \\ \Delta z_{AP}(t) \end{Bmatrix}_{ITRF00} = \begin{Bmatrix} x_P(t) - x_A(t) \\ y_P(t) - y_A(t) \\ z_P(t) - z_A(t) \end{Bmatrix}_{ITRF00} \qquad (2)$$

NGS typically stores processed vectors in its data base in some ITRF frame. Similarly, people familiar with NGS' ADJUST software know that this is the format required by the input G-file which contains all vector components of each session referred to a topocentric frame parallel to ITRFxx, located at the origin of the vector. The file also includes the standard error of each vector component and their correlation matrix. General matrix equations to rigorously transform vector components and their variance-covariance matrix at different epochs are given in Soler (2001).

Applying the theory described above, it is possible to transform the observed components at time *t* from ITRF00 to NAD 83. Notice that because the vector components defined by equation (2) involve the subtraction of two sets of coordinates, the shifts and shift rates in equation (1) will cancel. Substituting equation (1) into (2) twice, we arrive at the equation to transform ITRF00 components into NAD 83 components:

$$\begin{Bmatrix} \Delta x_{AP}(t) \\ \Delta y_{AP}(t) \\ \Delta z_{AP}(t) \end{Bmatrix}_{NAD83} = \begin{bmatrix} 1+s(t) & \varepsilon_z(t) & -\varepsilon_y(t) \\ -\varepsilon_z(t) & 1+s(t) & \varepsilon_x(t) \\ \varepsilon_y(t) & -\varepsilon_x(t) & 1+s(t) \end{bmatrix} \begin{Bmatrix} \Delta x_{AP}(t) \\ \Delta y_{AP}(t) \\ \Delta z_{AP}(t) \end{Bmatrix}_{ITRF00} \qquad (3)$$

where the values of the parameters required in the above equation were given previously. Equation (3), with the NAD 83 coordinates of the CORS stations, is used internally by NGS software OPUS (On-line Processing User Service; http://www.ngs.noaa.gov/OPUS/index.html) to obtain the final NAD 83 coordinates of the unknown point P. Thus, OPUS computes the coordinates of the unknown point P using the components of the vectors radiating to the three closest CORS stations.

The logic applied in OPUS is as follows. Assume that we want to know the coordinates of an arbitrary point P. Using IGS ephemeris and the RINEX data at three nearby CORS sites (name them A, B, and C) whose coordinates on the ITRF00 are known at the epoch of observation *t*, OPUS determines the coordinates of P at time *t* on the frame ITRF00.

These values are also given to the OPUS user on the output. However, most surveyors working on the United States need coordinates on the NAD 83. At this point, OPUS could have used program HTDP to convert ITRF00 coordinates into NAD 83 coordinates, however this approach would have assumed that the adopted 14 transformation parameters between ITRF00 and NAD 83 fit equally well in all areas of the country. This may not be the case due to the effect of small localized distortions. Thus, it was decided that instead of applying a general 14 parameter transformation using equations of the type (1), the propagation from the NAD 83 coordinates at the CORS stations A, B, and C, using actually observed vector components would be more realistic for the calculation of the NAD 83 coordinates at new unknown points P. It is at this stage that the components of the vectors are introduced.

Initially, the three components on the ITRF00 frame at time *t* of the three vectors $\overrightarrow{AP}, \overrightarrow{BP}$, and $\overrightarrow{CP}$ are computed: $\left\{\Delta x_{AP}(t) \quad \Delta y_{AP}(t) \quad \Delta z_{AP}(t)\right\}^T_{ITRF00}$, $\left\{\Delta x_{BP}(t) \quad \Delta y_{BP}(t) \quad \Delta z_{BP}(t)\right\}^T_{ITRF00}$, and $\left\{\Delta x_{CP}(t) \quad \Delta y_{CP}(t) \quad \Delta z_{CP}(t)\right\}^T_{ITRF00}$ where the superscript *T* stands for matrix transpose. The epoch of observation *t* is always given on the OPUS output (e.g., t = 2002.7696). Substituting these values in equations (3) after introducing the 14 transformation parameters described before, one obtains the components of the vectors $\overrightarrow{AP}, \overrightarrow{BP}$, and $\overrightarrow{CP}$ referred to the NAD 83 frame. Finally the adopted NAD 83 coordinates of the CORS stations (points A, B, and C) are invoked to determine the coordinates of the unknown point P also referred to the NAD 83:

$$\begin{Bmatrix} x_P(t) \\ y_P(t) \\ z_P(t) \end{Bmatrix}_{NAD83} = \begin{Bmatrix} x_A(t) \\ y_A(t) \\ z_A(t) \end{Bmatrix}_{NAD83} + \begin{Bmatrix} \Delta x_{AP}(t) \\ \Delta y_{AP}(t) \\ \Delta z_{AP}(t) \end{Bmatrix}_{NAD83} \tag{4}$$

Clearly, two similar equations could be obtained using the coordinates and vector components at the other two CORS stations B and C. Consequently, one ends up with three independent determinations of the unknown point P now referred to the NAD 83 frame. The final value given by the OPUS output is the average of the three independent determinations. As a measure of precision, OPUS prints the peak–to-peak error (maximum difference among the three determined values) for each component in linear units. Other ancillary information e.g. orthometric height using GEOID99 and NAVD 88, UTM northing and easting, baseline lengths, etc., are given.

CONCLUSIONS

The theory described above is the one applied by the OPUS software to compute NAD 83 coordinates. The program determines the coordinates of an arbitrary point P requiring as input the observation RINEX file (2 hours duration is the minimum allowed), the antenna part number (this is important because the software implements antenna phase pattern corrections), and the height of the ARP (antenna reference point) above the point of interest. The hands-off software uses the closest three CORS sites with good data to compute the coordinates of P. Depending on data quality, some of the CORS stations may be replaced by other nearby sites. Recently it was proved (Eckl et al., 2001) that baseline length is not critical to obtain accurate GPS-derived coordinates, consequently, the distance of the unknown P to the CORS stations does not have an important effect on the results and priority is given to the quality of the data to obtain accurate positioning.

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